Optimal Distributed All Pairs Shortest Paths

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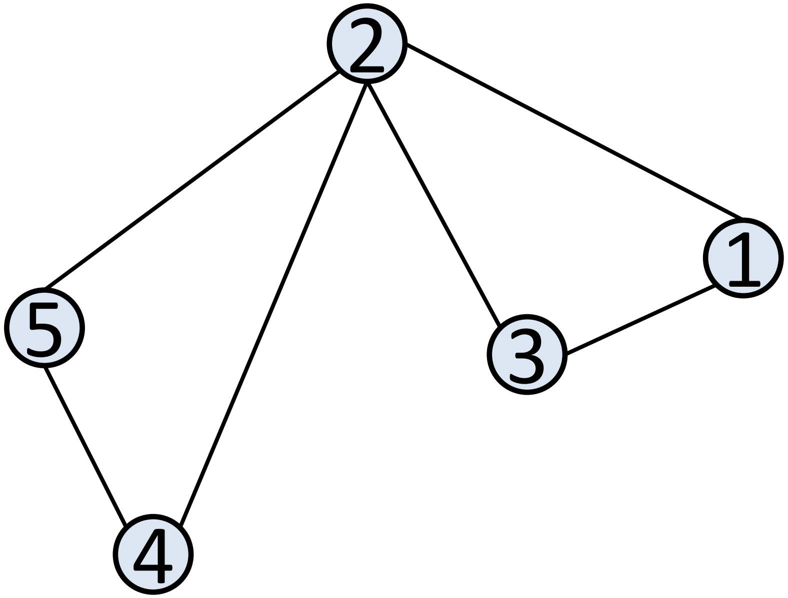
Distributed network
Graph G of n nodes

Distributed network
Graph G of n nodes

Unique IDs



Distributed network
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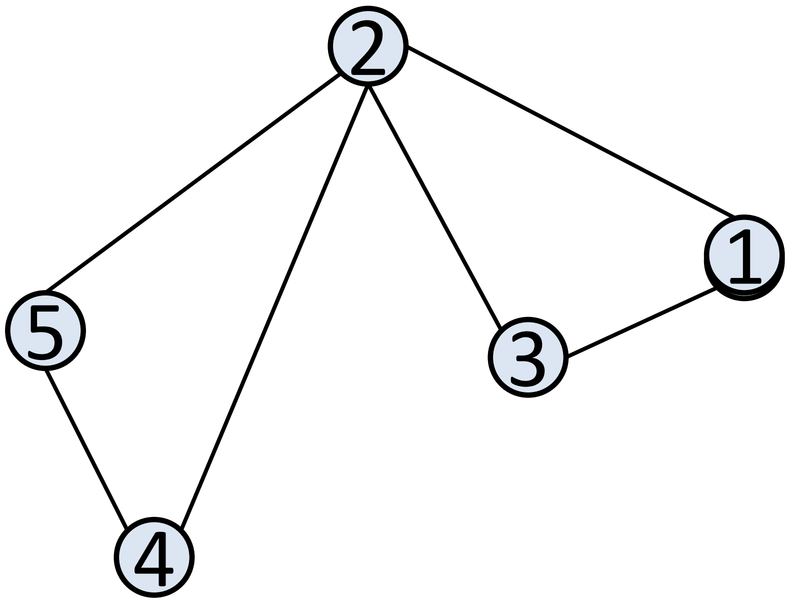

Local information only

Distributed network
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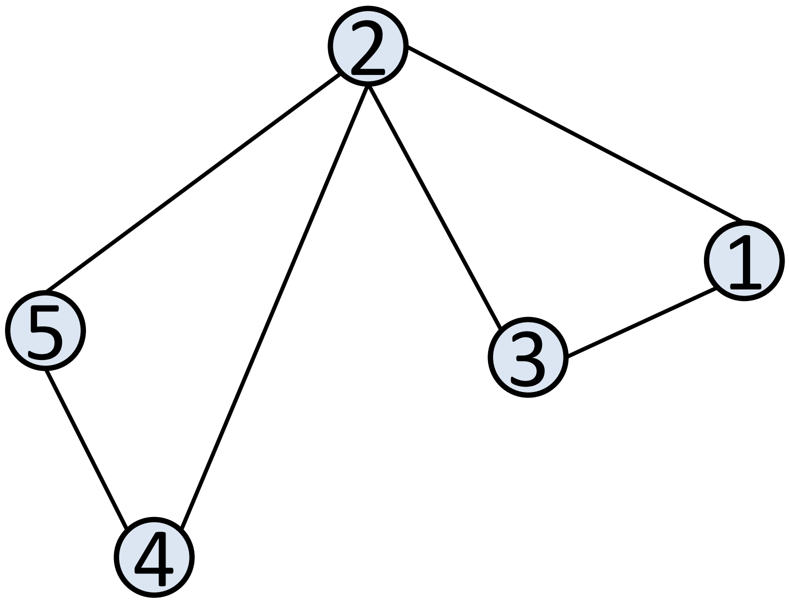

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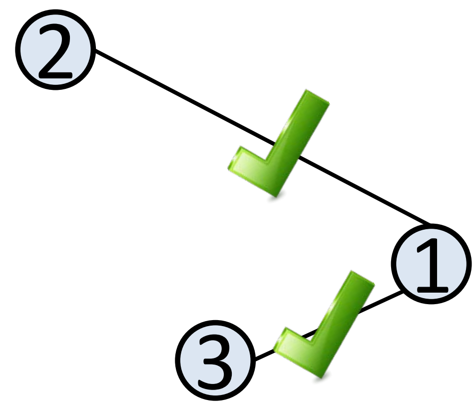

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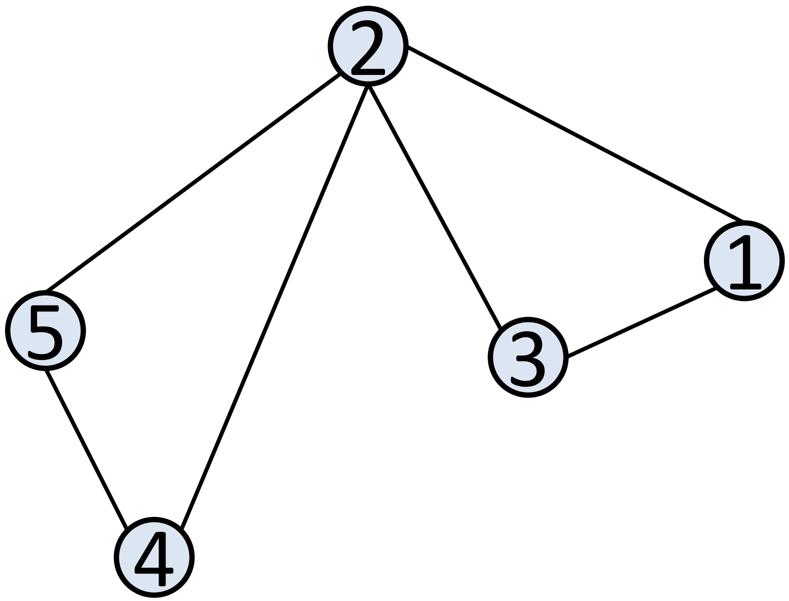

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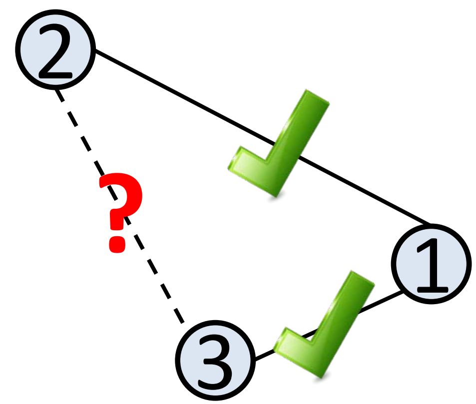

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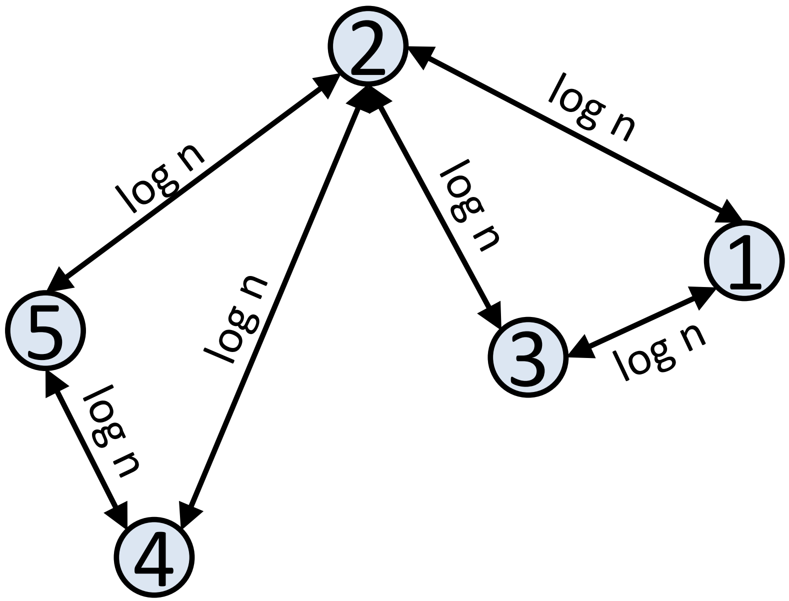
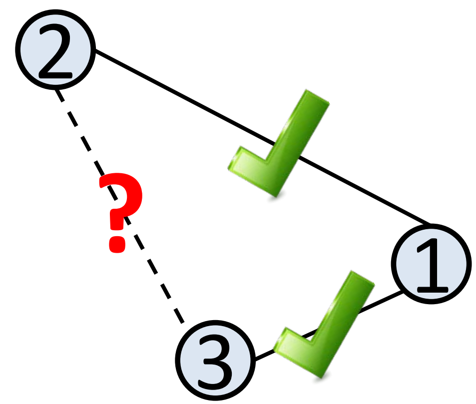


