Topology Control Meets SINR:
The Scheduling Complexity of Arbitrary Topologies

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Topology Control Meets SINR

3) What is the relationship between *topology* and *scheduling*?

Signal-to-interference-plus-noise-ratio

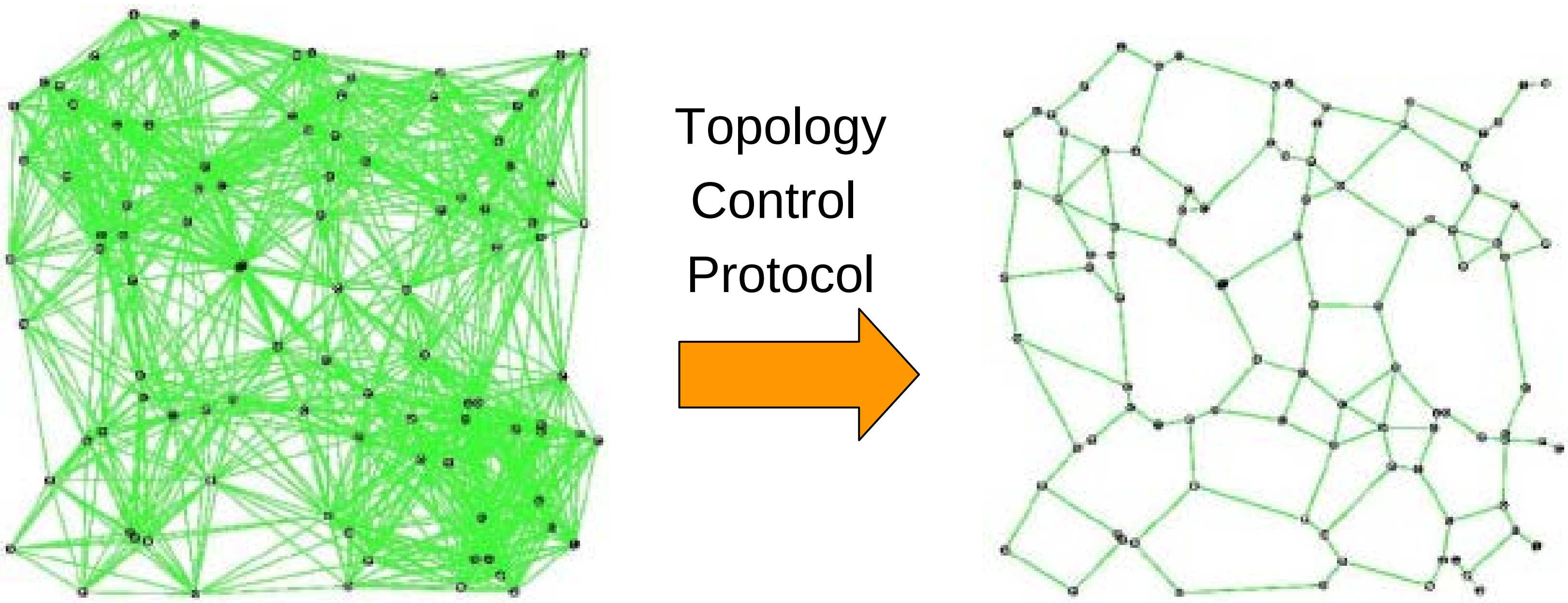
1) What is topology control ?

2) What is SINR ?
→ **Scheduling!**

Topology Control Meets SINR:
The Scheduling Complexity of Arbitrary Topologies

- Which topologies can be scheduled efficiently?
- How should requests/topologies be scheduled?
- Are currently used MAC-layer protocols good? (competitive compared to “optimal MAC protocol”)

What is topology control?



- Idea: Drop links to long-range neighbors
- Goal: Reduces energy and interference!

