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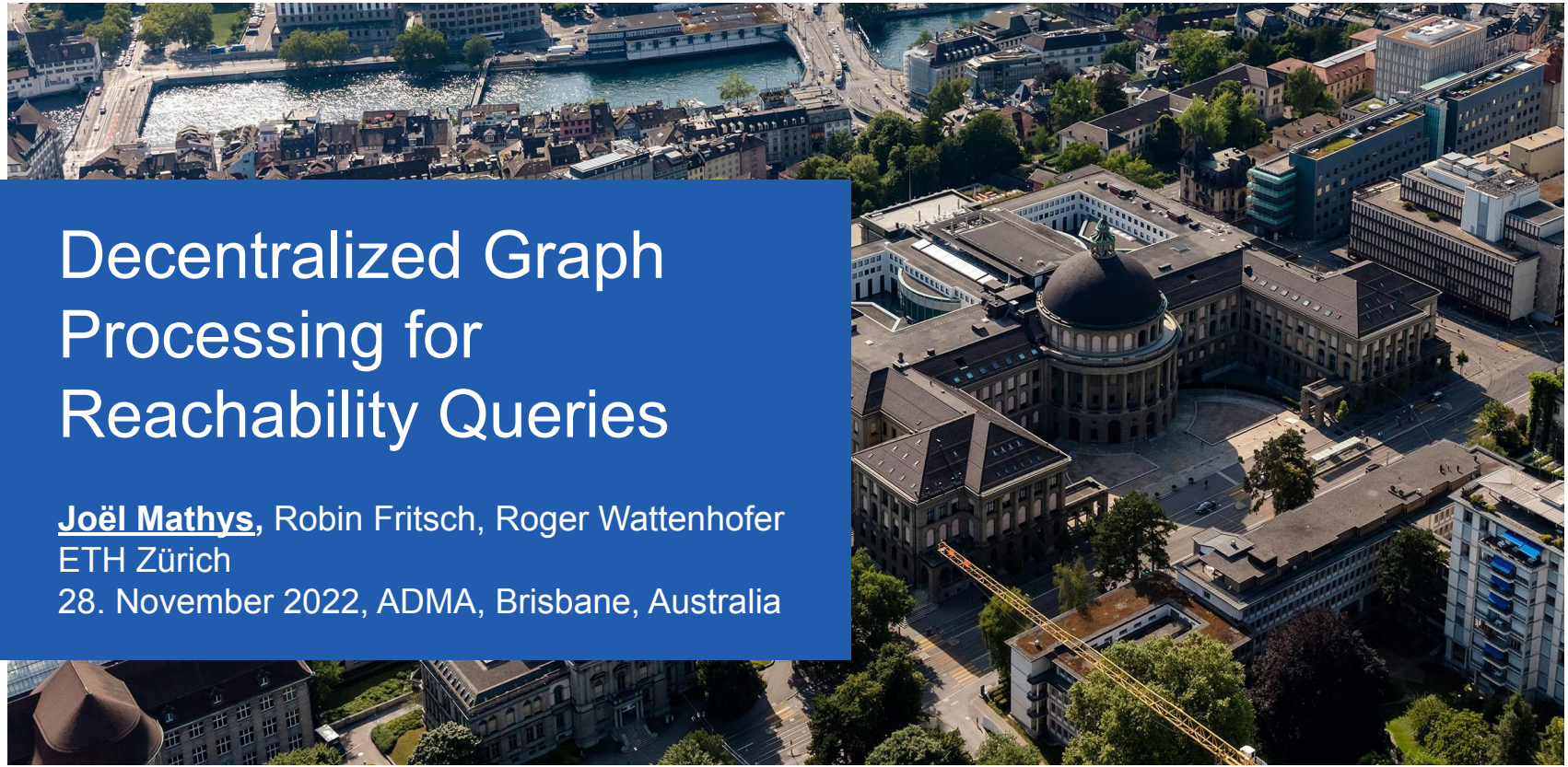
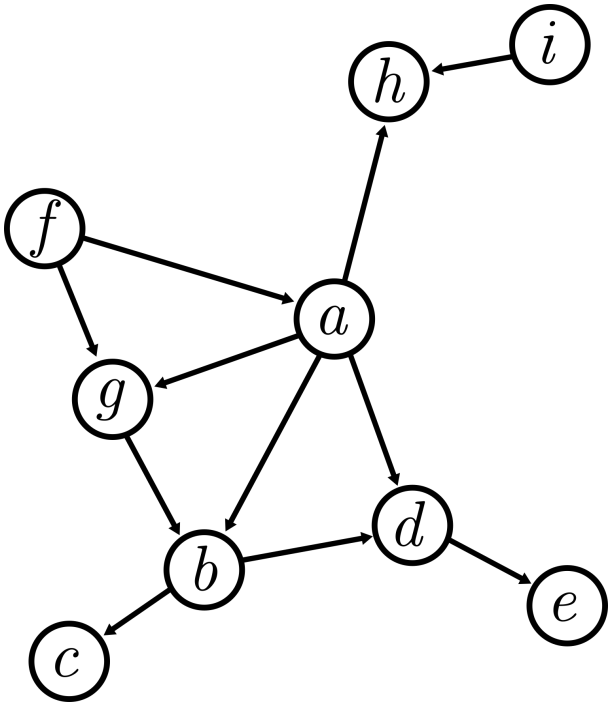


Decentralized Graph Processing for Reachability Queries

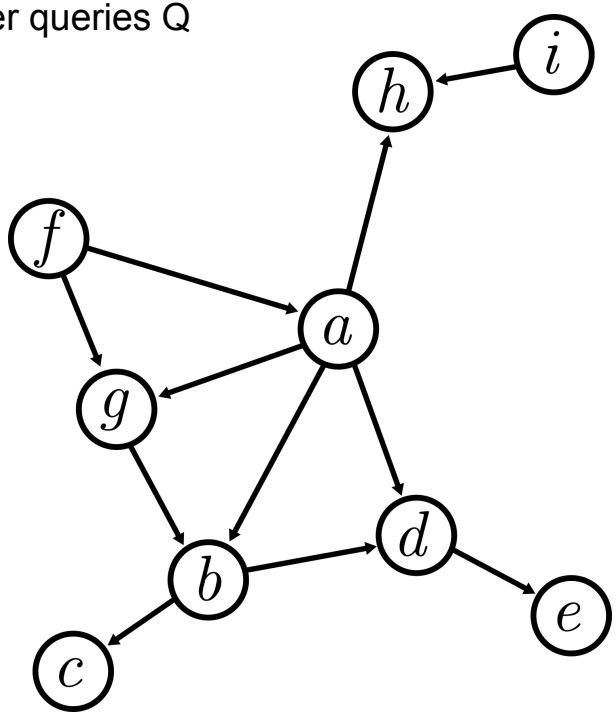
Joël Mathys, Robin Fritsch, Roger Wattenhofer
ETH Zürich
28. November 2022, ADMA, Brisbane, Australia