Limited bandwidth

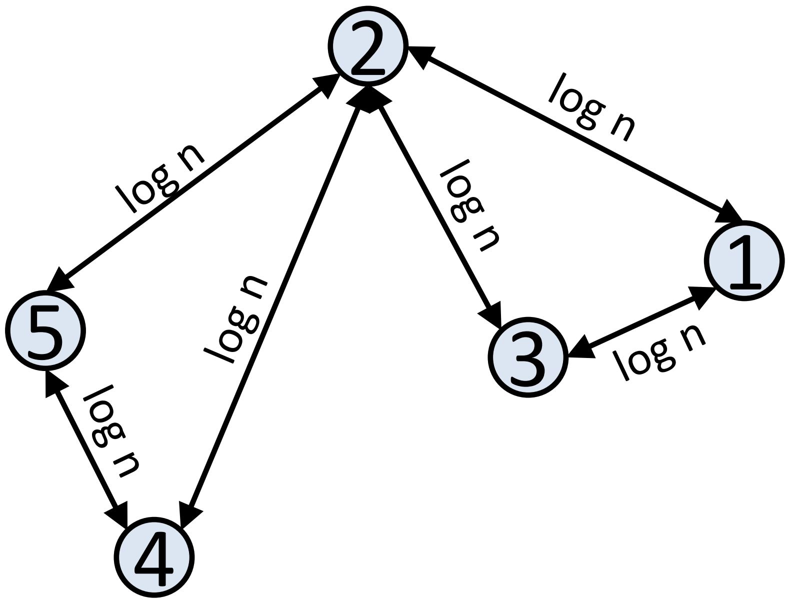
Local infor-
mation only



Distributed network

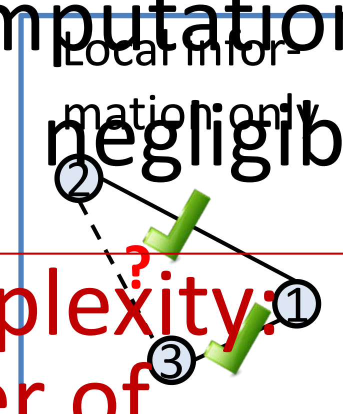
Graph G of n nodes

Unique IDs



Synchronized

Internal computations negligible

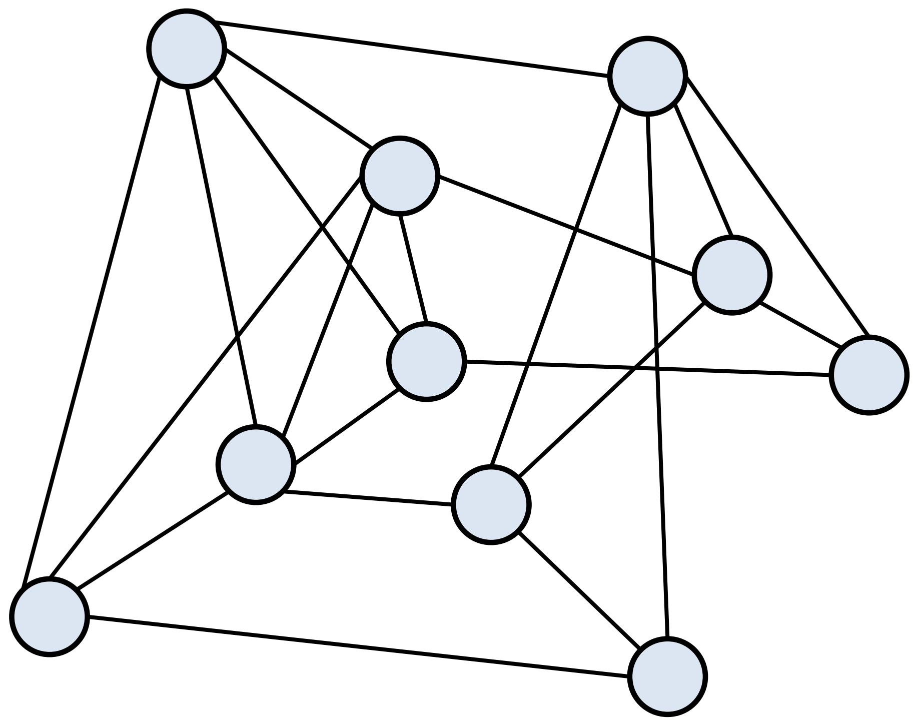


Limited bandwidth

Time complexity: number of communication rounds

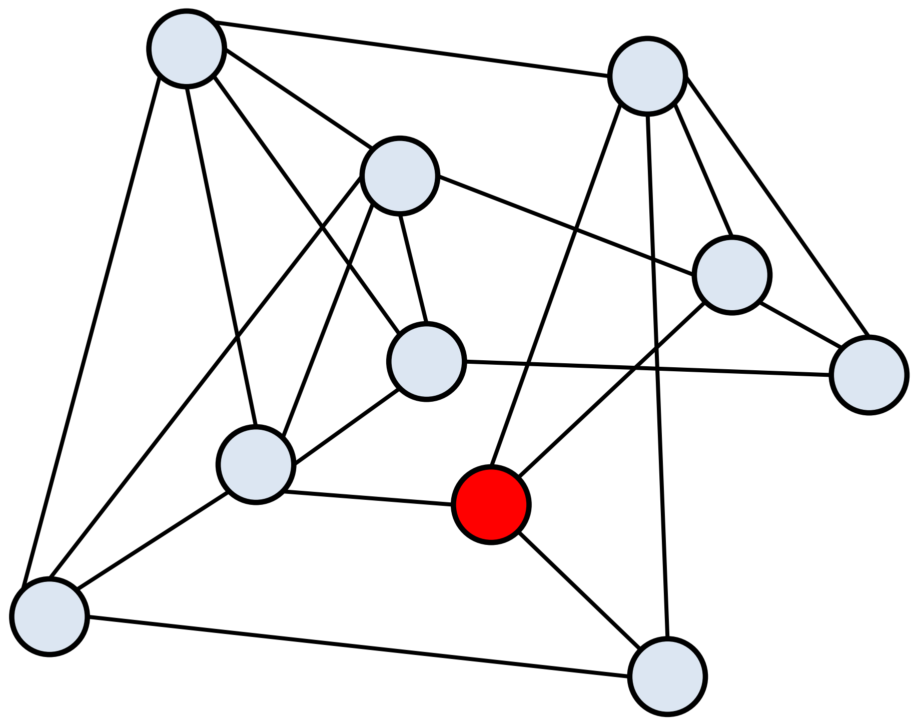
Distributed algorithms: a simple example

Count the nodes!



Count the nodes!

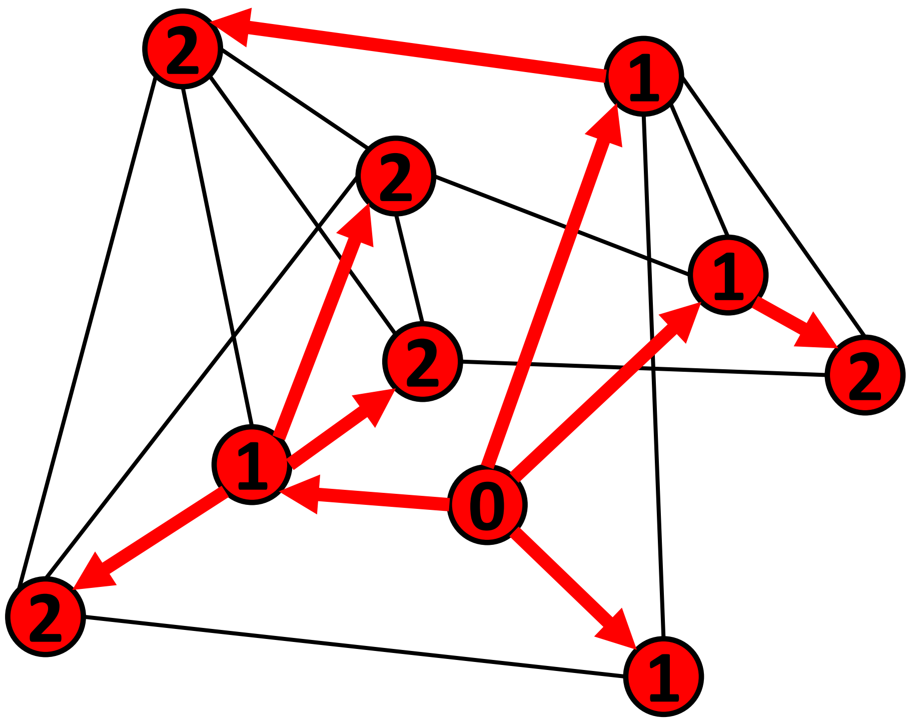
1. Compute BFS-Tree



Count the nodes!

1. Compute BFS-Tree

2. Count nodes in subtrees



Runtime: ?

Count the nodes!

1. Compute BFS-Tree

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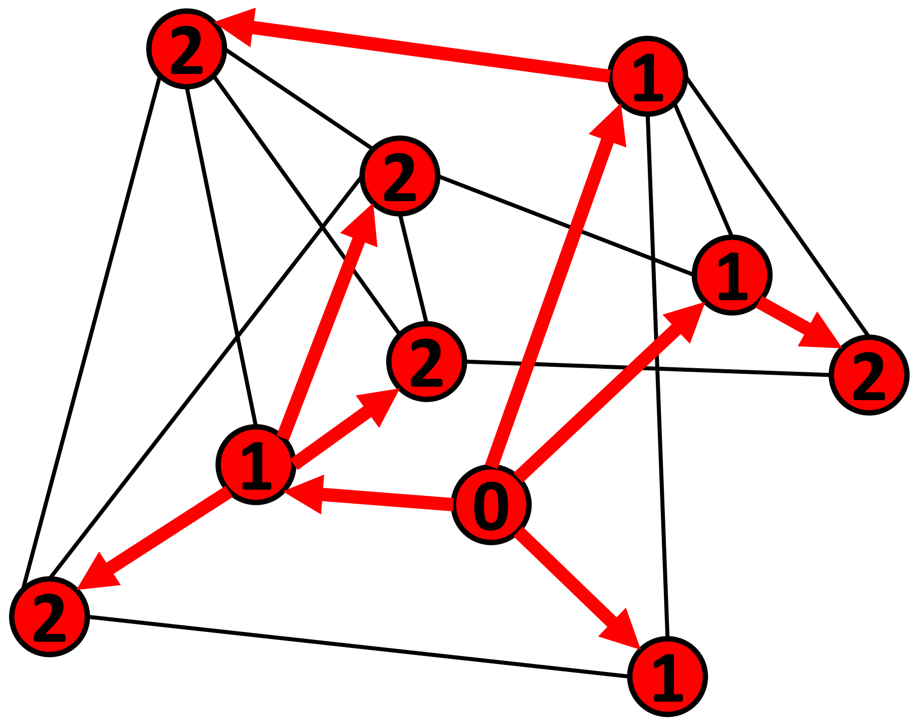
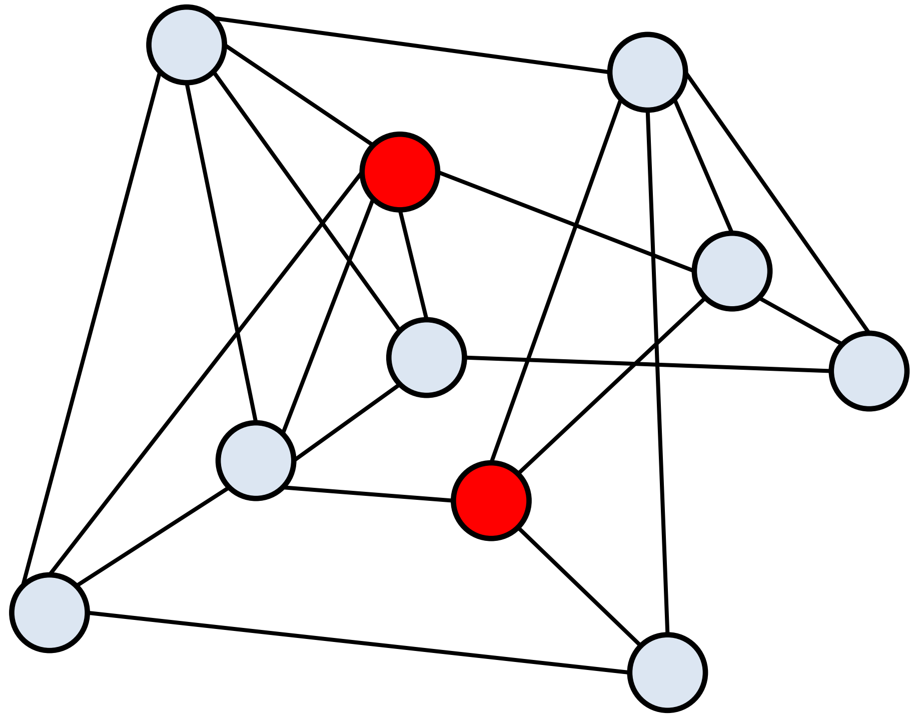