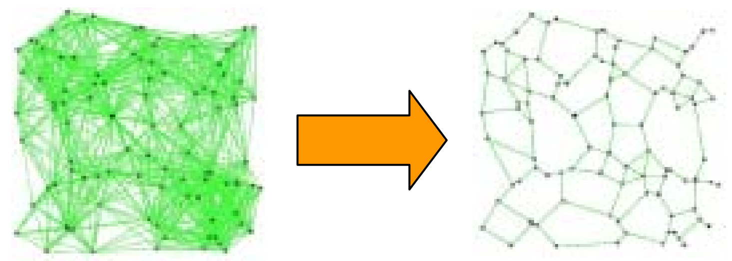
  But still stay connected (or even spanner)

What is topology control?

- Topology control papers argue that:

The selected topology should satisfy desirable properties beyond connectivity

- Spanner properties
- Low node degree
- Sparseness (few links)
- Low static interference
- Etc…



Some related work:

[Takagi & Kleinrock 1984]
[Hou & Li 1986]
[Hu 1993]
[Ramanathan & Rosales-Hain INFOCOM 2000]
[Rodoplu & Meng J.Sel.Ar.Com 1999]
[Wattenhofer et al. INFOCOM 2000]
[Li et al. PODC 2001]
[Jia et al. SPAA 2003]
[Li et al. INFOCOM 2002]
[Li et al. MOBICOM 2005]
[Santi, 2005]

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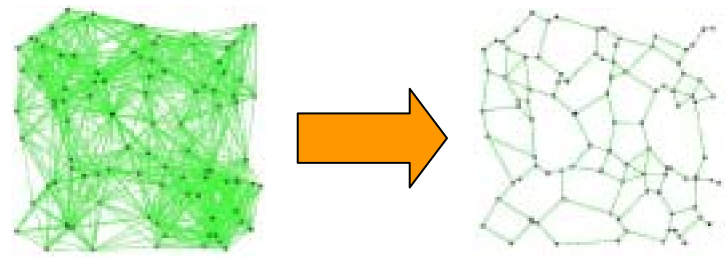

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- Low static interference
- Etc…

No node should be disturbed by many other nodes.

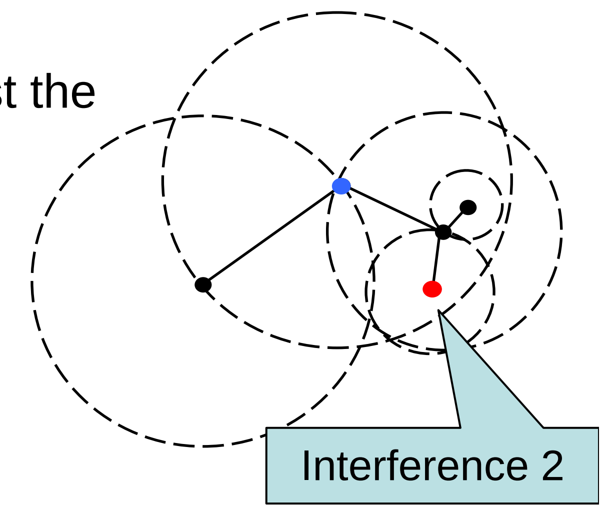
I_{in}: Measuring a topology‘s interference [von Rickenbach et al., WMAN‘05]

- Given a topology (or a set of communication requests) T
- I_{in} is the maximum number of nodes by which a receiver can potentially be disturbed.

Interference arises at the receiver!

Coverage of Node u

- Formally,
  - Node *u* may disturb all nodes closer than its farthest neighbor
    Draw a disk around each node with radius = longest outgoing link
  - Interference of node *u* =
    #nodes whose distance to *u* is at most the distance to their farthest neighbors
    #disks by which u is covered - 1
  - I_{in} Interference of topology or set of requests T =
    maximum interference over all nodes



Eventually, links must be scheduled…

- Topology control papers argue that:

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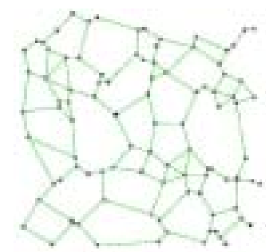
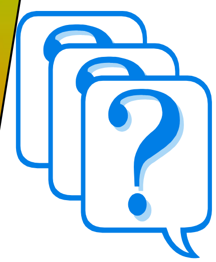
- Low static interference ← I_{in}
- Etc…