Introduction



Labeling Scheme

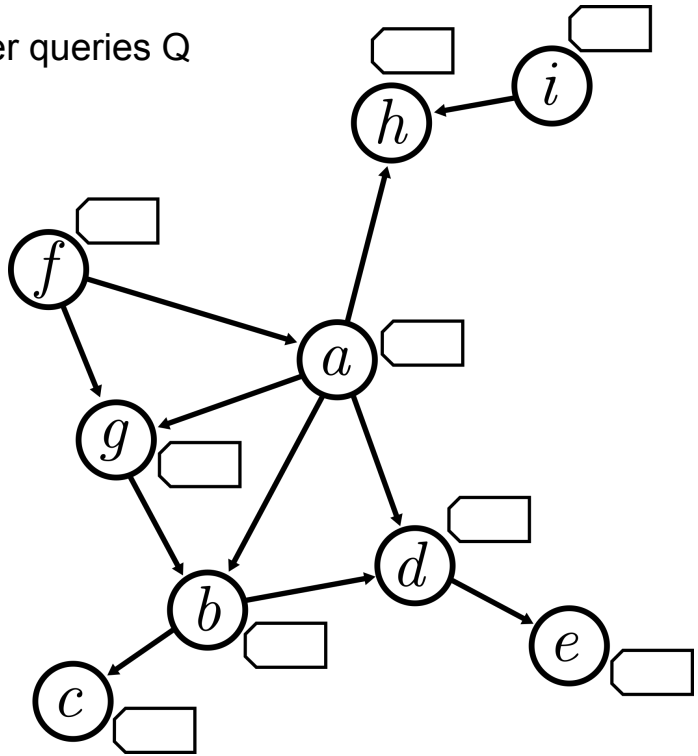
Given a Graph G, want to answer queries Q



Labeling Scheme

Given a Graph G, want to answer queries Q
Encoder l, Decoder d

Q(u, v) = d(l(u), l(v))



Labeling Scheme

Given a Graph G, want to answer queries Q
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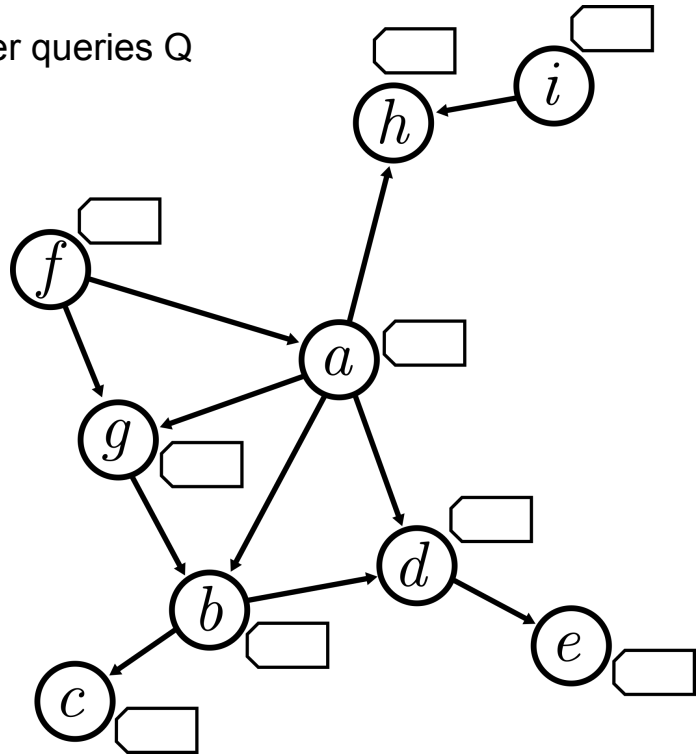
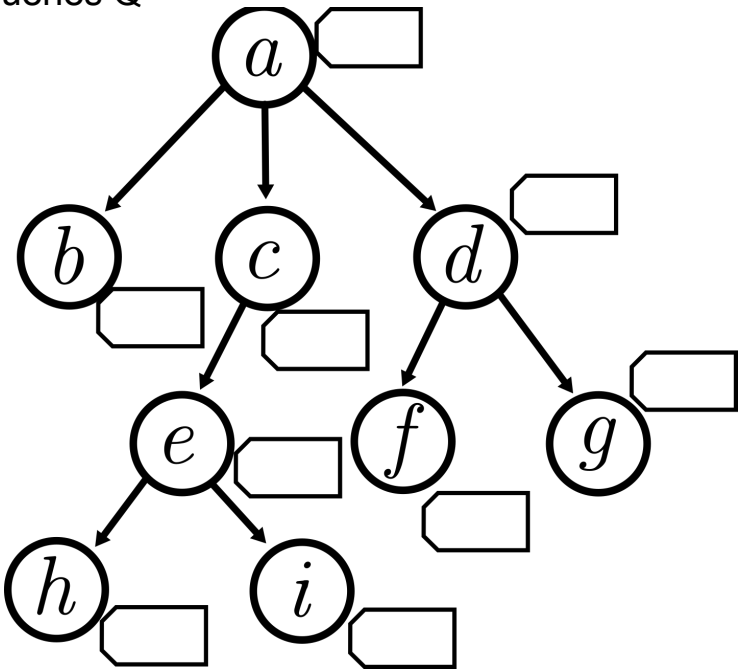
Q(u, v) = d(l(u), l(v))

Minimize max. label size [bits]