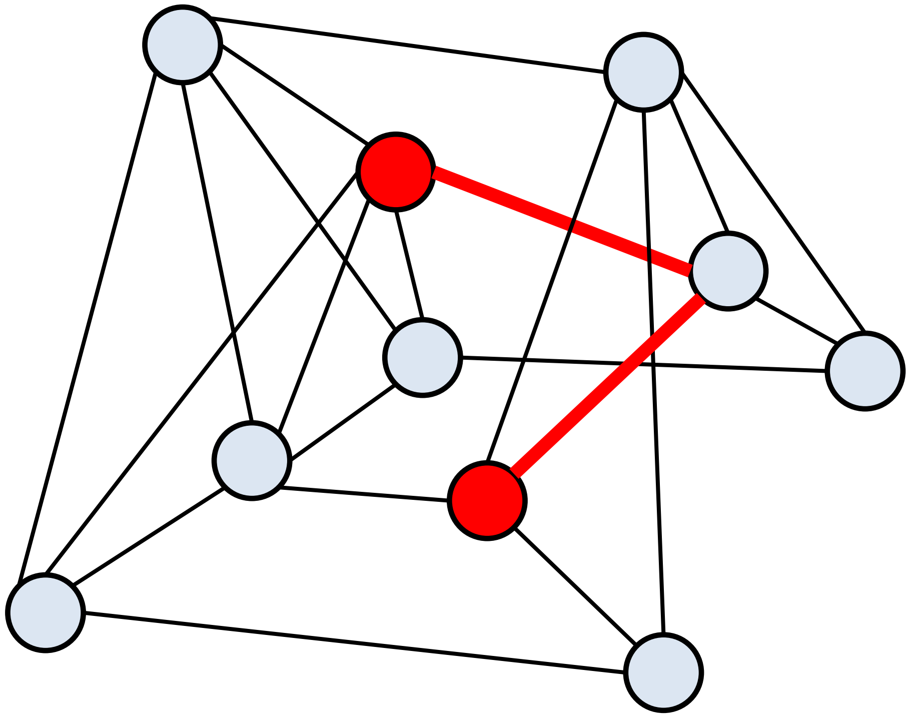
Runtime: Diameter

Diameter of a network



- Distance between two nodes = Number of hops of shortest path

Diameter of a network



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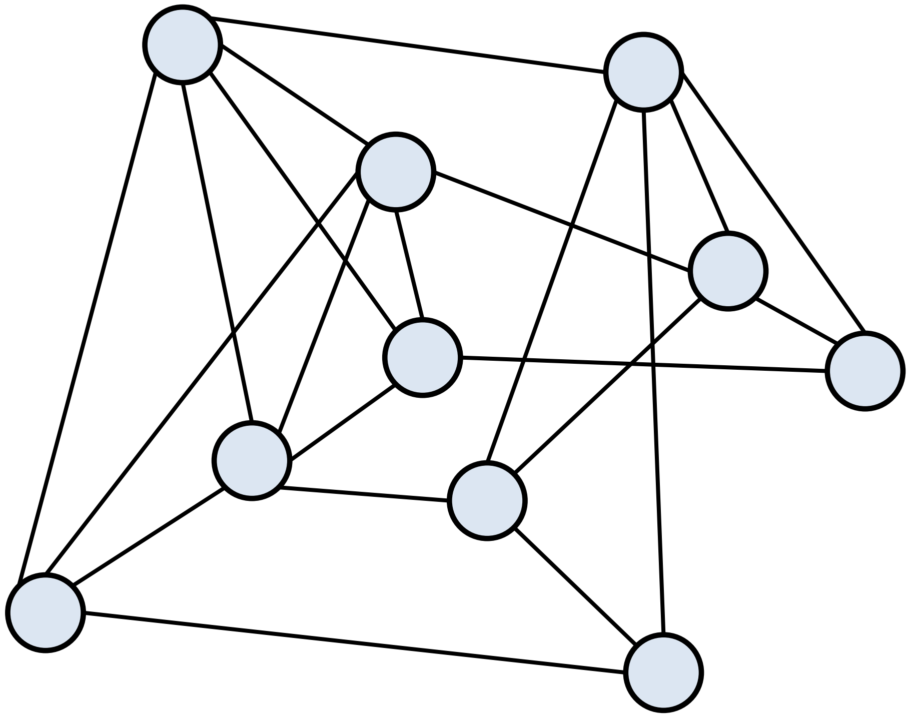
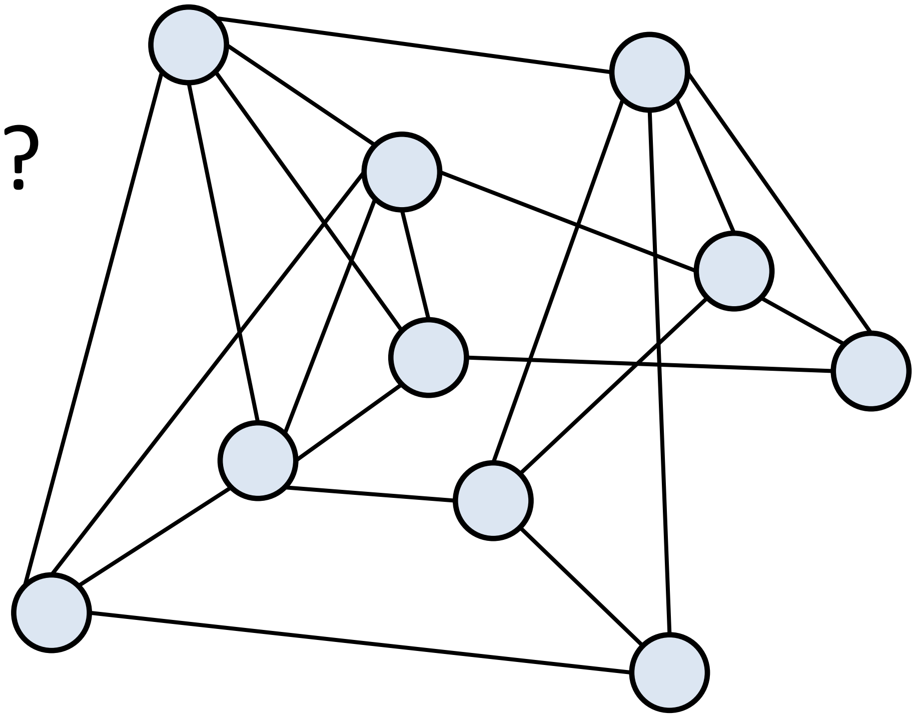


- Distance between two nodes = Number of hops of shortest path
- Diameter of network = Maximum distance, between any two nodes

Diameter of a network

Diameter of this network?



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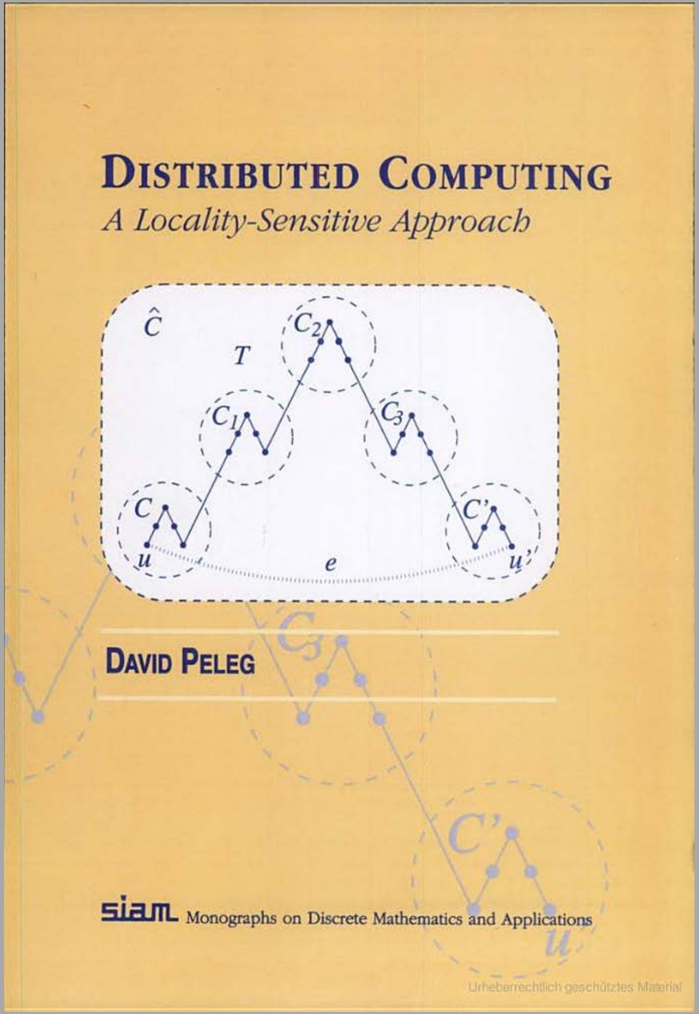
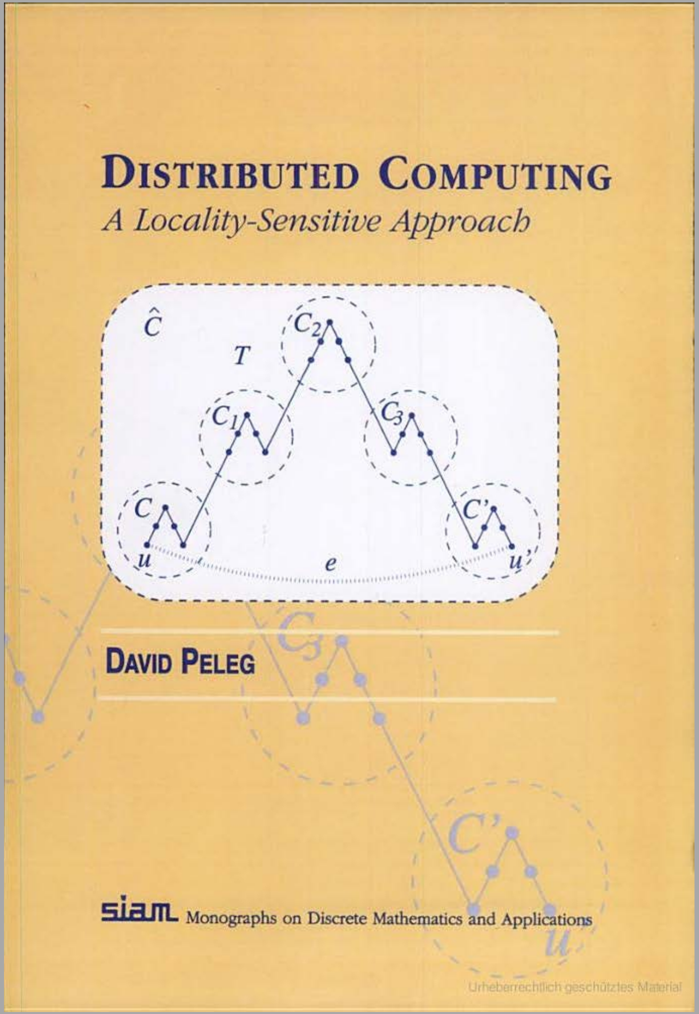
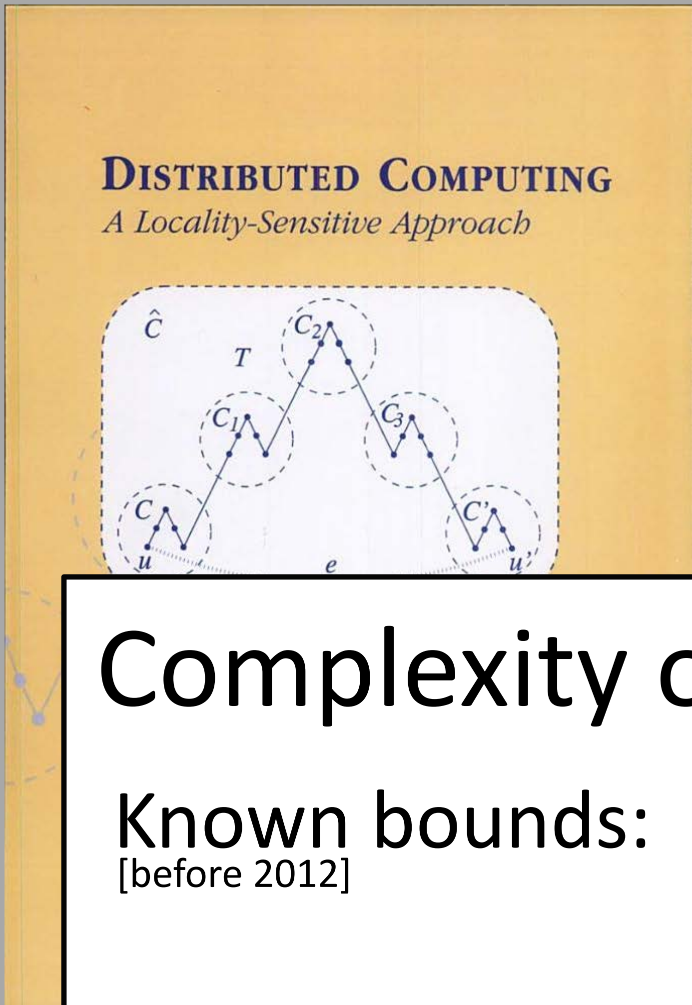


- Diameter appears frequently in distributed computing



- Diameter appears frequently in distributed computing
- E.g. local vs. global



measuring the distance between u and w looking at G as an unweighted graph, i.e., it is the minimum number of hops necessary to get from u to w.

1. Formal definition?

Throughout, we denote $\Lambda = \lceil \log Diam(G) \rceil$.

In a weighted graph G, *let* $Diam^{un}(G)$ *denote the* unweighted diameter *of* G, *i.e., the maximum unweighted distance between any two vertices of* G.

Definition 2.1.2 [Radius and center]: *For a vertex* $v \in V$, *let* $Rad(v, G)$ *denote the distance from* v *to the vertex farthest away from it in the graph* G:

$$Rad(v, G) = \max_{w \in V}\{dist_G(v, w)\}.$$

Let $Rad(G)$ *denote the* radius *of the network, i.e.,*

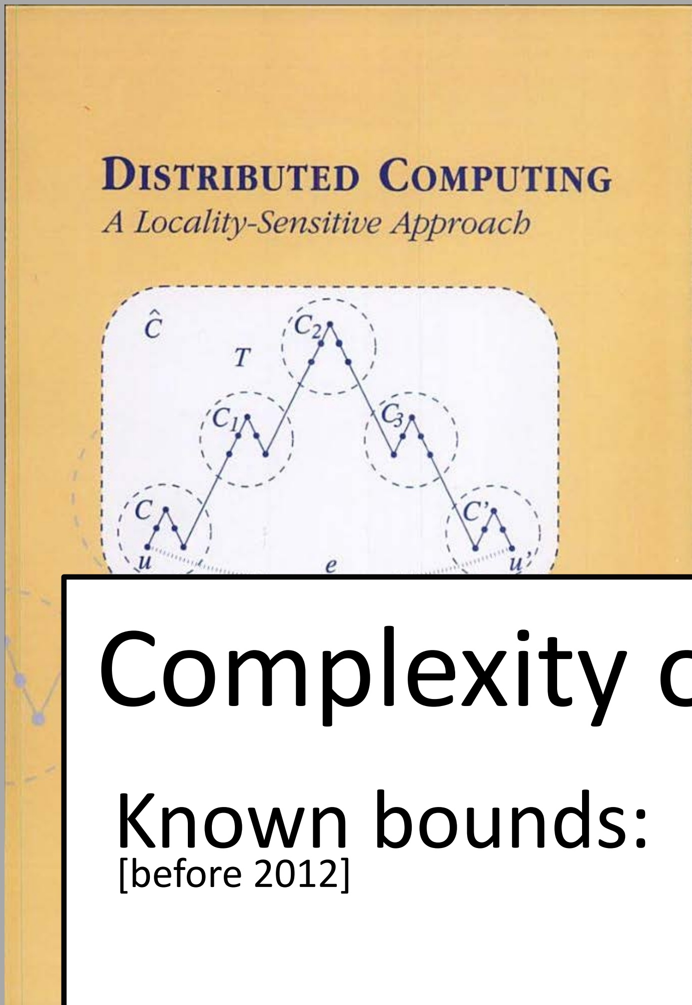
$$Rad(G) = \min_{v \in V}\{Rad(v, G)\}.$$

A center *of* G *is any vertex* v *realizing the radius of* G *(i.e., such that* $Rad(v, G) = Rad(G)$*). In order to simplify some of the following definitions, we avoid problems arising from 0-*diameter *or 0-radius graphs, by defining* $Rad(G) = Diam(G) = 1$ *for the single-vertex*

Complexity of computing D?