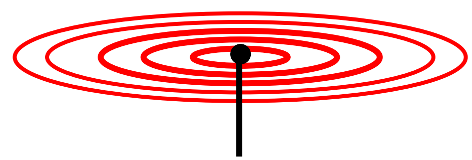
Topology Control is based on a *graph-based* model of wireless communication !

spanners, node degree…





Goal: Facilitate Scheduling!



radio signals, signal propagation

Physical SINR Model

- Scheduling is a low-level task → requires low-level model.
- Physical message reception determined by the signal-to-noise-plus-interference (SINR) ratio!
- Message arrives if SINR is larger than β at receiver

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N + \sum_{w \in V \setminus \{u\}} \frac{P_w}{d(w,v)^\alpha}} \geq \beta$$

Received signal power from sender

Power level of sender *u*

Path-loss exponent

Minimum signal-to-interference ratio

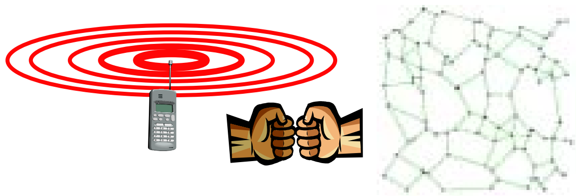
Noise

Received signal power from all other nodes (=interference)

Distance between two nodes

Graph-based Topology vs. Physical Scheduling?

- Fundamenal question:



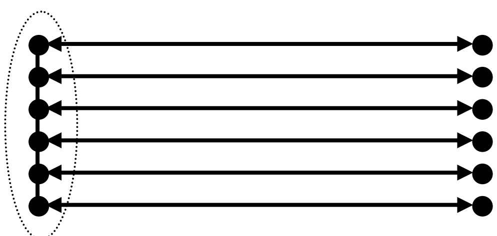
What is the relationship between topology control and physical scheduling?

structure of topology (set of comm. requests)

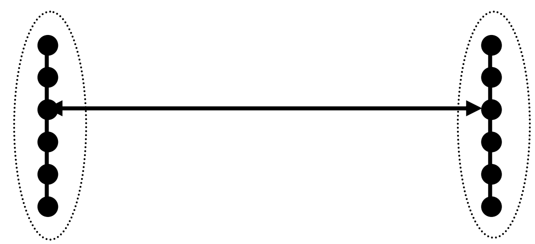
Relation

difficulty of scheduling

Simple examples of a connected topology:



- Scheduling requires ≥ n/2 time
- I_{in} of this topology is high

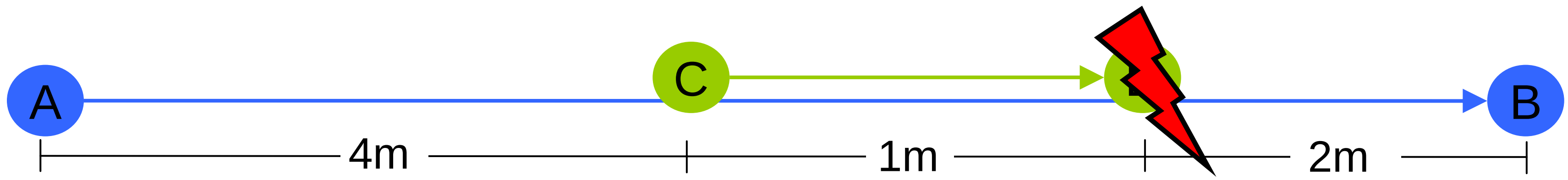


- Scheduling requires O(1) time
- I_{in} of this topology is low

→ Is this a law of nature… or just a lucky example…?

Good topology or bad topology…?

A wants to sent to B, C wants to send to D



Can A and C send simultaneously...?

No, they cannot!

