Leveraging Linial's Locality Limit

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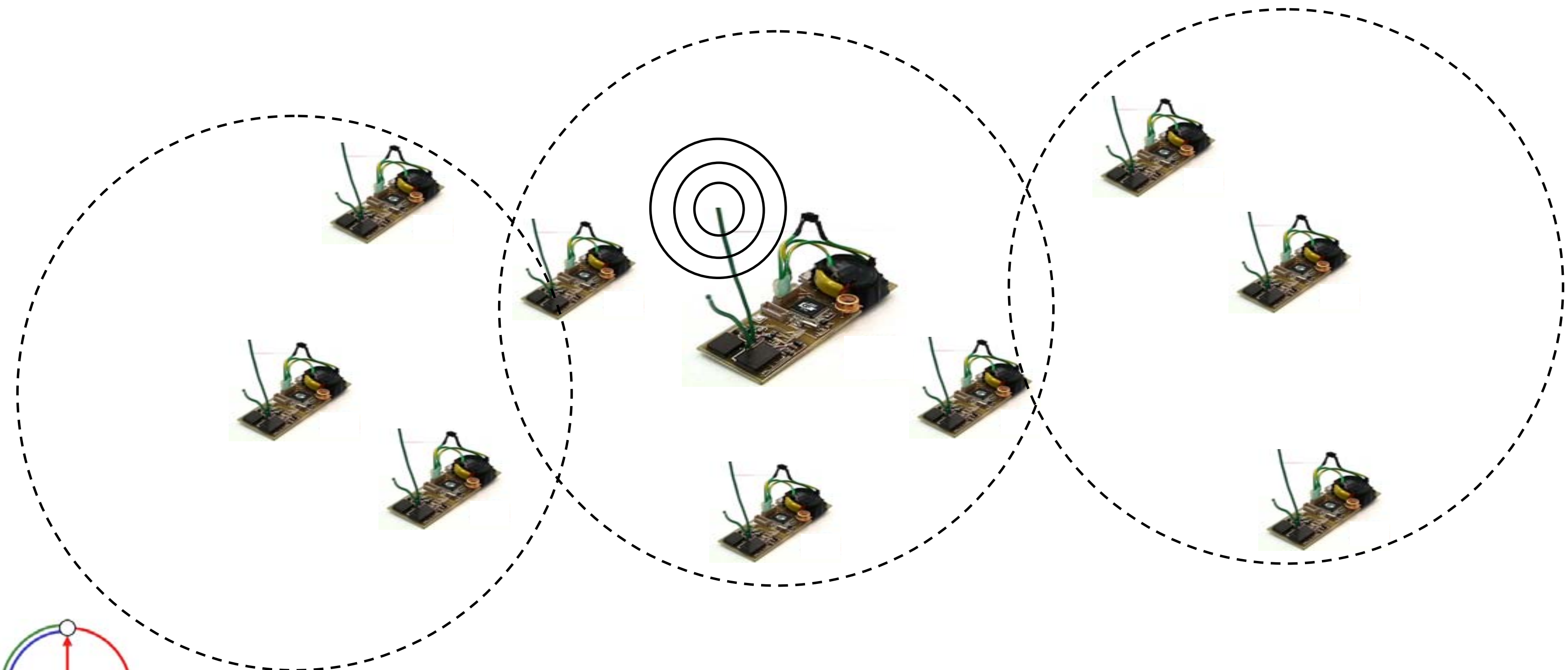
The problem