Known bounds: [before 2012]	O(nD)	Ω (D)
	≈O(n^2)	≈Ω(1)

- E.g. local vs. global



1. Formal definition?

Diameter, radius and depth

Definition 2.1.1 [Diameter] *Let $Diam(G)$ denote the (weighted or unweighted) diameter of the network G, i.e., the maximal distance between any two vertices in it:*

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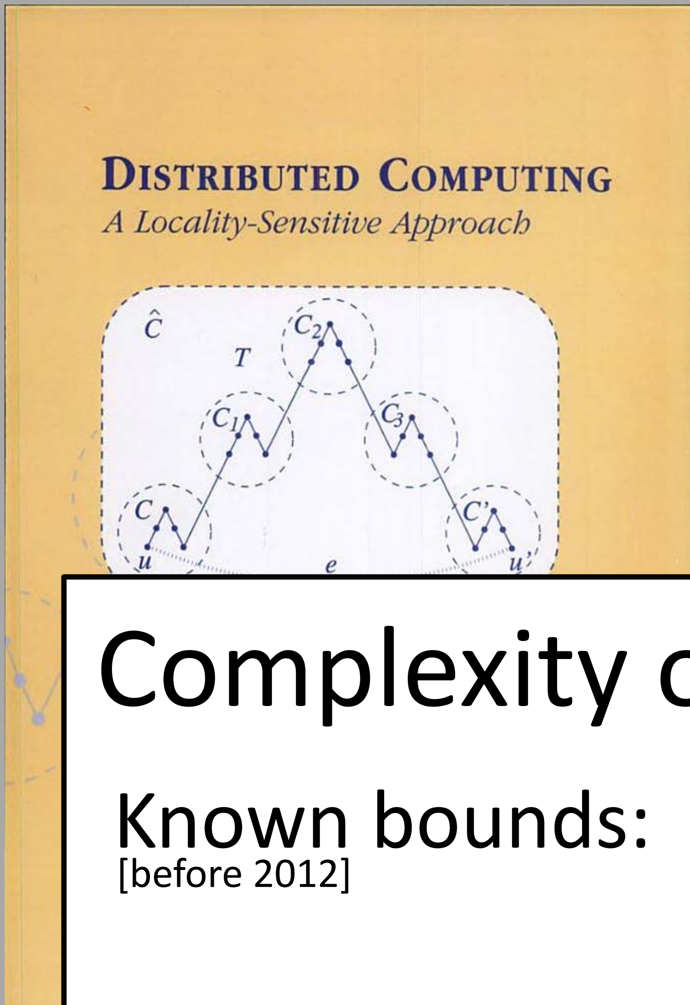
New bounds: [2012]		Ω (n) [FHW 2012]

Even if D = 5

- E.g. local vs. global



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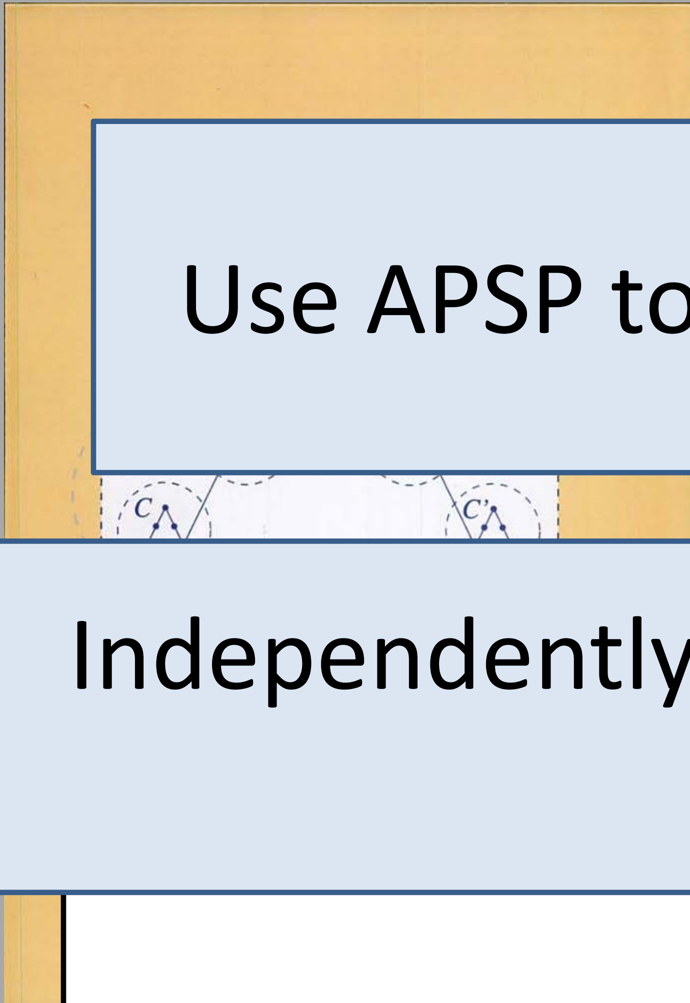
New bounds:	O(n) [here]	Ω (n) [FHW 2012]

Even if D = ~~5~~ 3

- E.g. local vs. global



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$Rad(G) = \min_{v \in V}\{Rad(v,G)\}.$

A center of G is any vertex v realizing the radius of G (i.e., such that $Rad(v,G) = Rad(G)$).

Use APSP to Compute Diameter

Independently: [Peleg, Roditty, Tal 2012]

≈O(n²) ≈Ω(1)

Even if D = 5 3

New bounds:	O(n) [here]	Ω (n) [FHW 2012]

- E.g. local vs. global

Extentions

Problem	exact	$(+,1)$	$(\times, 1+\varepsilon)$	$(\times, 3/2-\varepsilon)$	$(\times, 3/2)$	$(\times, 2)$
APSP	$\tilde{\Theta}(n)$ [16]	$\tilde{\Theta}(n)$ [1,13]	$\tilde{\Theta}(n)$ [1,13]	$\tilde{\Theta}(n)$ [1,13]	–	–
eccentricity	$\tilde{\Theta}(n)$ [5,11]	$\Omega(\frac{n}{D \cdot \log n} + D)$ [11]	$\mathcal{O}(\frac{n}{D} + D)$ [3]	$\Omega\left(\frac{\sqrt{n}}{\log n} + D\right)$ [11]	–	$\Theta(D)$ [18]
diameter	$\tilde{\Theta}(n)$ [6,20]	$\Omega(\frac{n}{D \cdot \log n} + D)$ [2]	$\mathcal{O}(\frac{n}{D} + D)$ [17]	$\Omega\left(\frac{\sqrt{n}}{\log n} + D\right)$ [21]*	$\mathcal{O}(n^{3/4} + D)$ [14]	$\Theta(D)$ [18]
radius	$\mathcal{O}(n)$ [8]	–	$\mathcal{O}(\frac{n}{D} + D)$ [17]	–	–	$\Theta(D)$ [18]
center	$\tilde{\Theta}(n)$ [9,12]	$\Omega(\frac{n}{D \cdot \log n} + D)$ [12]	$\mathcal{O}(\frac{n}{D} + D)$ [17]	$\Omega\left(\frac{\sqrt{n}}{\log n} + D\right)$ [12]	–	0 [19]
p. vertices	$\tilde{\Theta}(n)$ [10,11]	$\Omega(\frac{n}{D \cdot \log n} + D)$ [11]	$\mathcal{O}(\frac{n}{D} + D)$ [17]	$\Omega\left(\frac{\sqrt{n}}{\log n} + D\right)$ [11]	–	0 [19]
girth	$\mathcal{O}(n)$ [7]	–	$\mathcal{O}\left(\min\left\{n/g + D \cdot \log \frac{D}{g}\ ,\ n\right\}\right)$ [4]		–	–

For the girth, two additional ratios are of interest:

Problem	$(\times, 2-\varepsilon)$	$(\times, 2-1/g)$
girth	$\Omega\left(\frac{\sqrt{n}}{\log n} + D\right)$ [22]*	$\mathcal{O}\left(n^{2/3} + D \cdot \log \frac{D}{g}\right)$ [15]

Extentions

Routing tables

Social networks

Fighting spam

Problem	exact	$(+,1)$	$(\times,1+\varepsilon)$	$(\times,3/2-\varepsilon)$	$(\times,3/2)$	$(\times,2)$
APSP	$\tilde{\Theta}(n)^{16)}$	$\tilde{\Theta}(n)^{1,13)}$	$\tilde{\Theta}(n)^{1,13)}$	$\tilde{\Theta}(n)^{1,13)}$	–	–
eccentricity	$\tilde{\Theta}(n)$ [illegible]	[illegible]	[illegible] $+ D)^{3)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{11)}$	–	$\Theta(D)^{18)}$
diameter	$\tilde{\Theta}(n)$ [illegible]	$\Omega(\frac{n}{D\cdot\log n}+D)$ [illegible]	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{21)*}$	$\mathcal{O}(n^{3/4}+D)^{14)}$	$\Theta(D)^{18)}$
radius	$\mathcal{O}(n)^{8)}$	–	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	–	–	$\Theta(D)^{18)}$
center	$\Theta(n)^{9,12)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{12)}$	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{12)}$	–	$0^{19)}$
p. vertices	$\tilde{\Theta}(n)^{10,11)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{11)}$	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{11)}$	–	$0^{19)}$
girth	$\mathcal{O}(n)^{7)}$	[illegible] $\left(\min\left\{n/g+D\cdot\log\frac{D}{g}\ ,\ n\right\}\right)^{4)}$			–	–

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Problem	$(\times,2-\varepsilon)$	$(\times,2-1/g)$
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Extentions

Problem	exact	(+, 1)	(×, 1 + ε)	(×, 3/2 − ε)	(×, 3/2)	(×, 2)
APSP	$\tilde{\Theta}(n)$[16]	$\tilde{\Theta}(n)$[1,13]	$\tilde{\Theta}(n)$[1,13]	$\tilde{\Theta}(n)$[1,13]	–	–
eccentricity	$\tilde{\Theta}(n)$[5,11]	$\Omega(\frac{n}{D \cdot \log n} + D)$[11]	$\mathcal{O}(\frac{n}{D} + D)$[3]	$\Omega\left(\frac{\sqrt{n}}{\log n} + D\right)$[11]	–	$\Theta(D)$[18]
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Problem	(×, 2 − ε)	(×, 2 − 1/g)
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Combination with: [Peleg, Roditty, Tal 2012]

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eccentricity	$\tilde{\Theta}(n)^{5,11)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{11)}$	$\mathcal{O}(\frac{n}{D}+D)^{3)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{11)}$	–	$\Theta(D)^{18)}$
diameter	$\tilde{\Theta}(n)^{6,20)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{2)}$	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{21)*}$	$\mathcal{O}(n^{3/4}+D)^{14)}$	$\Theta(D)^{18)}$
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Combined with [Peleg, Roditty, Tal 2012]

APSP in O(n)

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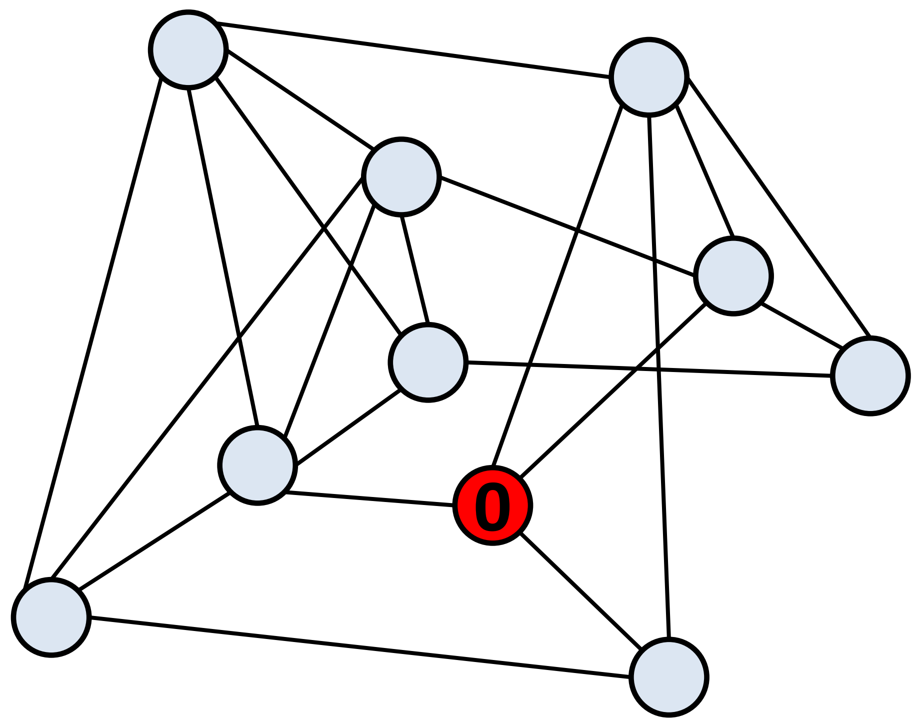
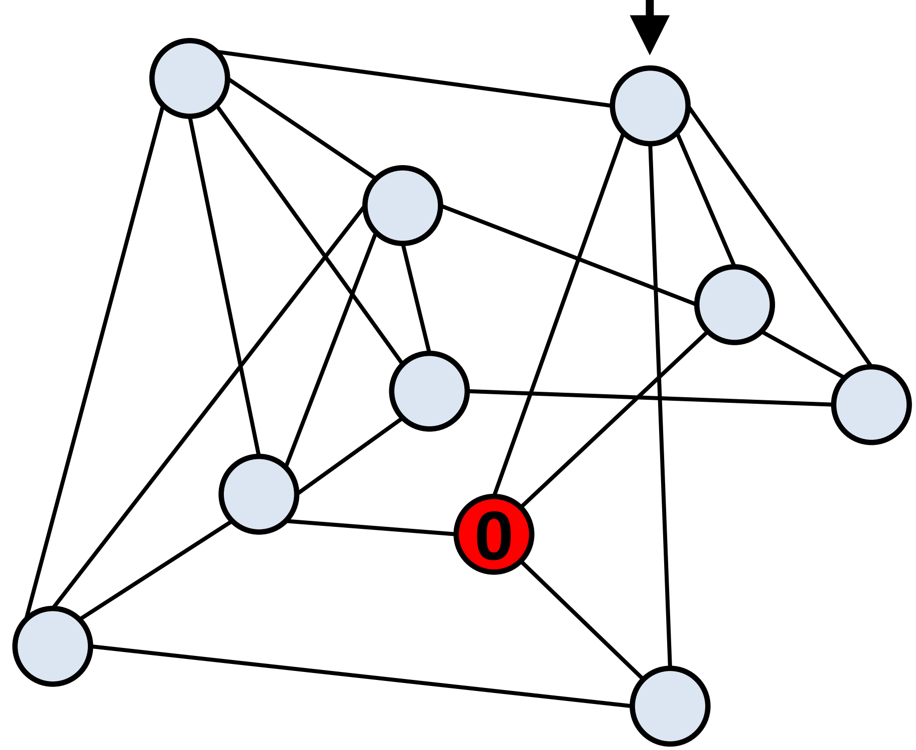
Compute All Pairs Shortest Paths