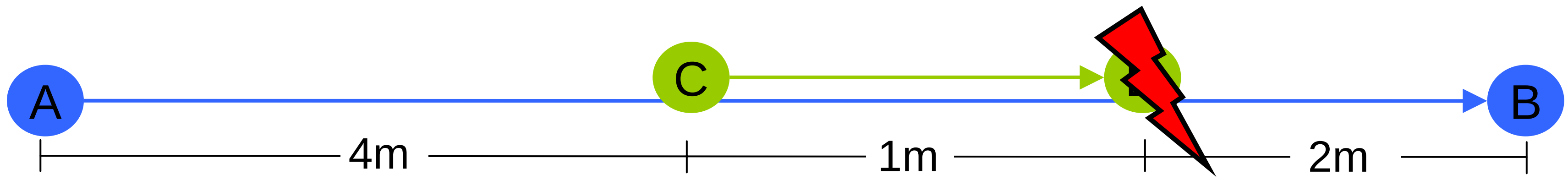
D is inside A's transmission range!

Interference causes a collision at D!

it seems…

Good topology or bad topology…?

A wants to sent to B, C wants to send to D



- Let α=3, β=3, and N=10nW
- Set the transmission powers as follows P_C= -15 dBm and P_A= 1 dBm
- SINR at D is: $$\frac{1.26mW/(7m)^3}{0.01\mu W + 31.6\mu W/(3m)^3} \approx 3.11 \geq \beta$$ 
- SINR at B is: $$\frac{31.6\mu W/(1m)^3}{0.01\mu W + 1.26mW/(5m)^3} \approx 3.13 \geq \beta$$ 

Simultaneous transmission *is* possible !

Scheduling – Some Related Work

- There is a lot of related work on scheduling
  → numerous practical scheduling protocols
  → wireless MAC layer protocols
- Capacity of wireless networks [Gupta, Kumar, Trans.Inf.Theory'00]
- Combined power assignment and scheduling problems
  [Behzad, Rubin, Infocom'05], [Jain, Padhye, Padmanabhan, Qiu, Mobicom'03], [Bjorklund, Varbrand, Yuan, Infocom'03], etc…
- Specifically SINR based scheduling protocols
  [Ephremides,Truong,Trans.Comm'90], [ElBatt, Ephremides, Infocom'02], [Cruz, Santhanam, Infocom'03], etc…
- Comparison between graph-based and SINR-based scheduling
  [Gronkvist, Hansson,Mobihoc'01], etc…

Capturing the difficulty of scheduling...?

Graph-based topology vs. SINR-based scheduling?

Scheduling in Wireless Networks

Relationship between a topology and scheduling is not trivial!

→ Often counter-intuitive!

1) There are topologies with high I_{in} that can be scheduled quickly!
2) There are topologies with low I_{in} that are difficult to schedule!

→ Big discrepancy between graph-based and SINR-based models

- → Interference created by simultaneous senders cumulates
- → Power may not be chosen uniformly

→ Power assignment policy is decisive!

We need a measure that captures how quickly a topology can be scheduled

Not clear whether topology control helps in scheduling!

***Scheduling Complexity* in Wireless Networks**

Outline

- Topology control
- Scheduling in SINR-environments
- Graph-based protocol design vs. physical interference!
- The *scheduling complexity* of wireless networks
  - Intuitive, but inefficient scheduling protocols
  - A note on the energy metric
  - Our efficient $O(I_{in} \cdot \log^2(n))$ protocol
- Topologies with low I_{in}
  - Symmetric versus asymetric links
- Conclusions

The Scheduling Complexity of Wireless Networks

- n nodes in 2D Euclidean plane (arbitrary, possibly worst-case position)
- An arbitrary topology T (analogous: a set of communication requests)
- Nodes can choose power levels
- Message successfully received if SINR at receiver sufficient

Scheduling Complexity S(T)

The minimum number of time slots required until all links in T have been successfully scheduled at least once!