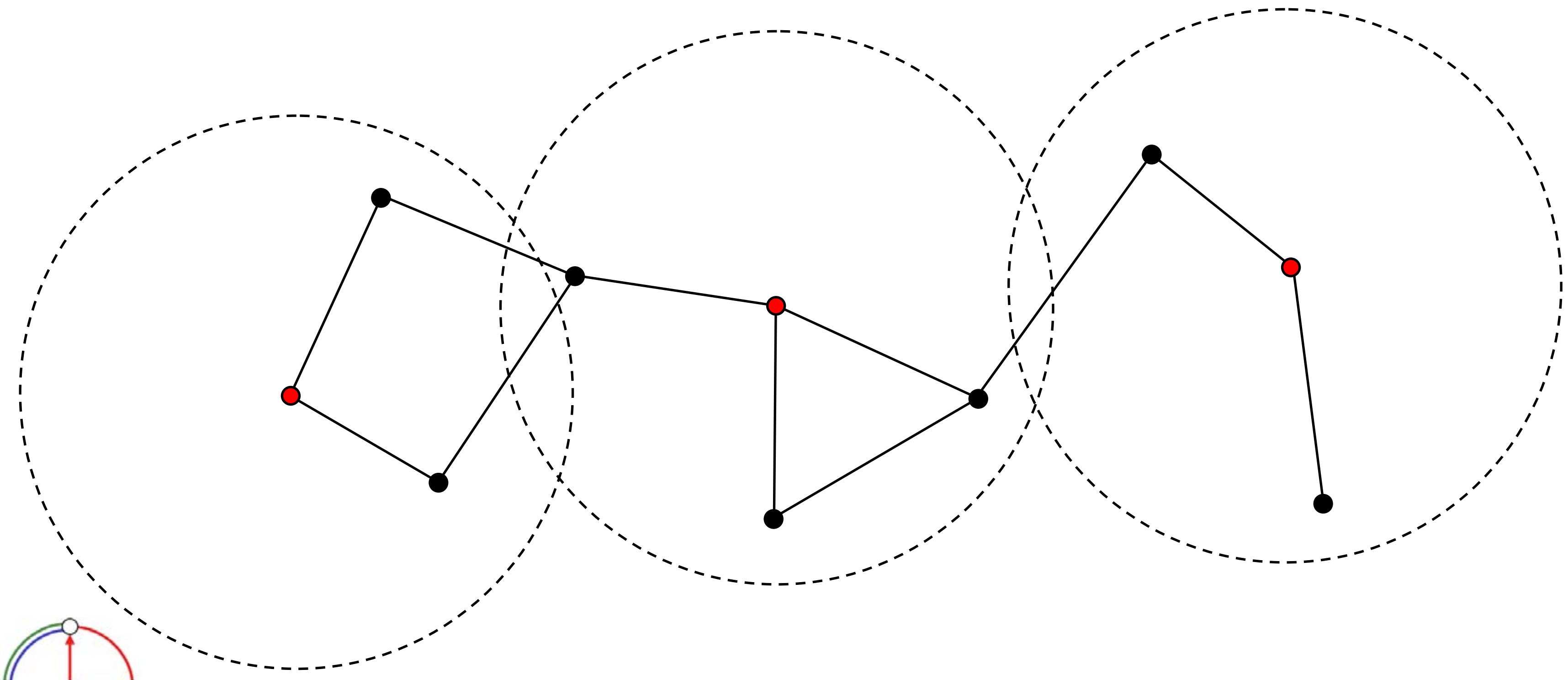
- we have a sensor network, where nodes communicate by radio
- all radios have the same (normalized) range
- we want to minimize energy consumption for communication

⇒ we want a small subset of the nodes to cover the network



Minimum Dominating Sets (MDS)