Warmup Tree

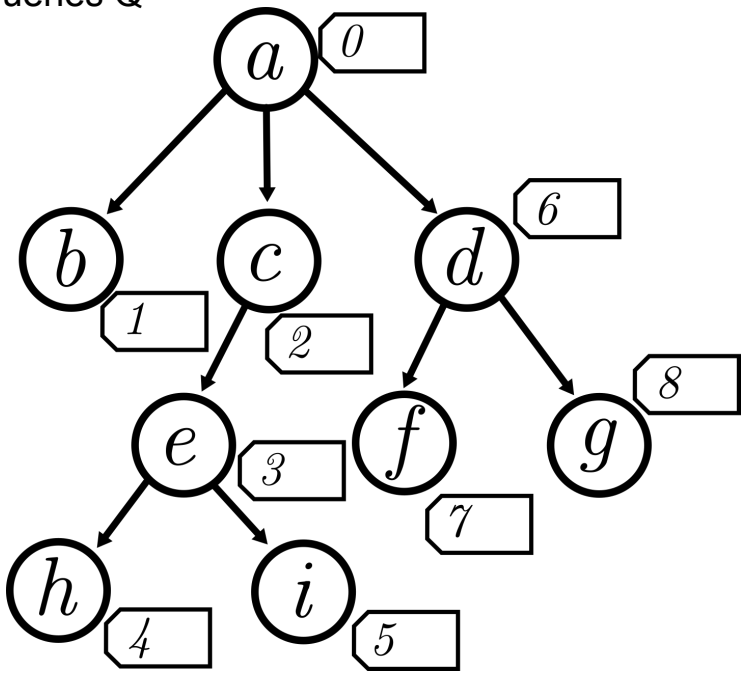
Tree T, answer reachability queries Q



Warmup Tree

Tree T, answer reachability queries Q

DFS enumeration

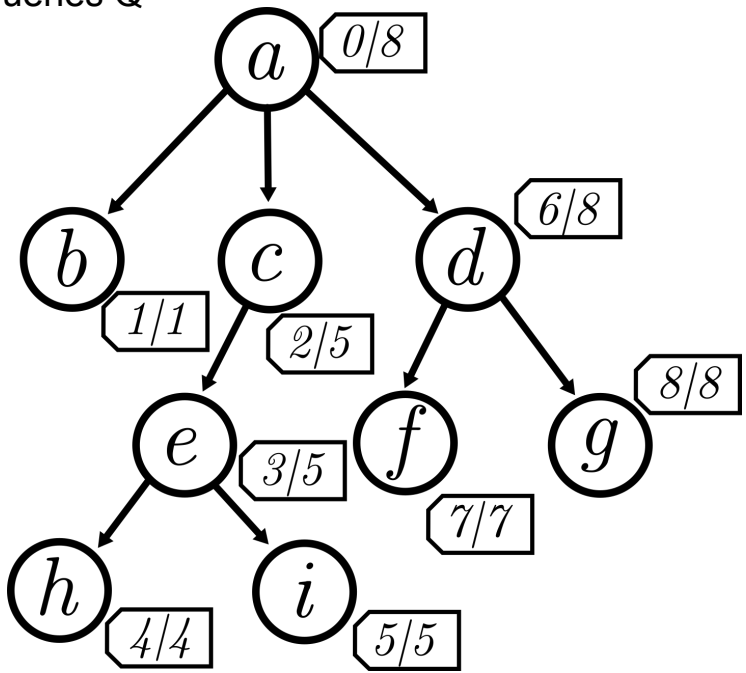


Warmup Tree

Tree T, answer reachability queries Q

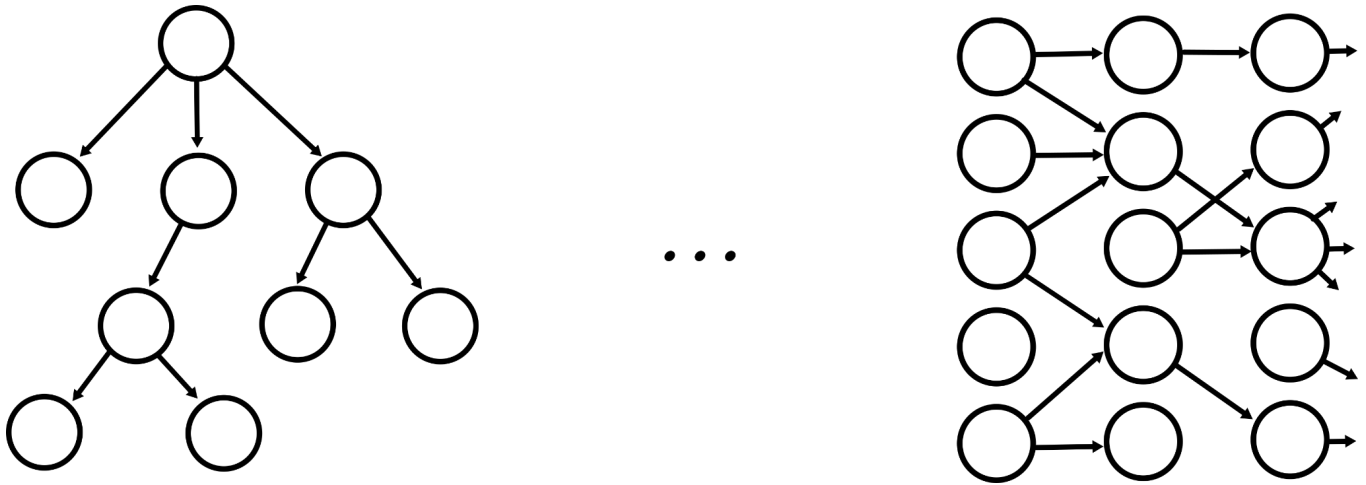
DFS enumeration

Reachable range, log(n)



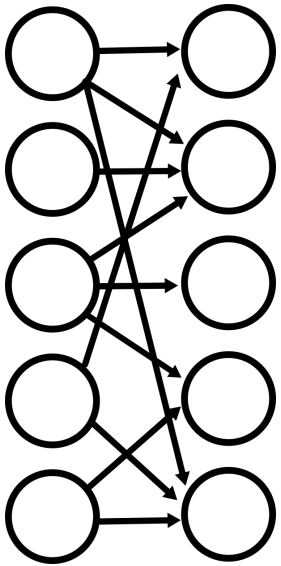
Our Goal

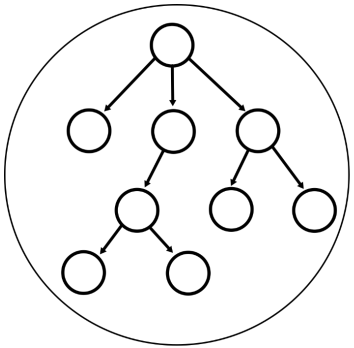
Given DAG G, answer reachability queries Q

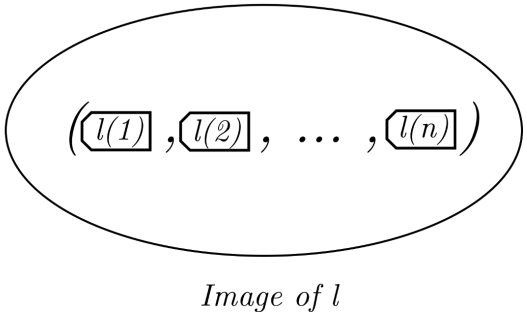


General Graphs

Given DAG G, answer reachability queries Q → L ≥ Ω(n)