Moscibroda, Wattenhofer, Infocom 2006

What is known…

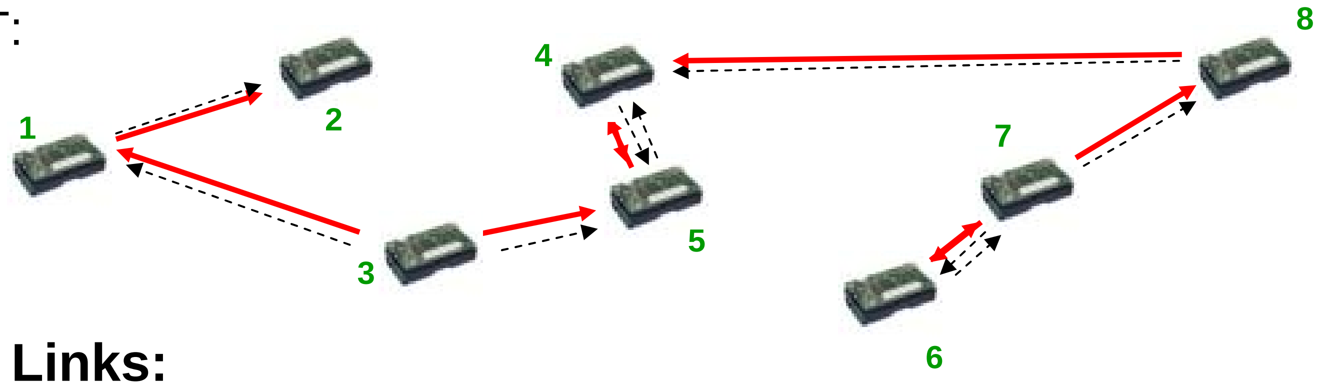
Clearly, $S(T) \leq n$ (if broadcast allowed)

Scheduling Complexity of Strong Connectivity: $S(T) \leq O(\log^4 n)$

General Topologies?

Scheduling Complexity – Example

Consider topology T:



Time-Slot	**Links:**
t_1:	1→2, 4→5, 6→7
t_2:	3→1, 5→4, 7→6
t_3:	7→8, 3→5
t_4:	8→4

→ **Scheduling complexity of T is at most 4 !**

Do good topologies have a small scheduling complexity ?

graph-based topology control

SINR-based scheduling

Our Results

In the paper we prove the following theorem:

Theorem:

Scheduling Complexity of any topology T with in-interference I_{in} is at most $S(T) \in O(I_{in} \cdot \log^2 n)$

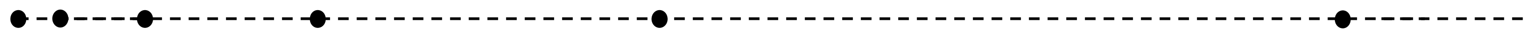
- This result hold in every (even worst-case) networks
- Theoretically, good static topologies can be scheduled eficiently → no fundamental scaling problem in scheduling
- This implies that topology control (reducing I_{in}) helps!

- But, achieving this result requires highly non-trivial power assignments and scheduling !

Bad Scheduling in SINR

[Moscibroda, Wattenhofer, Infocom 2006]

- Consider the exponential chain:



Bad Scheduling in SINR

- Consider the exponential chain:

[Moscibroda, Wattenhofer, Infocom 2006]

- This topology has interference $I_{in} = 1$

Not trivial…

- All links can be scheduled in O(1) time!

By a factor Θ(n) slower!

- But, it can be shown that:
  - Any protocol with uniform power assignment has time Ω(n)
  - Any protocol with power according to $P \sim O(d^{\alpha})$ has time Ω(n)

Transmitting according to **energy-metric** implies **slow scheduling**!

Energy-Metric !

Our Protocol

- How can we break the $\Omega(n)$ barrier...?
- Observation: Scheduling a set of links of roughly the same length is easy...
  - → Partition the set of links in length-classes
  - → Schedule each length-class independently one after the other...
- The problem is...
  - → there may be up to n different length-classes
  - → We must schedule links of different lengths simultaneously!