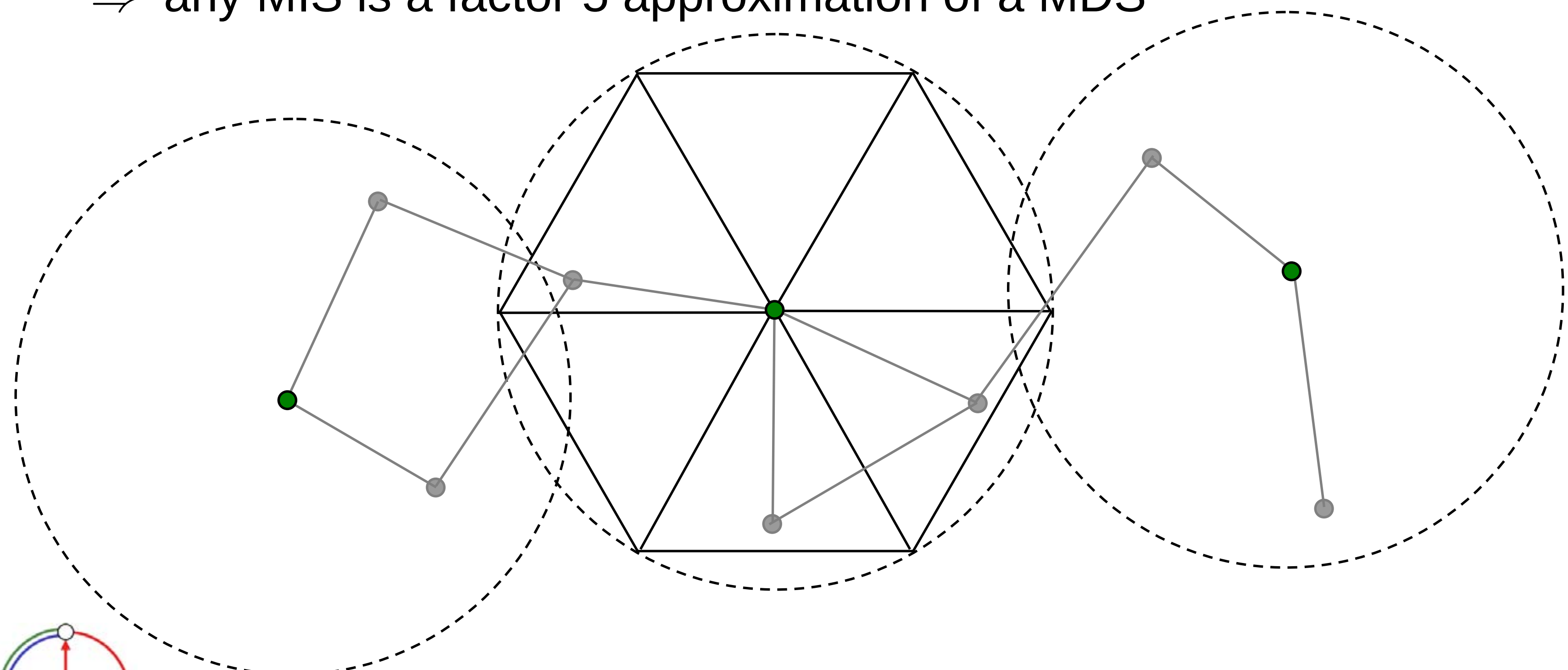
- this is the minimum dominating set (MDS) problem on unit disk graphs (UDG's)
- nodes have positions in the Euclidian plane
- two nodes are joined by an edge iff their distance is at most 1
- MDS: minimum subset of vertices covering the graph



Maximal Independent Sets (MIS)

- maximal independent set (MIS): maximal subset of nodes containing no neighbors
- MDS and MIS are closely related on UDG's
- neighborhood of any MIS is the whole graph
- only 5 independent neighbors in a UDG