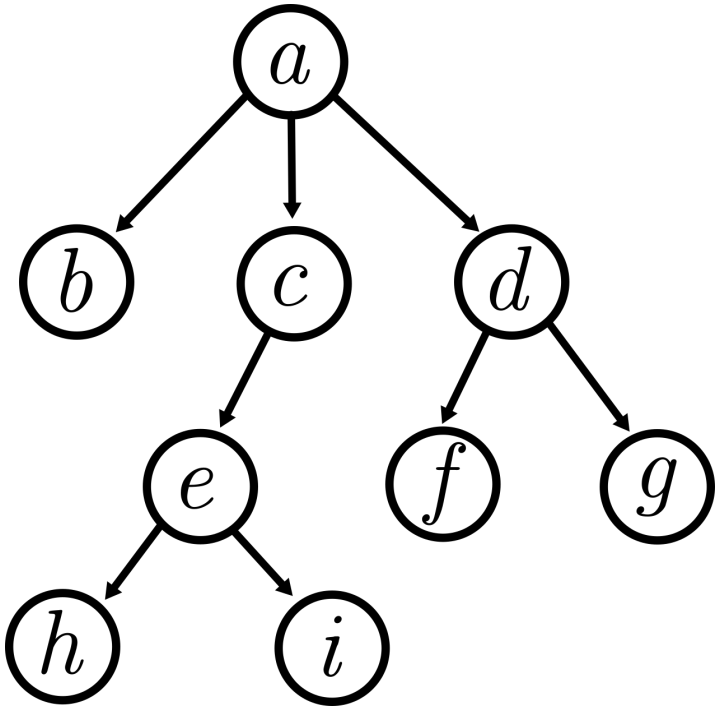




Graphs of Bounded Degree

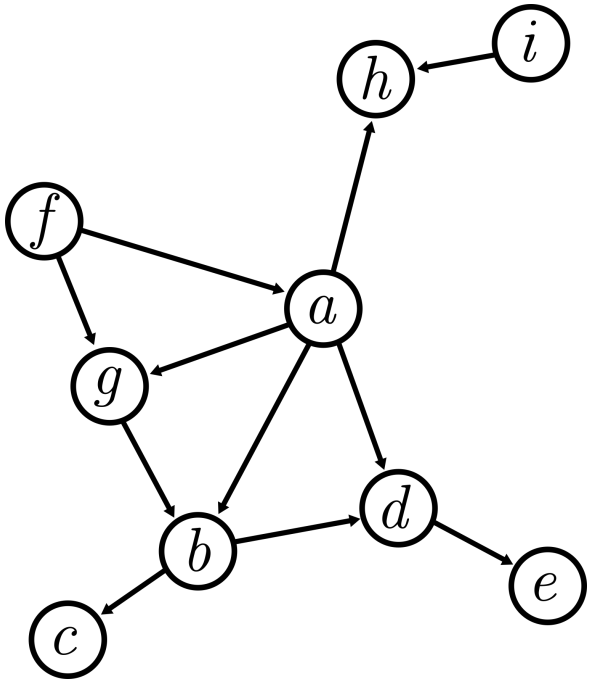
Bounded Degree Graphs

Δ at most 1



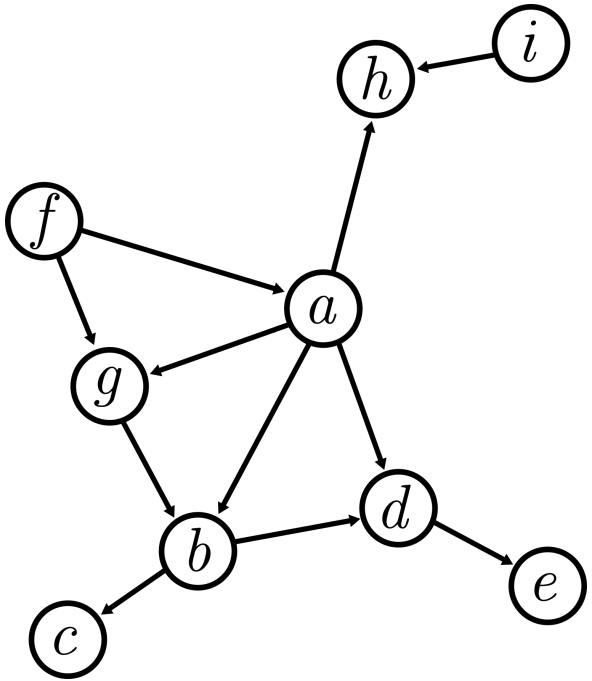
Bounded Degree Graphs

Δ at most 2



Bounded Degree Graphs

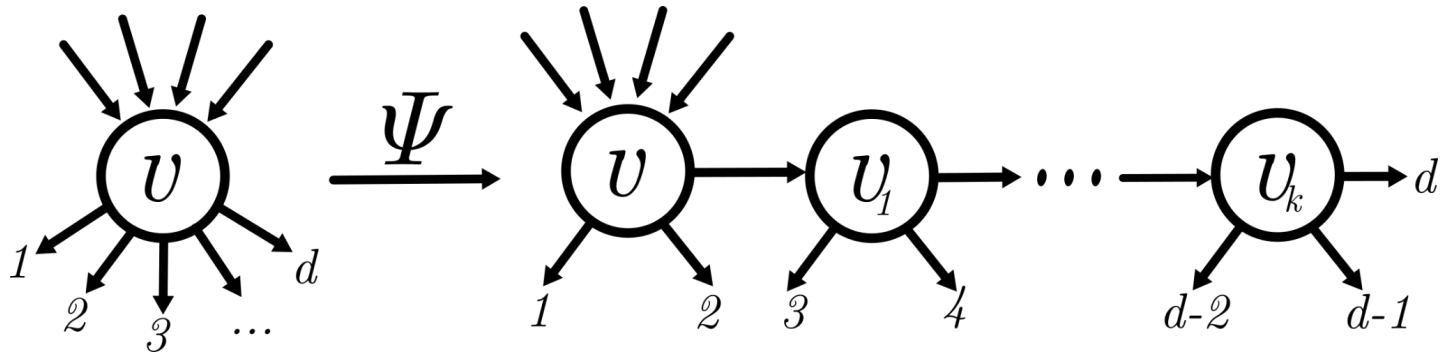
Δ at most 2
ψ Graph Transformation



Bounded Degree Graphs

Δ at most 2

ψ Graph Transformation, preserves reachability, adds at most n / Δ nodes



Bounded Degree Graphs

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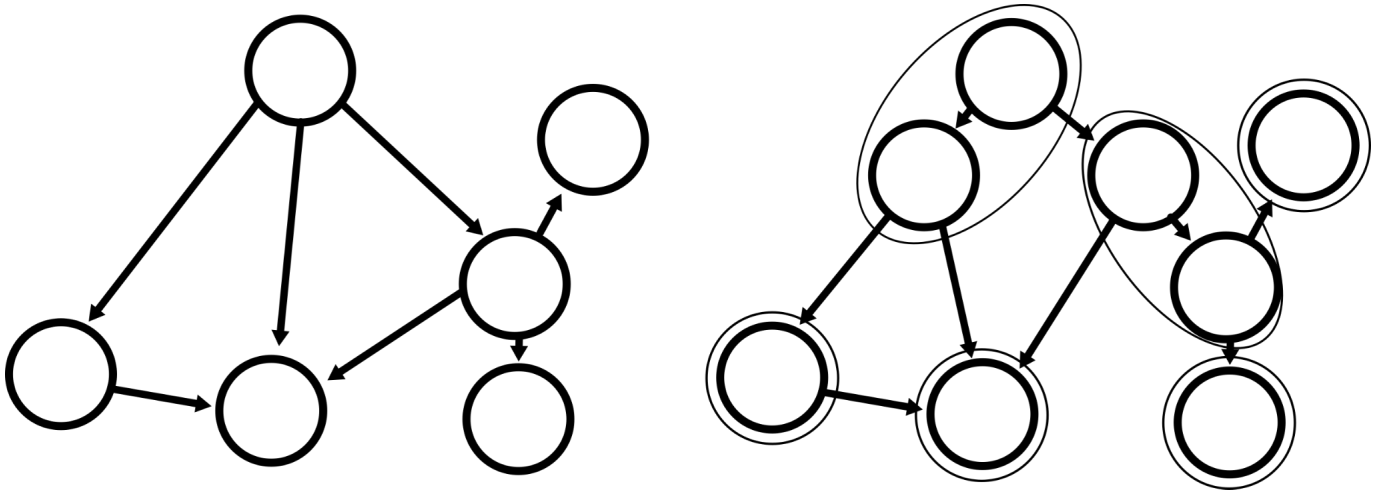