e.g. uniform and ~d^{α} examples before

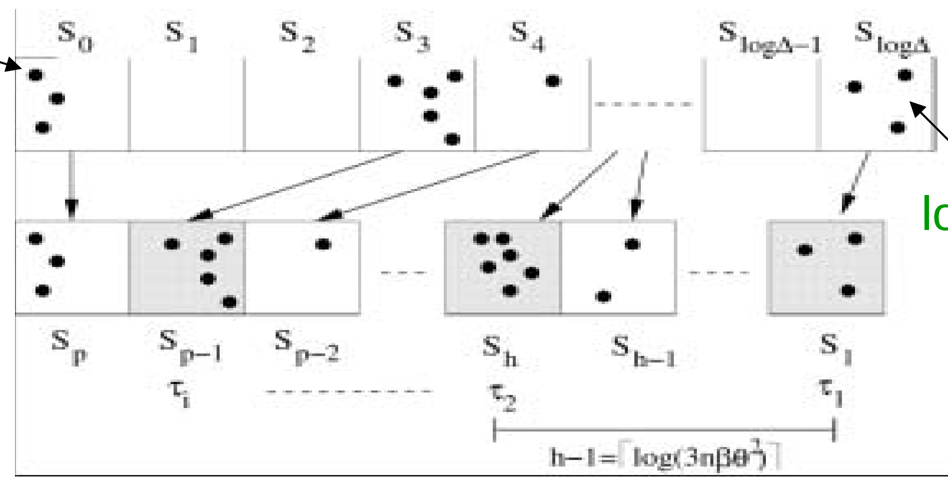
- How can we assign powers to nodes?
  - → Making the transmission power dependent on the length of link is bad!
- We must make the power assigned to simultaneous links dependent on their relative position of the length class!

Our Protocol – Power Assignment

shortest links

Each point is a link
A box is a length class

Within a length class:
Length of links within factor 2



longest links

- A node v in length-class τ and a link of length d transmit roughly with a power of

$$P(v) \approx (n\beta)^{\tau} \cdot d^{\alpha}$$

This would be $P \sim O(d^{\alpha})$ assignment

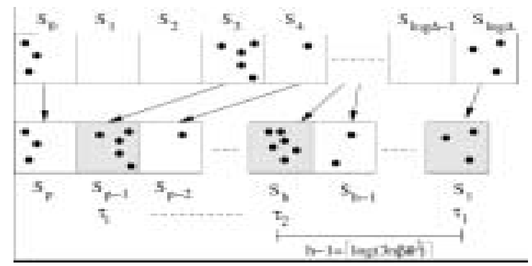
Intuitively, nodes with small links must *overpower* their receivers!

- But now, short links disturb distant long links!!!
- Therefore, we also need to carefully select the transmitting nodes!

Ooops, now it gets complicated...!

Our Protocol – Scheduling Links

- Short links are "overpowered"
  → create much more interference
  → this precludes simple geometric arguments!



- Partition the set of nodes into sets, according to their longest link
- In each iteration k=0…log(3βn)-1, consider nodes in sets

$$\underbrace{S_k,\ S_{\log(3\beta n)+k},\ S_{2\log(3\beta n)+k},\ \ldots,\ S_{x\log(3\beta n)+k}}_{\left\lceil \frac{n}{\log(3\beta n)} \right\rceil}$$

Schedule links of very different length simultaneously.

- In each iteration, schedule all links belonging to nodes in these sets.

Our protocol achieves this in $O(I_{in} \cdot \log n)$ time slots.

Our Protocol – Scheduling Links

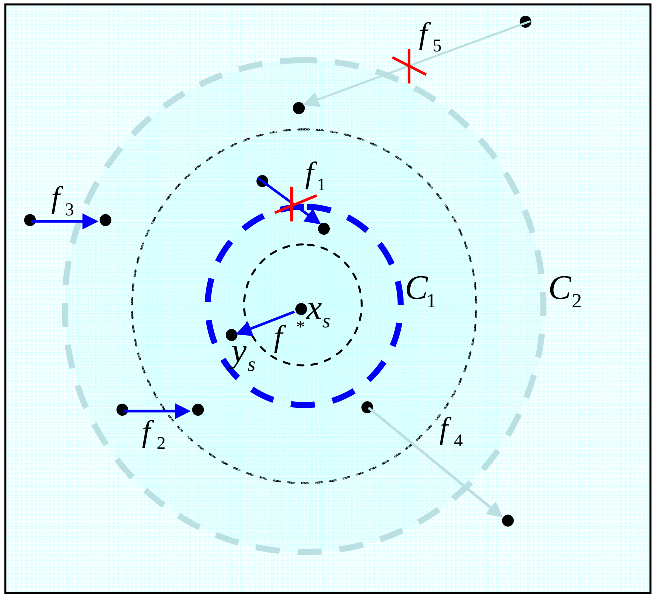
- Short links are "overpowered"
  → create much more interference
  → this precludes simple geometric arguments!