APSP in O(n)

Compute All Pairs Shortest Paths

Knows its distance to all other nodes



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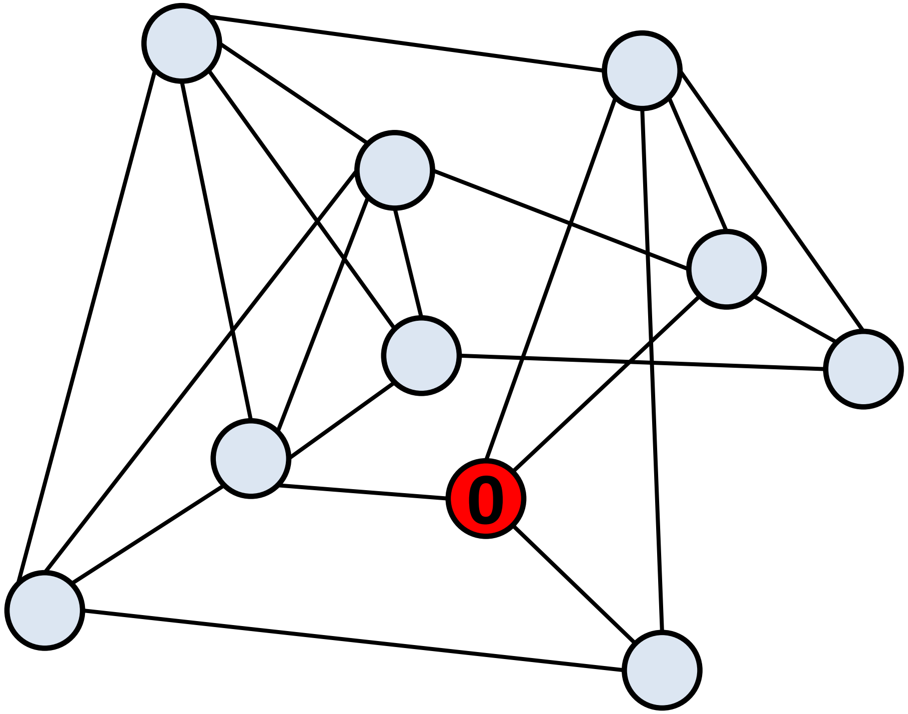
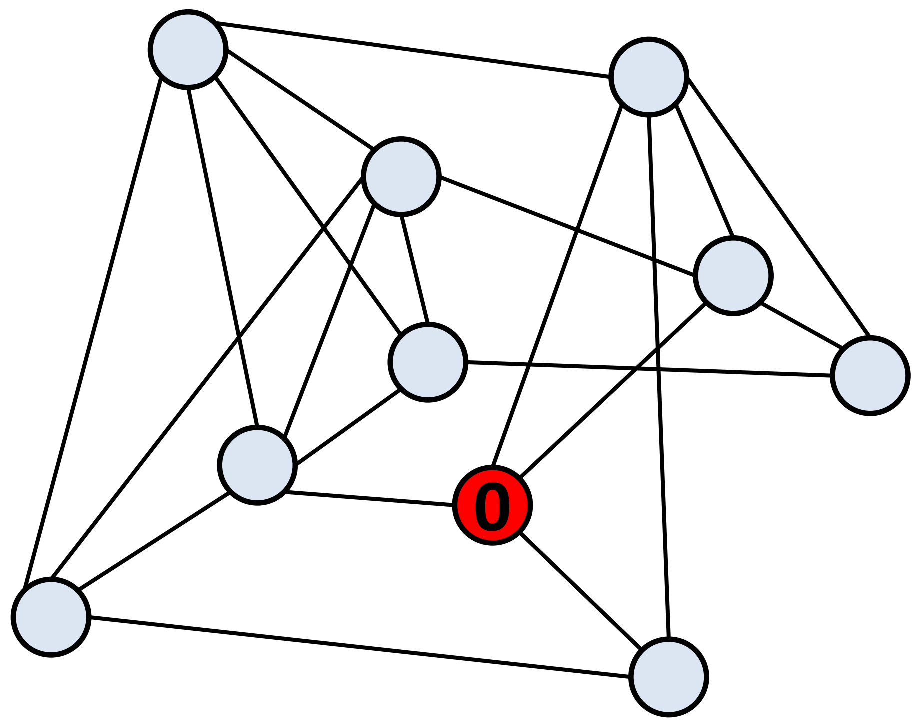


APSP in O(n)

Compute All Pairs Shortest Paths

```
For each node {
        compute distances to all other nodes;
}
```



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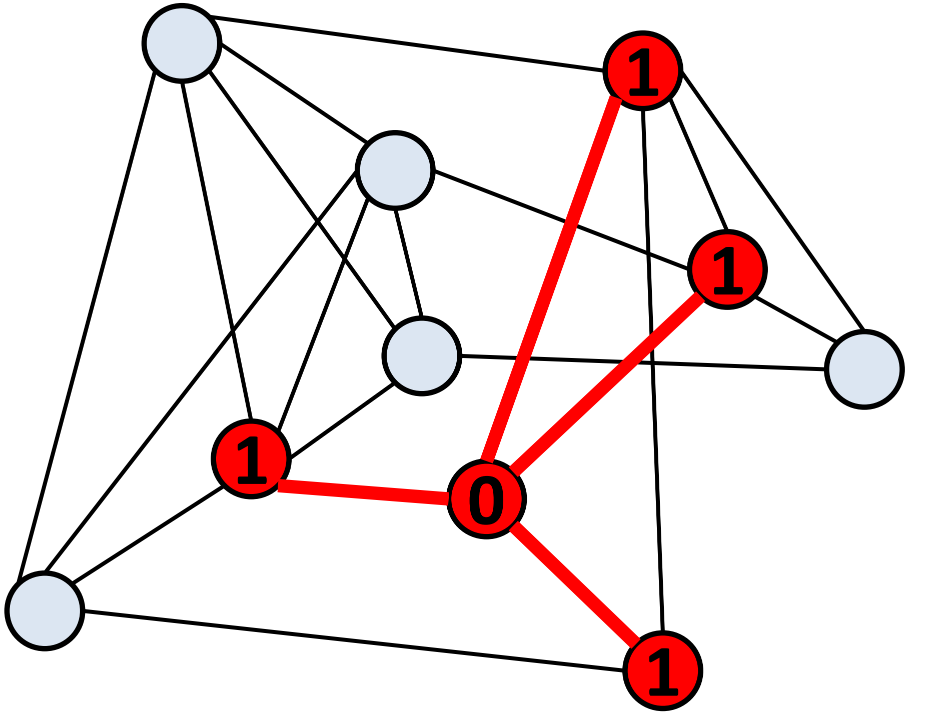

APSP in O(n)

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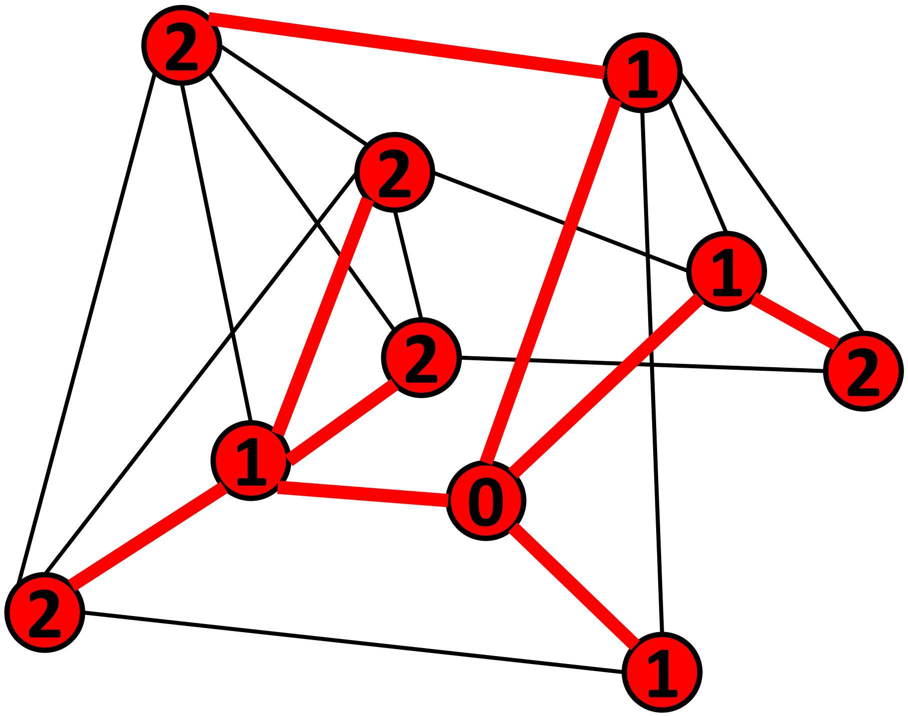


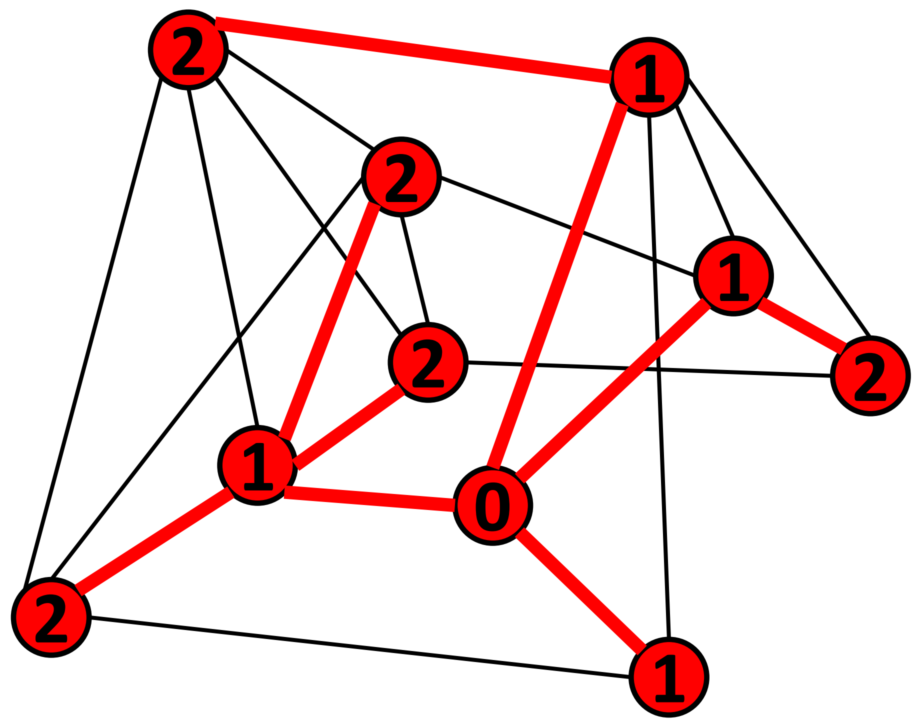
APSP in O(n)

Compute All Pairs Shortest Paths

For each node {

    compute distances to all other nodes;   O(D)

}



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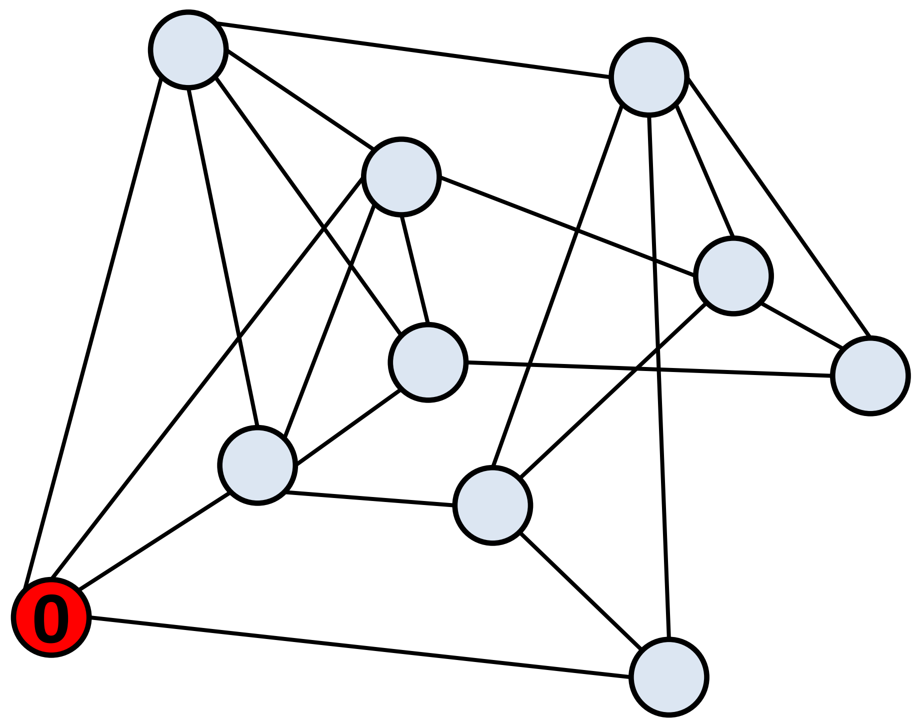