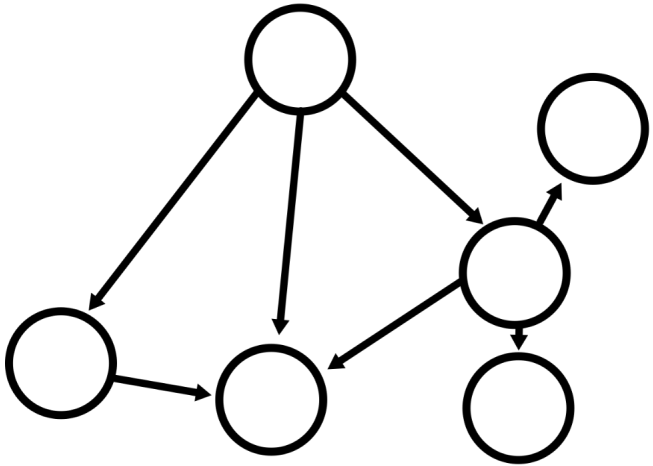


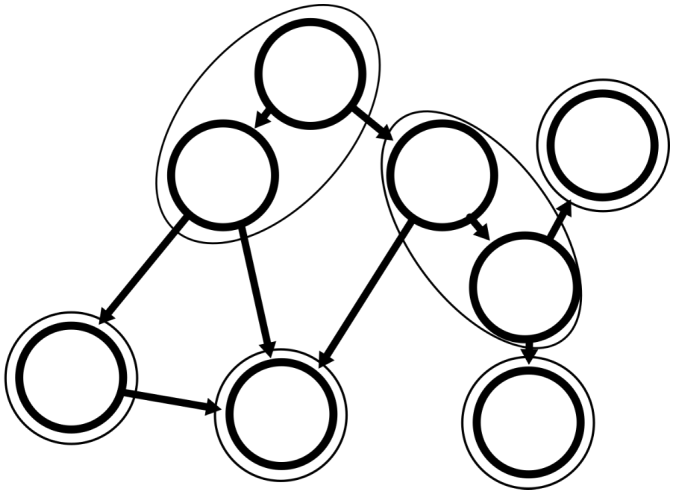
Bounded Degree Graphs

Δ at most 2

ψ Graph Transformation, preserves reachability, adds at most n / Δ nodes

$$o(\sqrt{\tfrac{n^2}{\Delta}\Delta}) = o(n) \Rightarrow \Omega(\sqrt{\Delta^+ n})$$



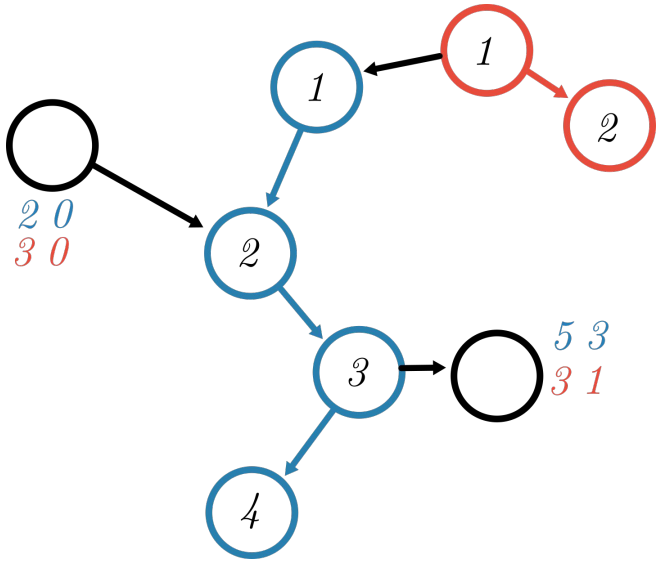


Graphs of Bounded Genus

Tool I: Path removal

A directed path p, store log(n) bits per path

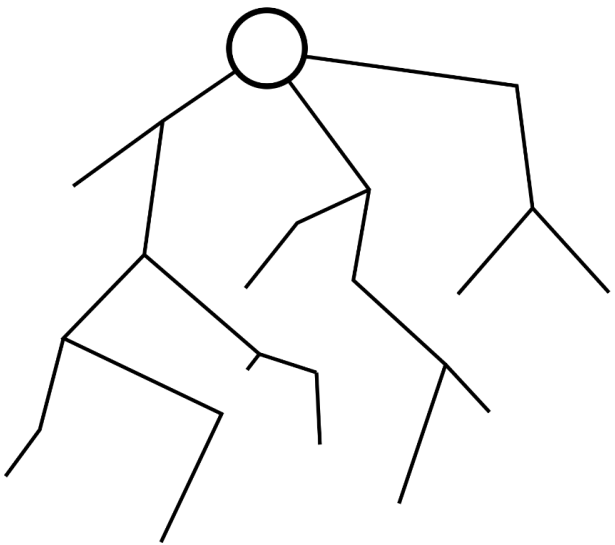
$$u \rightsquigarrow_G v \Leftrightarrow \mathrm{to}_p[u] \leq \mathrm{from}_p[v]$$