- In each time slot, consider all nodes in decreasing order of longest link
- Add a node to E_T if **allowed()** evaluates to true

$\mathrm{allowed}(\mathrm{v_i}, \mathrm{E_t})$

for each $v_j \in E_t$ **do**
    $\delta_{ij} := \tau(v_i) - \tau(v_j)$;
    if $\tau(v_i) = \tau(v_j)$ and $\mu \cdot r_i > d(v_i, v_j)$ **return false**
    else if $r_i \cdot (3n\beta)^{\frac{\delta_{ij}+1}{\alpha}} + r_j > d(v_i, v_j)$ **return false**
end for
return true

Please find details in the paper...



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What is the value of I_{in} ?

<u>Theorem</u>:

Scheduling Complexity of a topology T with in-interference I_{in} is at most $S(T) \in O(I_{in} \cdot \log^2 n)$

All current MAC protocols

Topology	$\mathbf{I_{in}}$	**our protocol**	**uniform power energy-metric**
nearest neighbor forest	≤ 5	$S(T) \in O(\log^2 n)$	$S(T) \in \Omega(n)$
exponential chain (directed)	1	$S(T) \in O(\log^2 n)$	$S(T) \in \Omega(n)$
strong connectivity			
- asymmetric links	$O(\log n)$	$S(T) \in O(\log^3 n)$	$S(T) \in \Omega(n)$

Improves the *scheduling complexity of connectivity*!

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strong connectivity			
- asymmetric links	$O(\log n)$	$S(T) \in O(\log^3 n)$	$S(T) \in \Omega(n)$
- symmetric links	$\Omega(\sqrt{n})$	$S(T) \in O(\sqrt{n} \log^{2.5} n)$	$S(T) \in \Omega(n)$
	$O(\sqrt{n} \log n)$		

Scheduling asymmetric vs. symmetric links!

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Conclusion - Our Contributions

1) Improved *“scheduling complexity of connectivity”*

→ from $O(\log^4 n)$ [Moscibroda, Wattenhofer, Infocom 2006] to $O(\log^3 n)$

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2) Scheduling symmetric vs. asymmetric links in topologies

→ using symmetric links has numerous practical advantages (ACK, ..)

→ but, asymmetric topologies can be scheduled much faster!

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3) Power assignment is crucial
   → uniform power assignment leads to extremely slow schedules!
   → "energy-metric" power assignment $P \sim d^{\alpha}$, too!

energy-spanner, energy minimum broadcast,…

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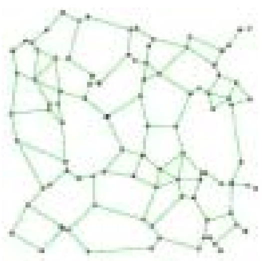
4) Bridge gap between information theoretic world (SINR) and protocol design (graph-based, topology control)

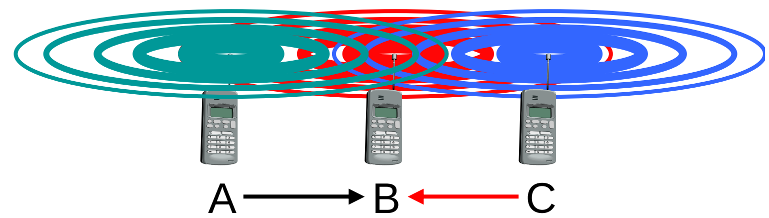
   → fundamental justification for topology control

Graph-based Protocol Design vs. SINR Scheduling?

Fundamenal question:

What is the relationship between topology control and physical scheduling?





<u>Graph-based topologies</u>

- Protocol designers use (various) graph models
- e.g. Topology control protocols

<u>SINR Scheduling</u>

- Information theoreticians use SINR (physical) models
- e.g. capacity of wireless networks

Topology Control helps in scheduling!

but, only if scheduling is done right!