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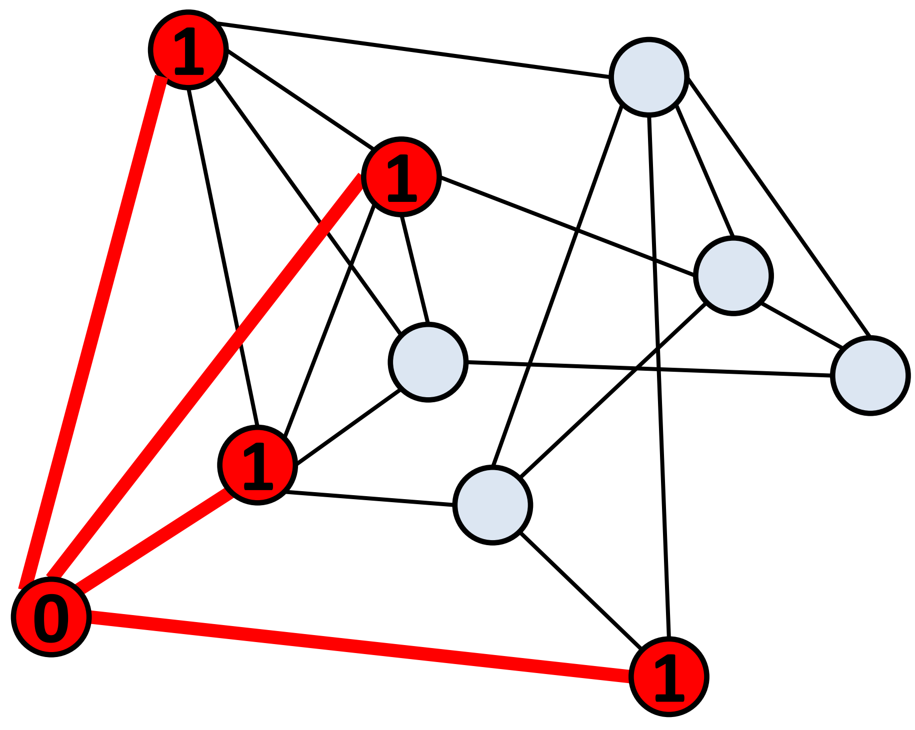
For each node {

compute distances to all other nodes; O(D)

}



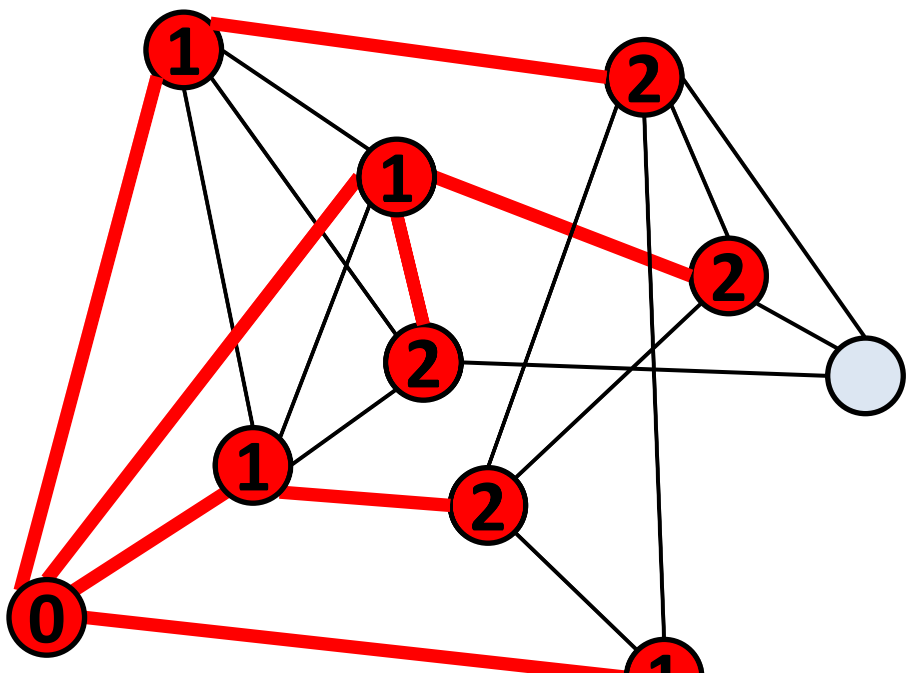
APSP in O(n)

Compute All Pairs Shortest Paths

For each node {

    compute distances to all other nodes;   O(D)

}



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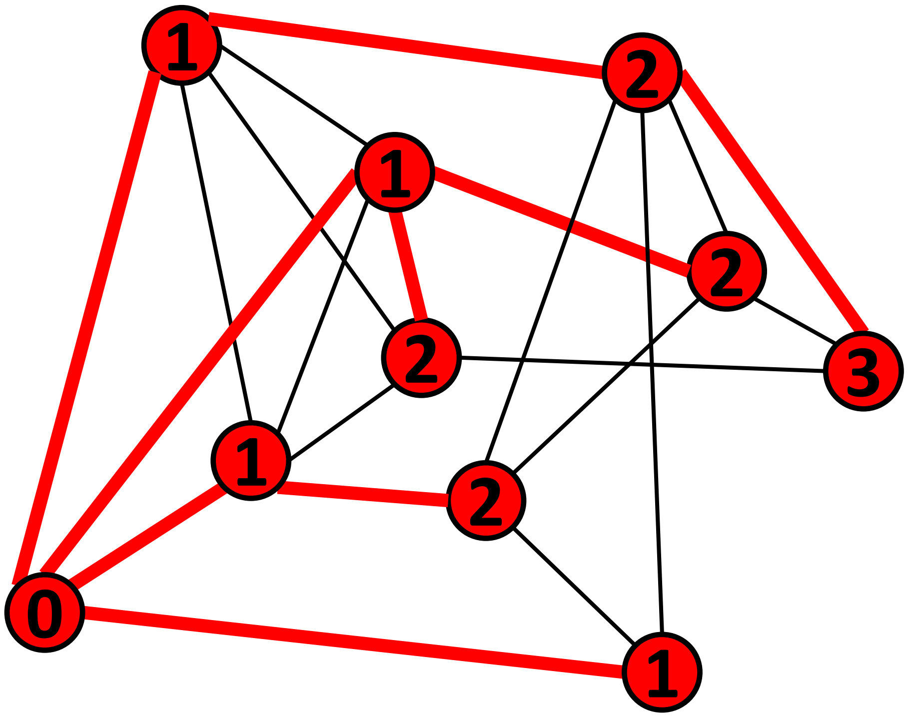

APSP in O(n)

Compute All Pairs Shortest Paths

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compute distances to all other nodes; O(D)

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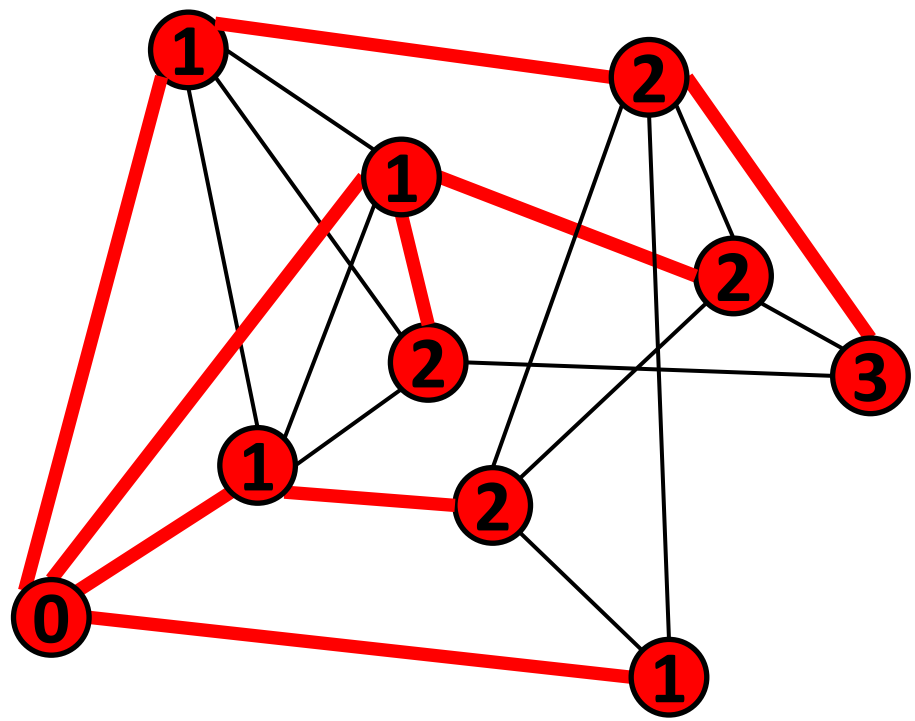


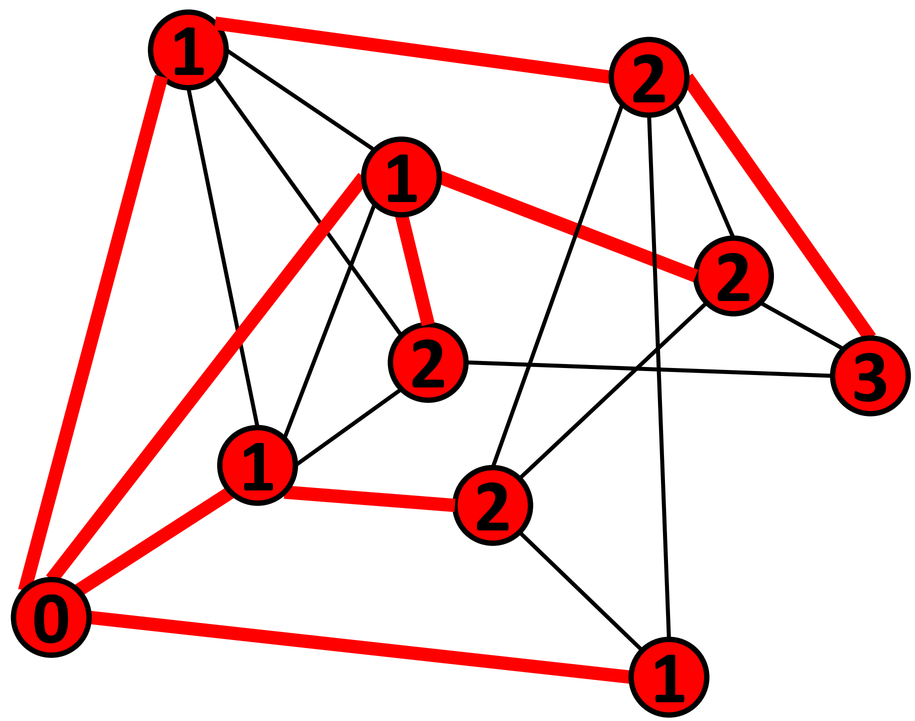
APSP in O(n)

Compute All Pairs Shortest Paths

For each node { O(n)

    compute distances to all other nodes; O(D)

} O(nD)



APSP in O(n)

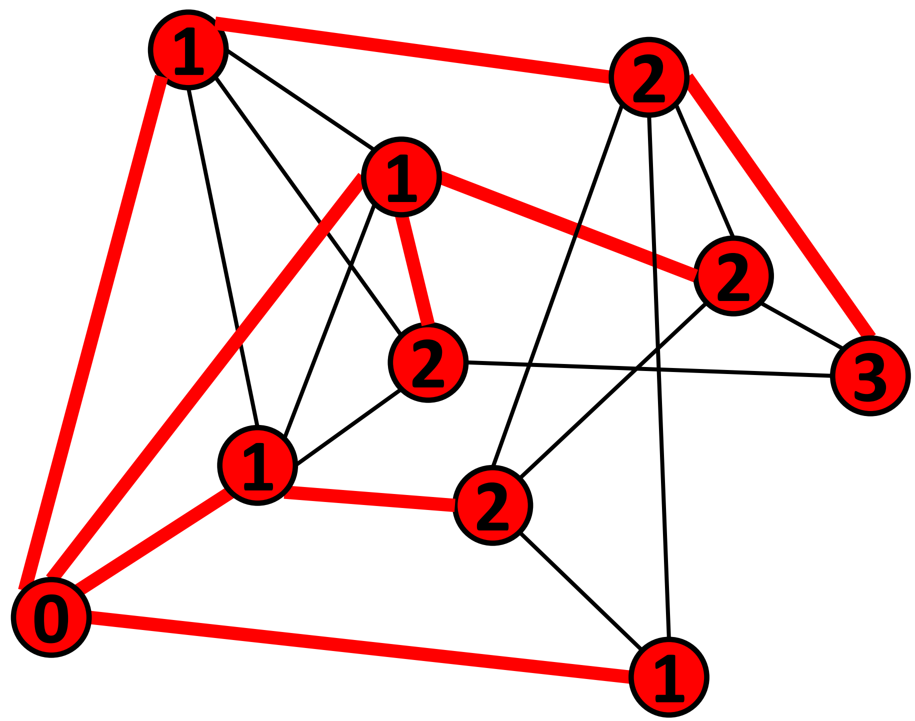
Compute All Pairs Shortest Paths

For each node { O(n)

compute distances to all other nodes; O(D)

} O(nD)

Limited parallelism:
Only some nodes active.