$\Rightarrow$ any MIS is a factor 5 approximation of a MDS

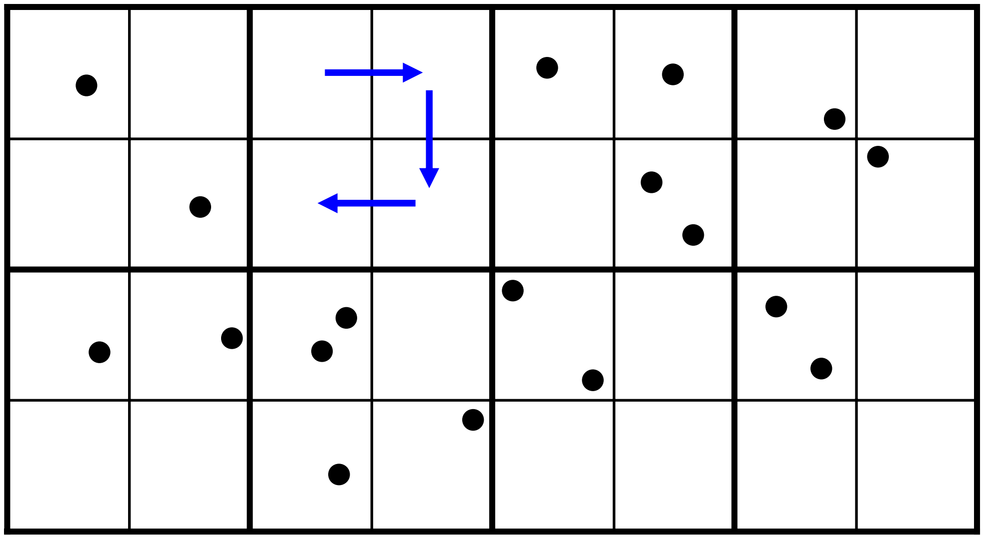


Geometry helps

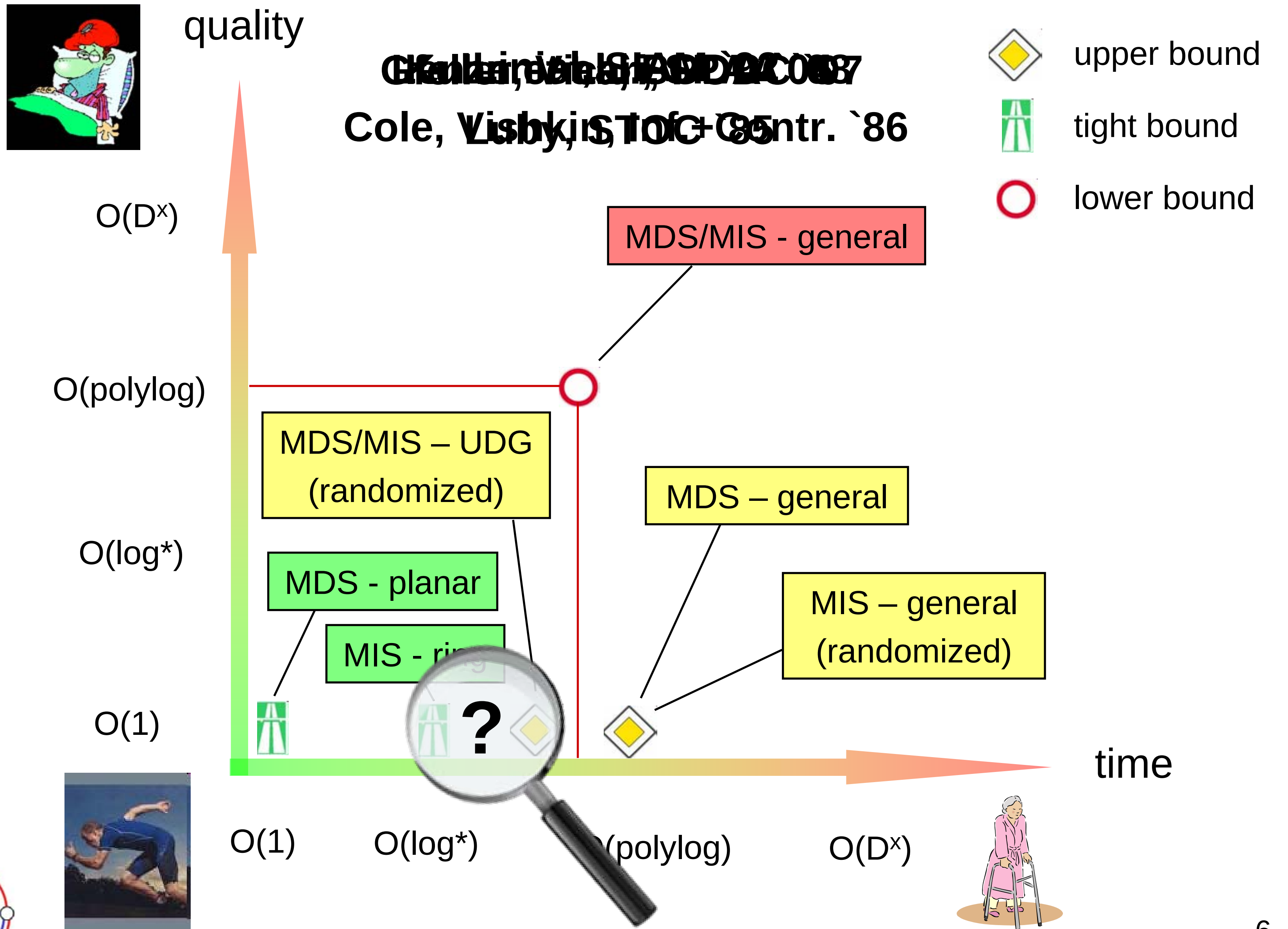
- model: local, deterministic, synchronous, unbounded message size, arbitrary computation, unique ID's, non-uniform
- both problems are easy with (global) positions
- e.g. Nieberg and Hurink (WAOA 2005): PTAS for MDS