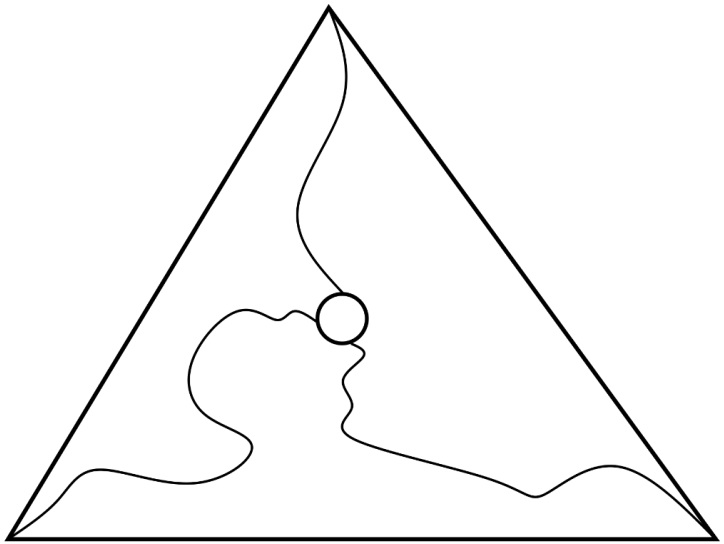


Tool II: Planar Graphs

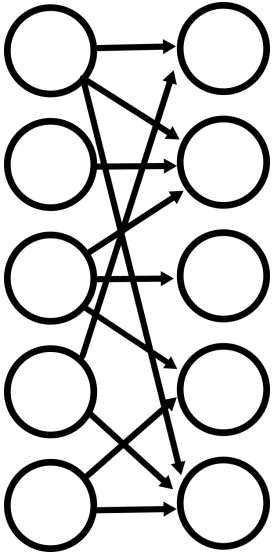
Construct a special tree T

Remove 6 paths to half the graph size, $\log^2(n)$ scheme

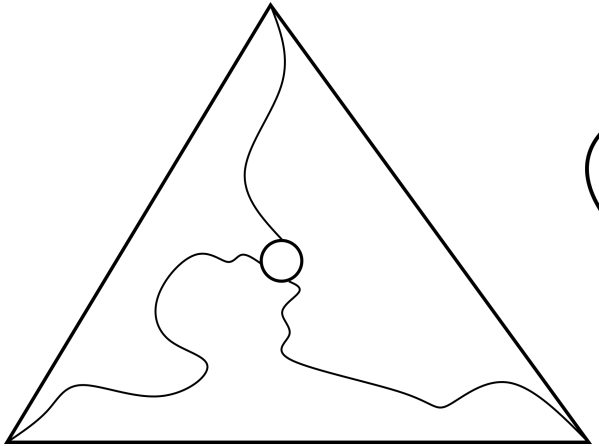




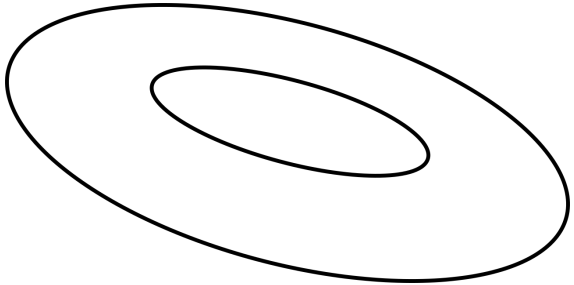
Graphs of Bounded Genus: Intuition



General

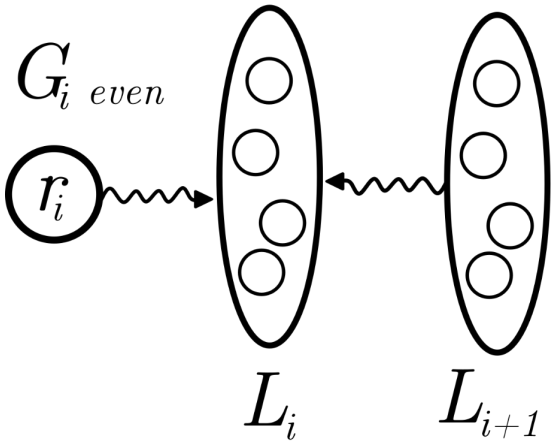


Planar

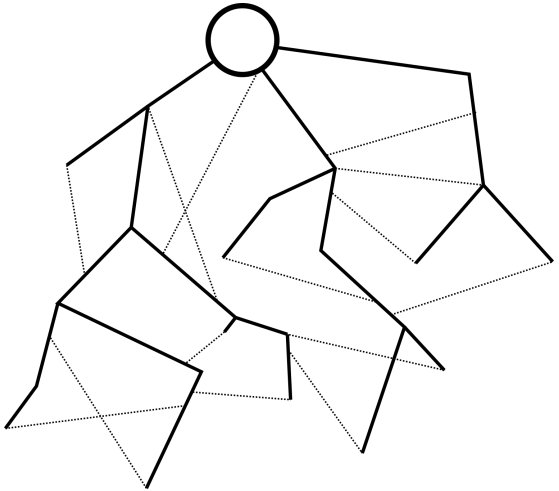


Bounded Genus

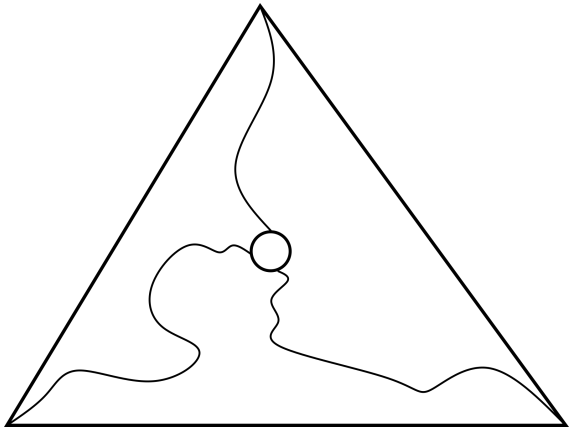