APSP in O(n)

Compute All Pairs Shortest Paths

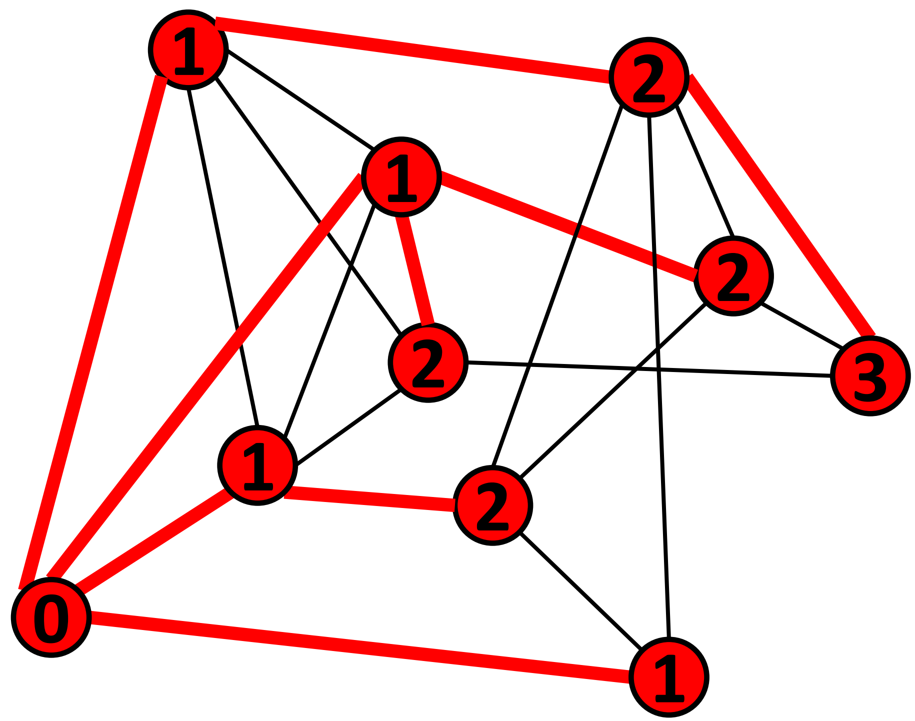
For each node { O(n)

compute distances to all other nodes; O(D)

} O(nD)

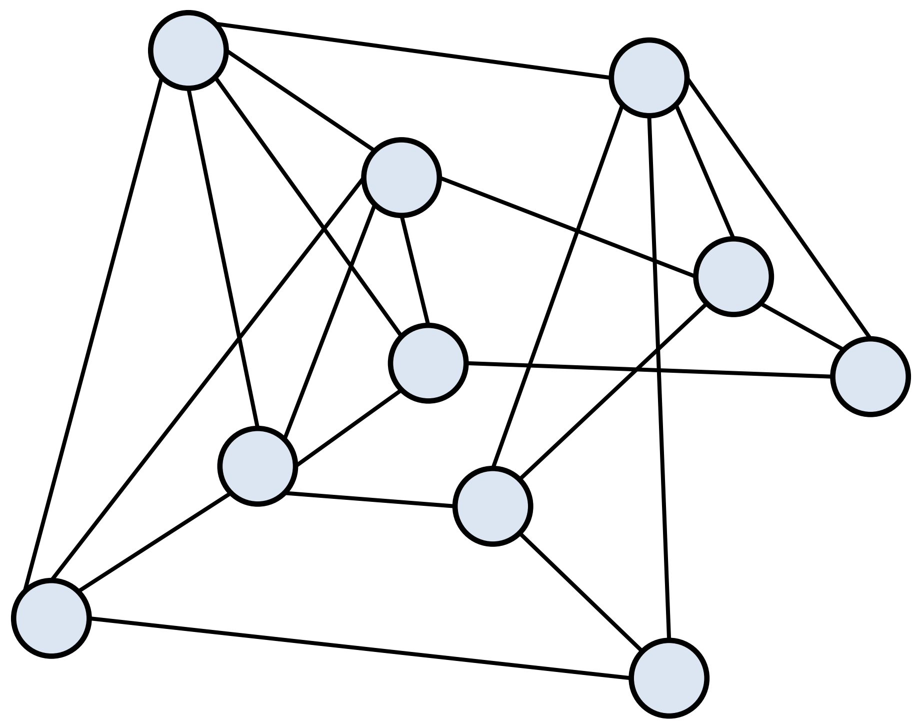
Limited parallelism:
Only some nodes active.

Wanted: All nodes active all the time!



APSP in O(n)

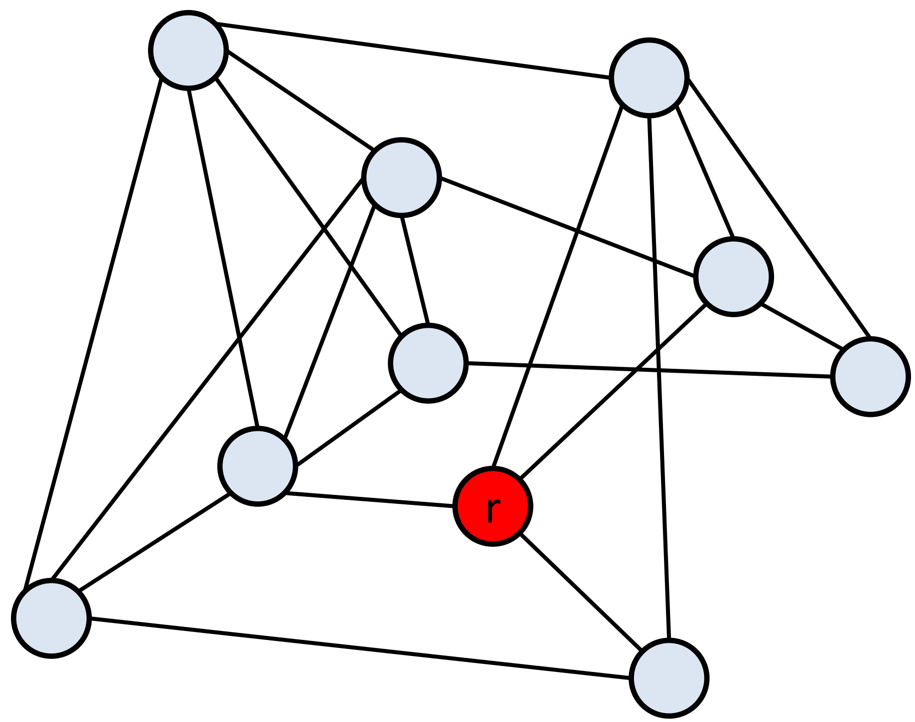
Compute All Pairs Shortest Paths



APSP in O(n)

Compute All Pairs Shortest Paths

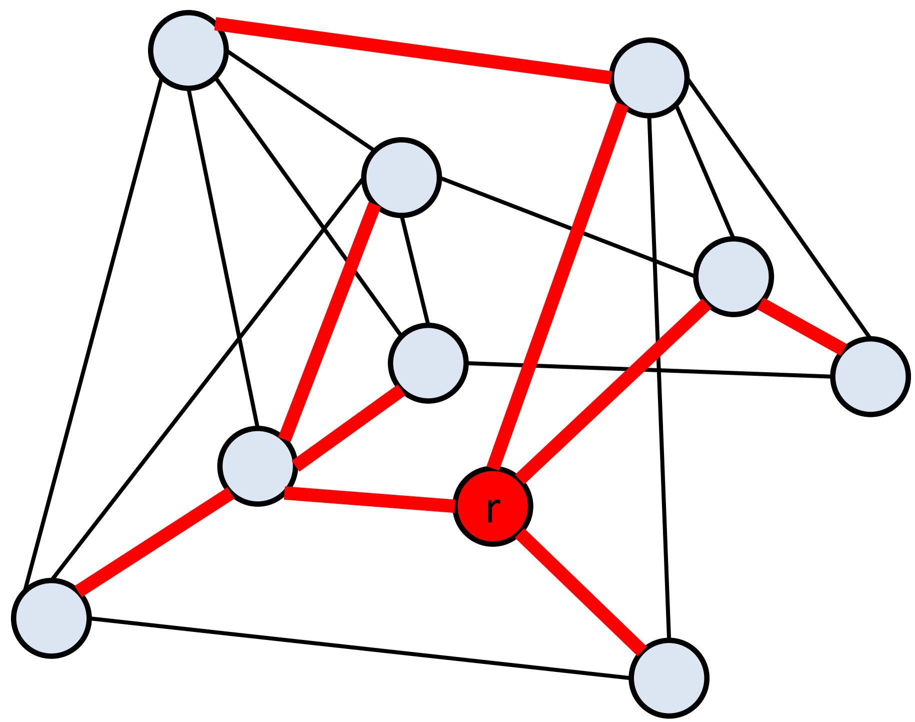
1. Pick a root-node r



APSP in O(n)

Compute All Pairs Shortest Paths

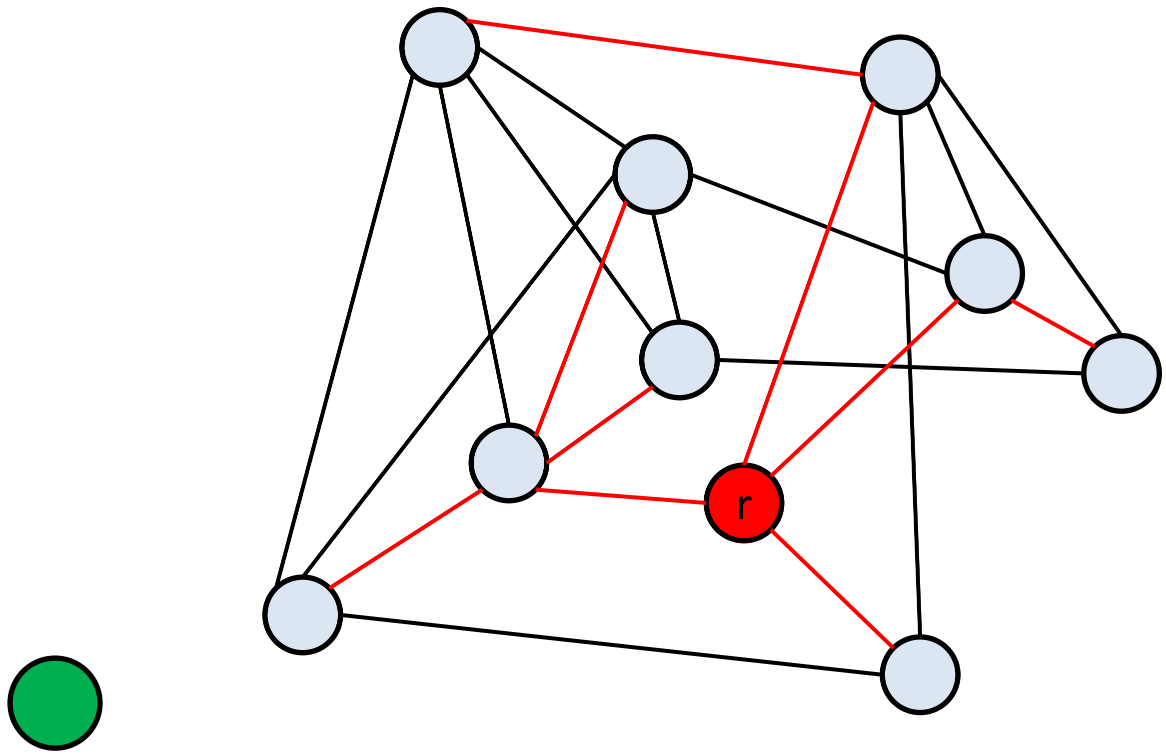
1. Pick a root-node r
2. **T** := BFS-Tree(r)



APSP in O(n)

Compute All Pairs Shortest Paths

1. Pick a root-node r;
2. **T** := BFS-Tree(r);
3. Pebble **P** traverses **T** in preorder;



APSP in O(n)

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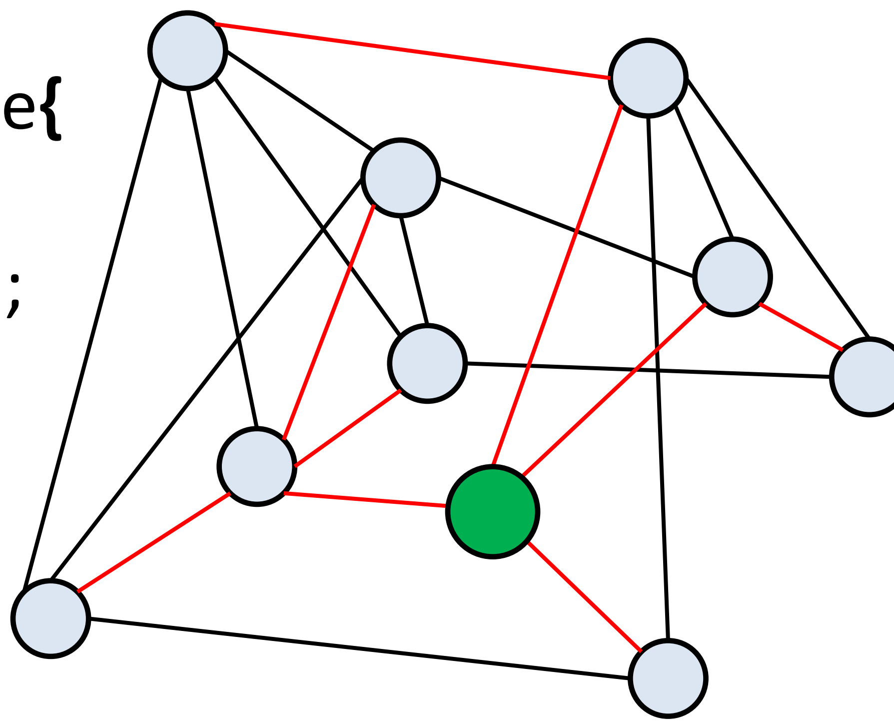