⇒ how fast can these problems be solved w/o positions?

1. subdivide the plane
2. choose leaders
3. cycle through subcells



An overview – previous results



Looking for a connection to Linial's bound...

- Linial (SIAM `92): MIS on the ring takes $\Omega(\log^* n)$ time

$\Rightarrow$ no algorithm can assign to each node one bit such that:

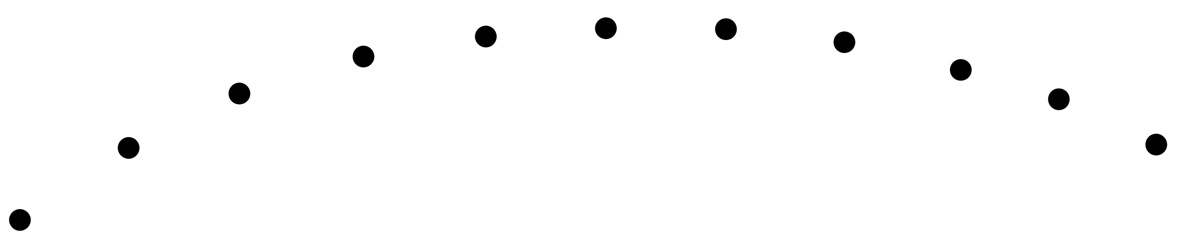
- only $o(\log^* n)$ consecutive 0's or 1's occur
- the algorithm has running time $o(\log^* n)$

- otherwise one could construct a MIS in $o(\log^* n)$ time:

$o(\log^* n)$ //compute bits

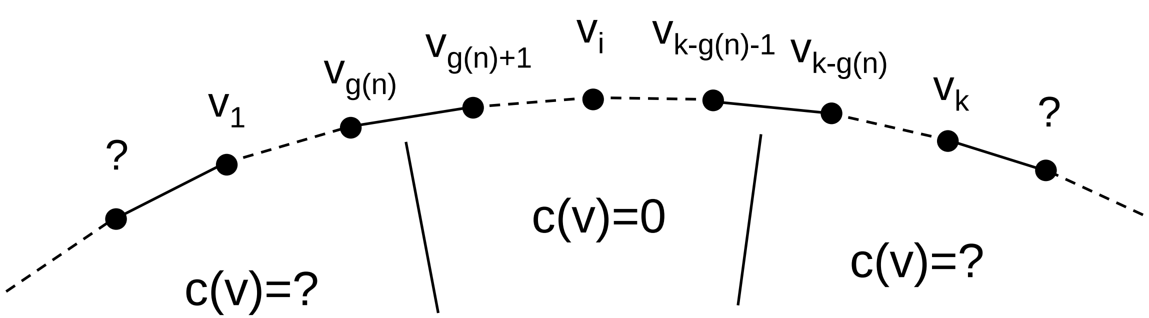
$\underline{+o(\log^* n)}$ //decide alternately, starting at 0-1 resp. 1-0 pairs