Graphs of Bounded Genus: Outline



Layering

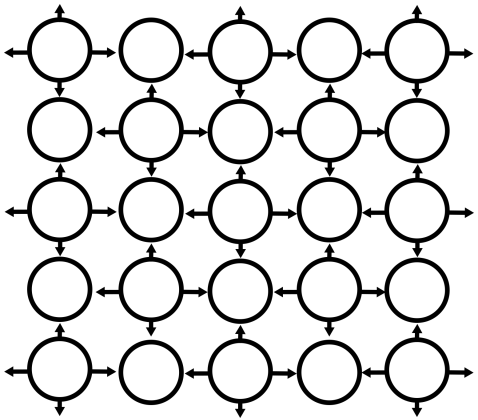


Planarizing

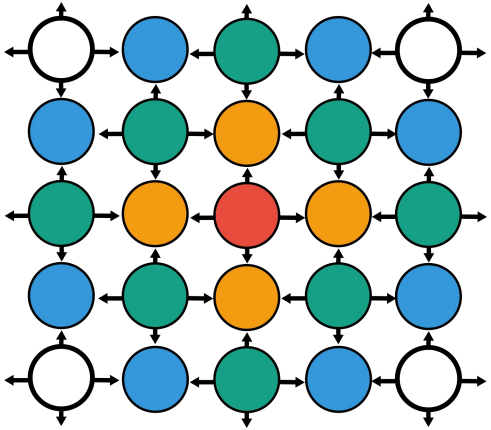


Planar Graphs

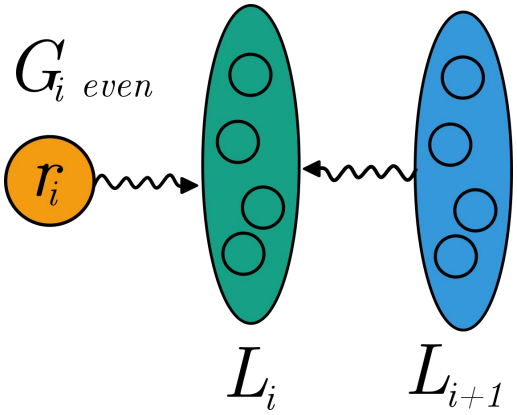
Graphs of Bounded Genus: Layering



Local Parts



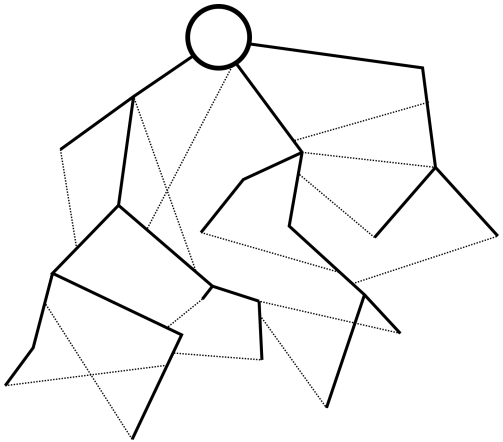
Layer Partition

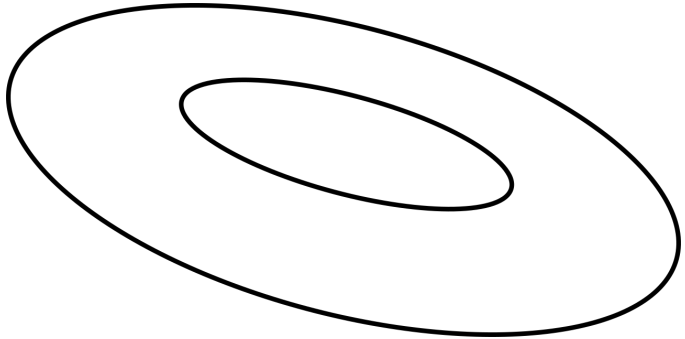


Digraph Sequence

Graphs of Bounded Genus: Planarizing

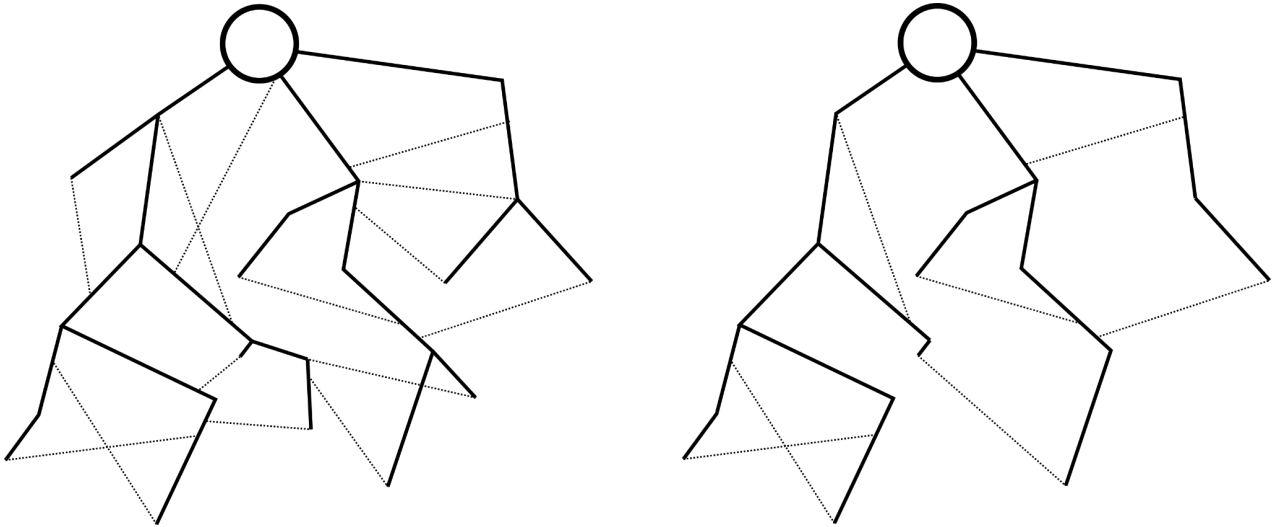
Layering preserves genus, now reduce genus to planar