$=o(\log^* n)$

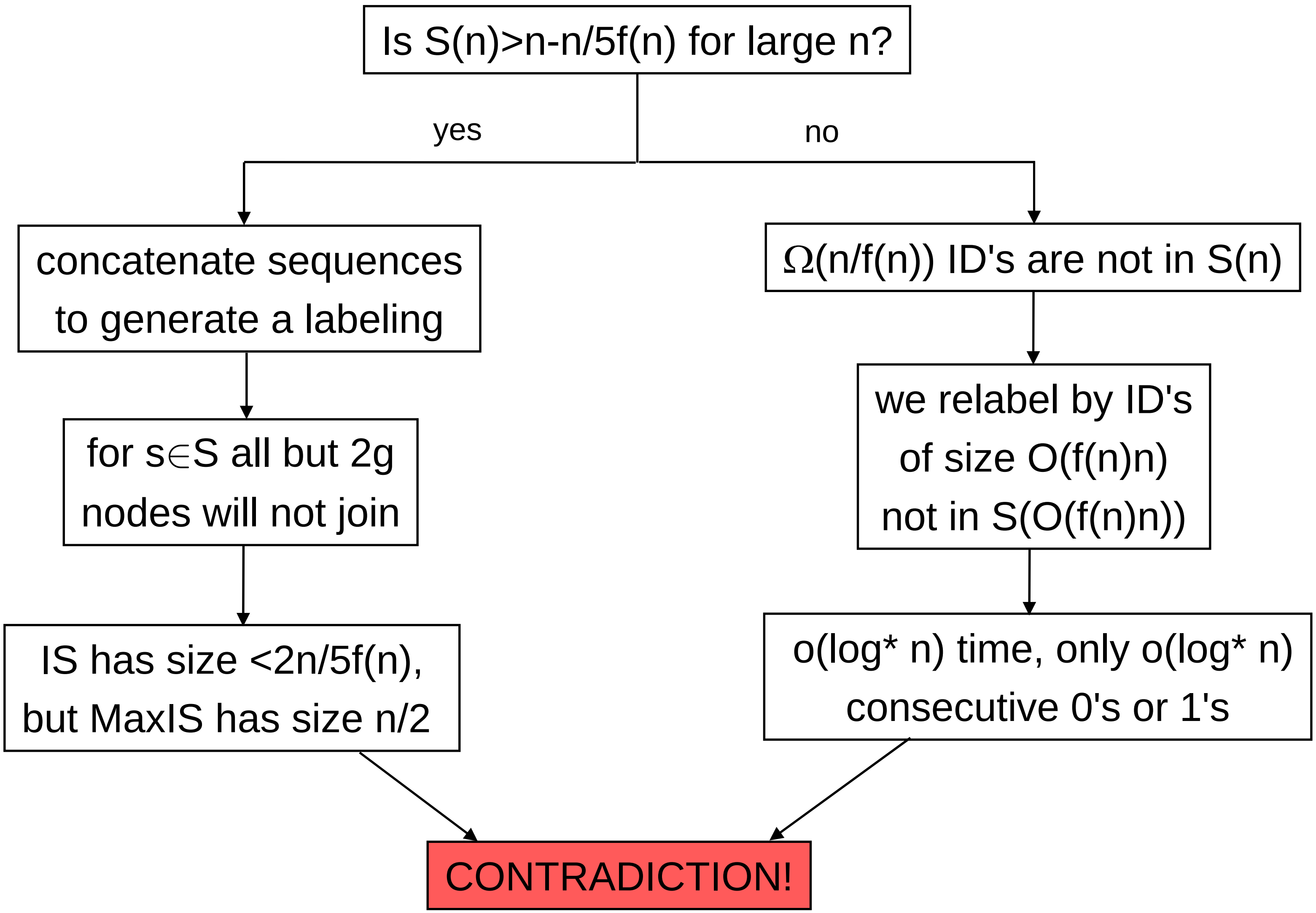


Do maximum independent set approximations do this?

- assume one finds an independent set (IS) at worst a factor f(n) smaller than the largest IS in g(n) time, $f(n)g(n) \in o(\log^* n)$
- no neighbors are both assigned 1 (since we have an IS)
- are long sequences of 0's possible?
- denote by S(n) a maximal subset of ID's forming disjoint sequences of length k=10f(n)g(n), where the inner nodes do not join the IS

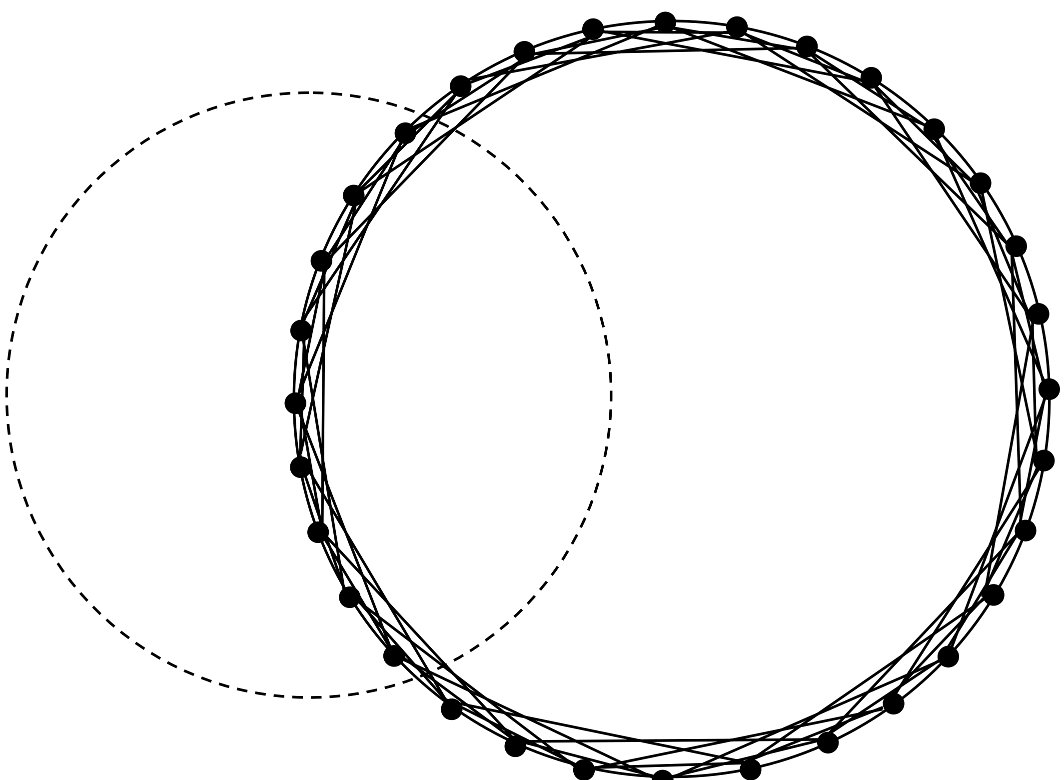


No maximum independent set approximations in o(log* n)!



But what about MDS – may be this can be solved faster?

- we take R_r, a ring where nodes are connected to their r next neighbors in each direction
- assume an f-approx. in g time on UDG's exists, $f g \in o(\log^* n)$
- only 2r nodes may get a 0, but now many 1's are problematic
- define for each r $S_r(n)$ similar to the MaxIS case



No minimum dominating set approximations in $o(\log^* n)$!

Exists r with $S_r(n) \leq n/2$ for all n?

yes

relabel the ring R_1 with ID's of size $2n$ not in $S_r(2n)$

simulate $R_1 = R_r$

$o(r \log^* 2n) = o(\log^* n)$ running time, $o(\log^* n)$ consecutive 0's or 1's