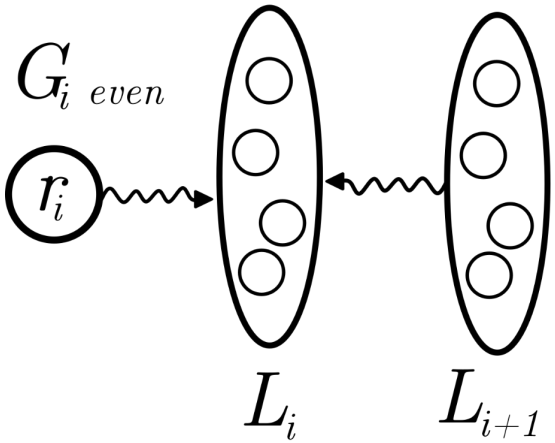
Graphs of Bounded Genus: Planarizing

Find rooted subgraph with at most 4g leaves, remove through paths

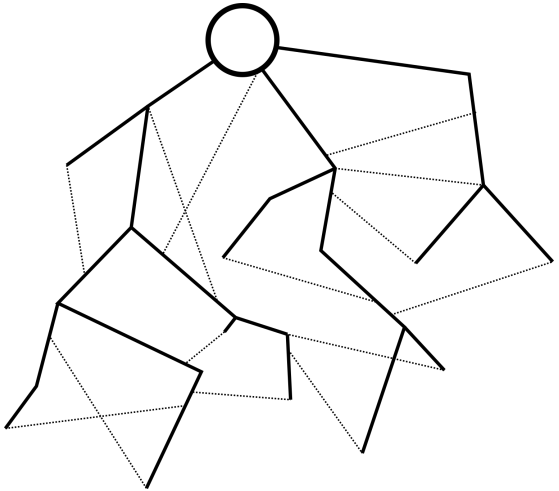


Graphs of Bounded Genus

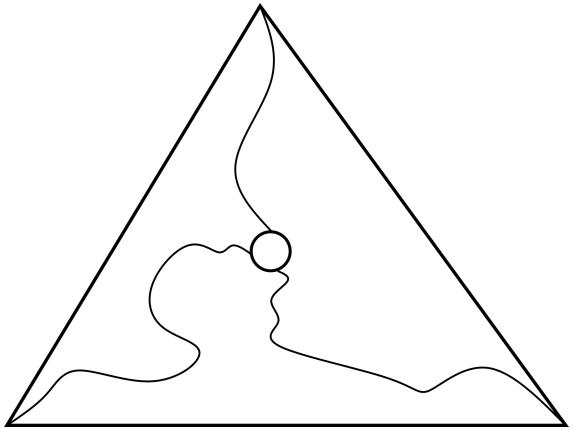
$$\Rightarrow \mathcal{O}(g \log(n) + \log^2(n))$$



Layering

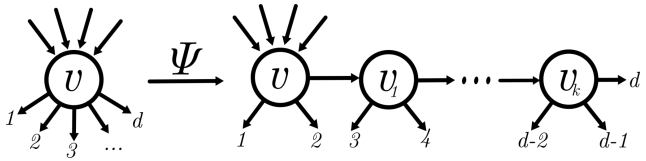


Remove g paths



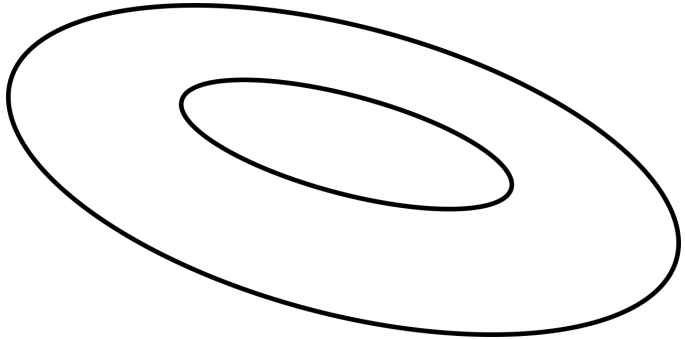
Planar Scheme

Conclusion



Bounded Degree

$\Omega(\sqrt{n\Delta})$



Bounded Genus

$\Rightarrow \mathcal{O}(g\log(n) + \log^2(n))$

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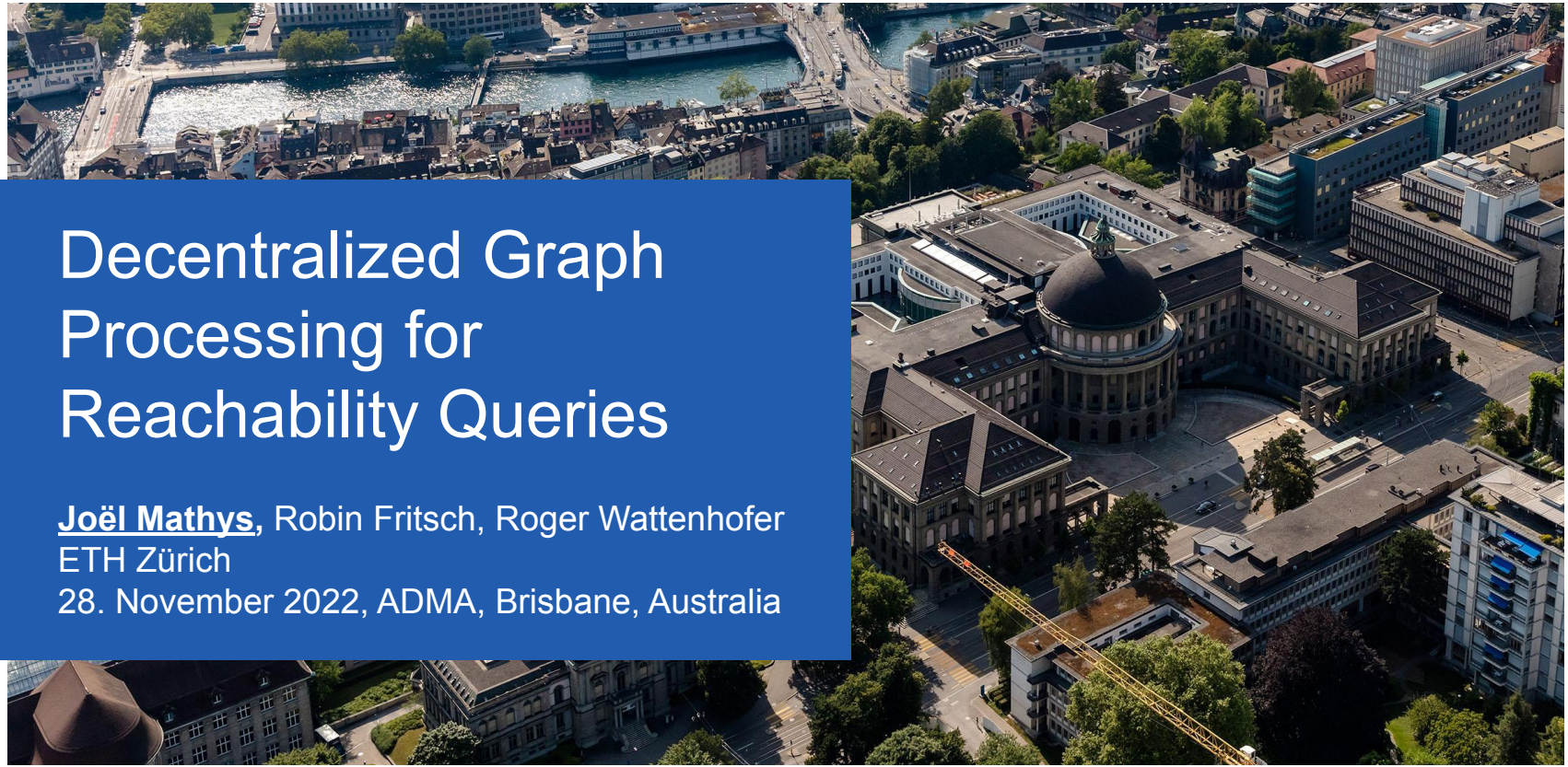

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