no

choose n_r min. with $S_r(n_r) > n_r/2$

concatenate ID's to label R_r $\Rightarrow \Omega(n_r)$ nodes enter the DS

$f(n_r) \in \Omega(r)$, as MDS is smaller than n/r

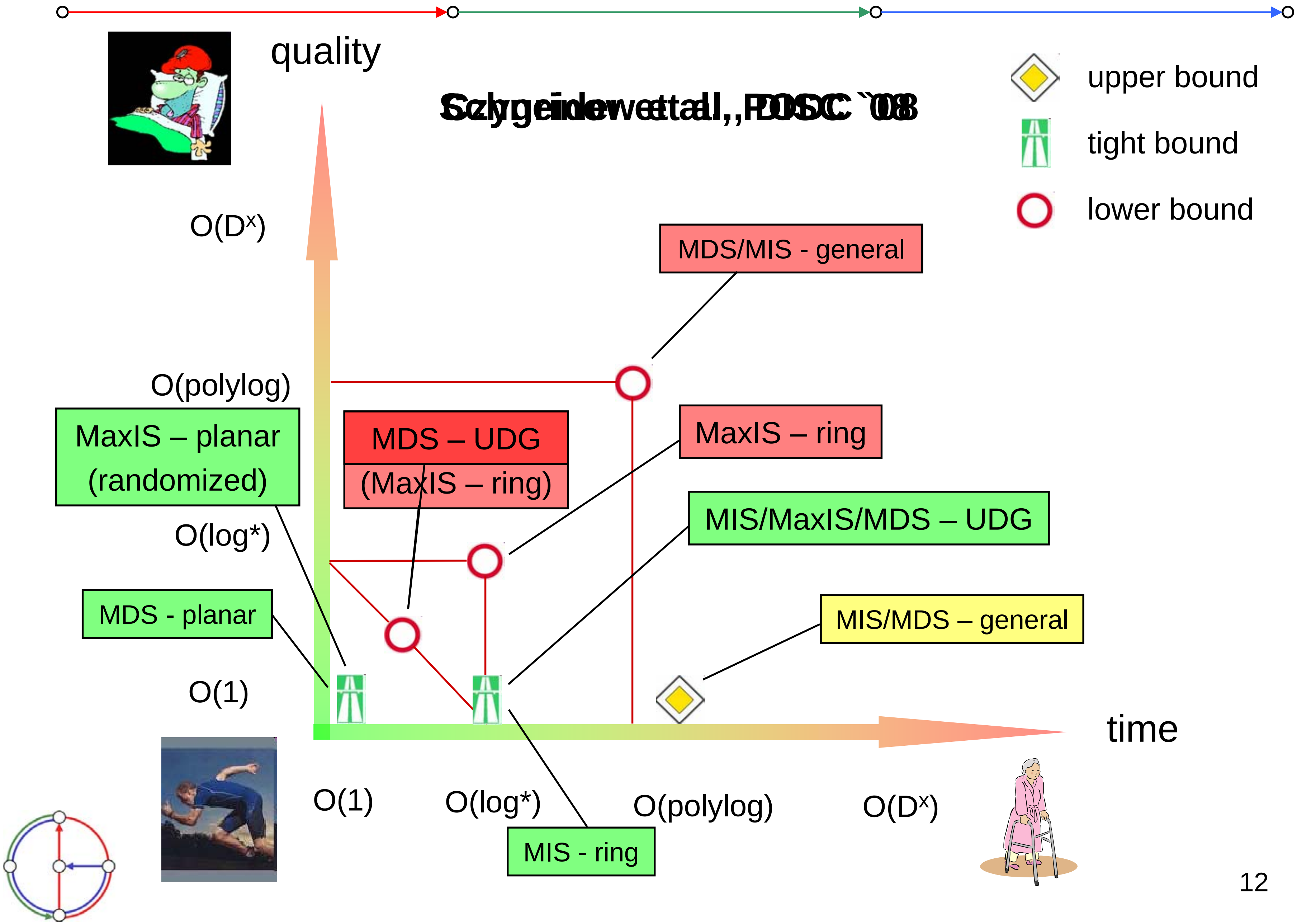
relabel by ID's not in $S_{r(n)}(n_{r(n)})$, where $n_{r(n)-1} \leq 2n < n_{r(n)}$

simulate $R_1 = R_{r(n)}$

$O(r(n) g(n_{r(n)}) \subseteq O(f(n) g(n))$ $\subset o(\log^* n)$ running time, $o(\log^* n)$ cons. 0's or 1's

CONTRADICTION!

An overview – new results



Any questions or comments?

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