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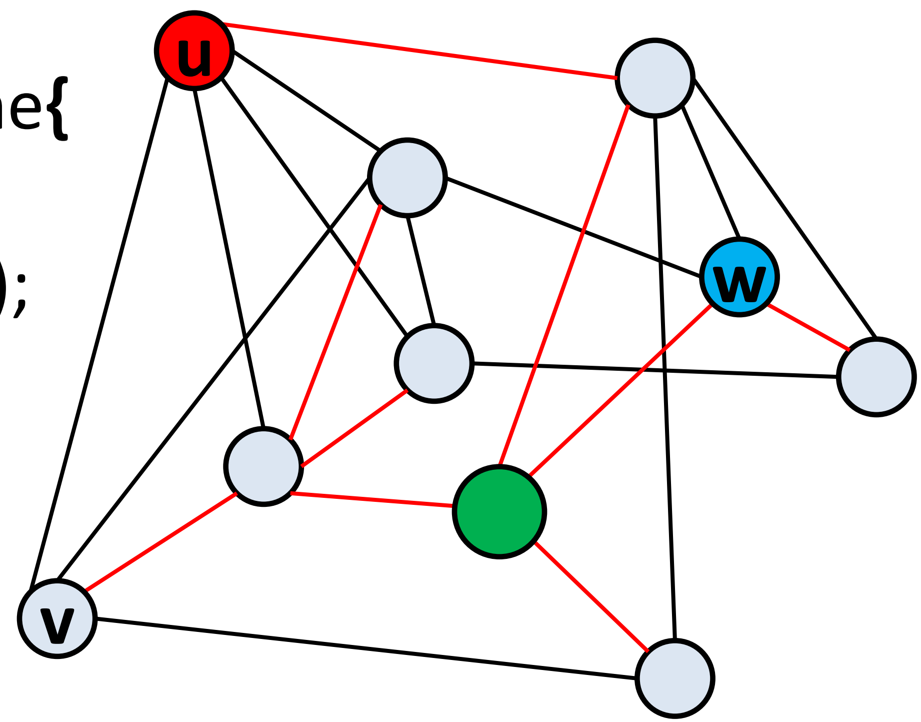

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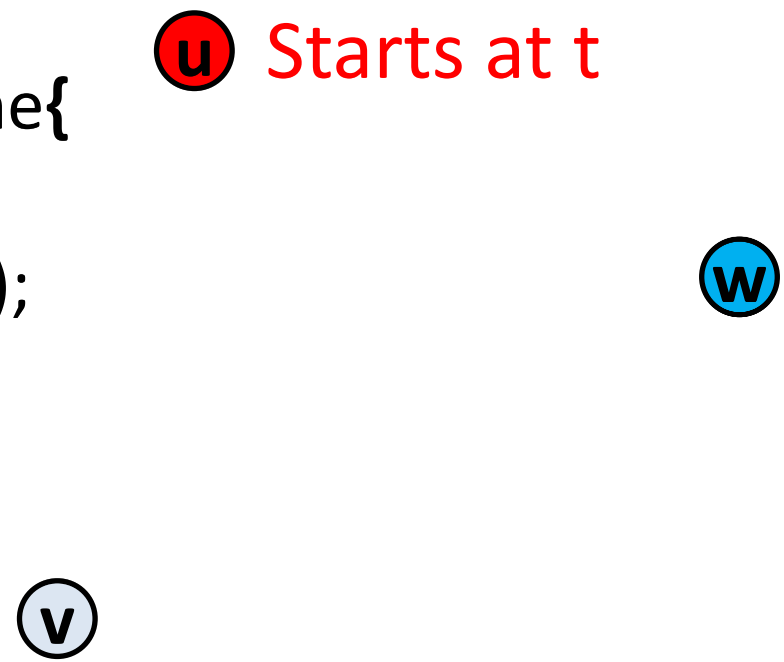

APSP in O(n)

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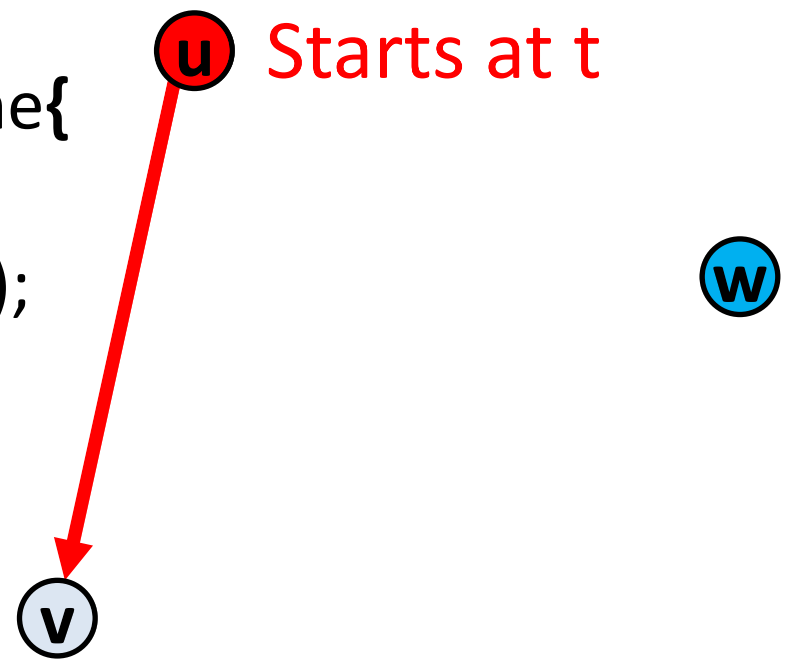

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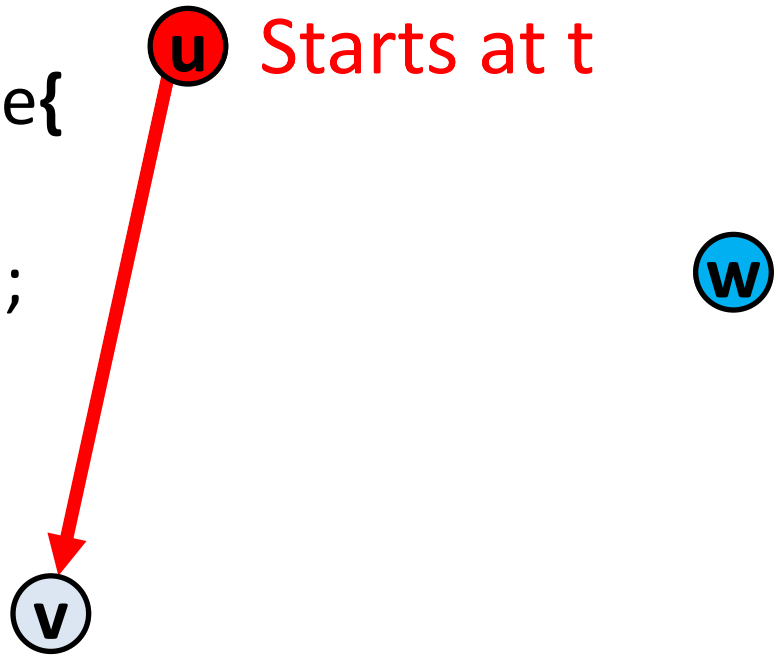

Arrives at $t + d(u, v)$

APSP in O(n)

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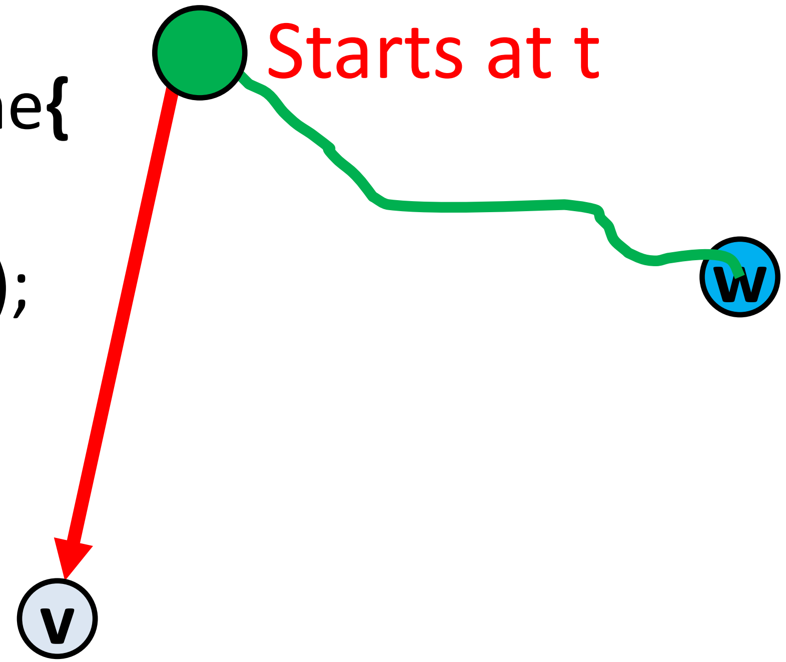

APSP in O(n)

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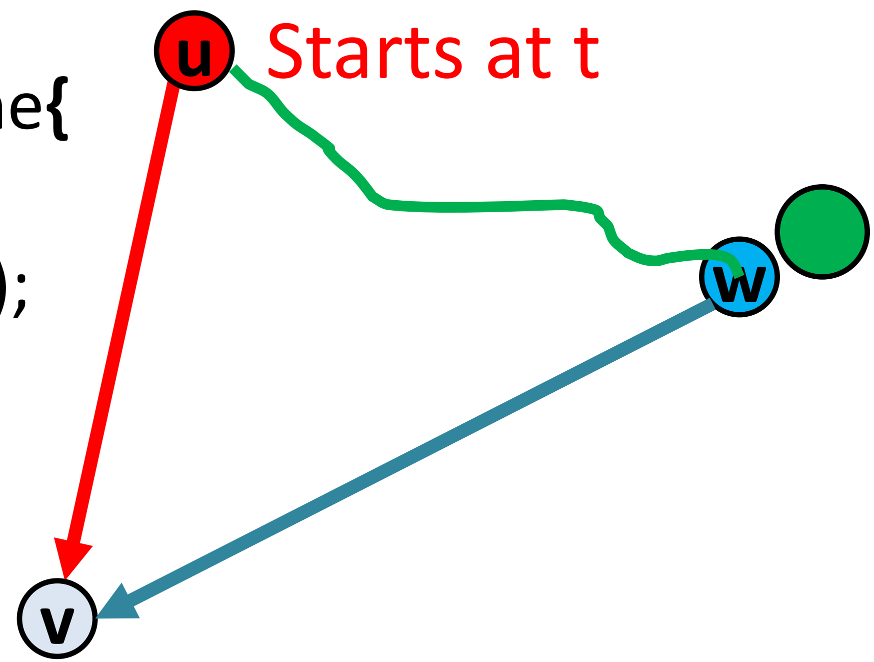

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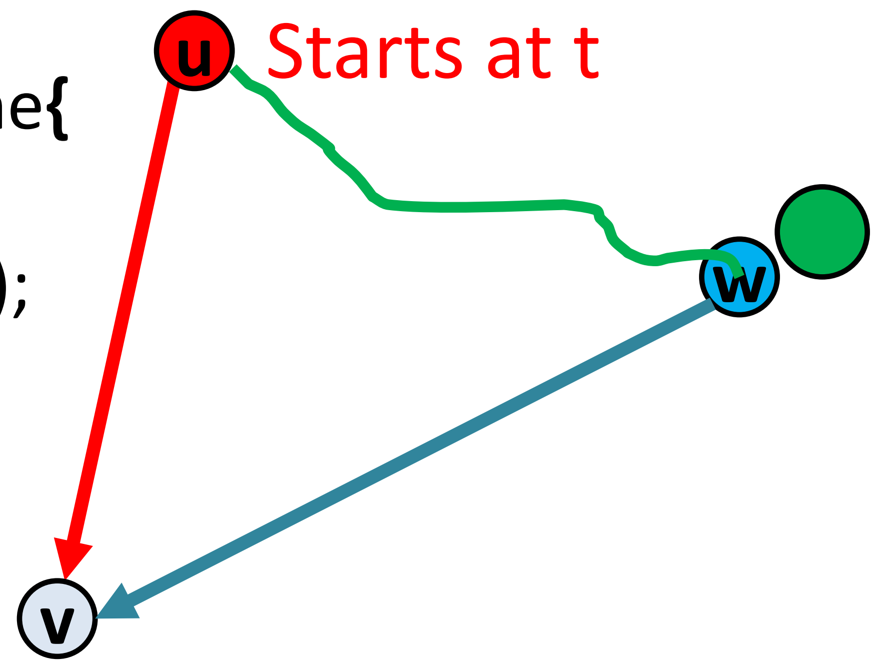

Arrives at $t + d(u, v)$

Arrives at $\geq t + d(u, v)$

APSP in O(n)

Compute All Pairs Shortest Paths

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3. Pebble **P** traverses **T** in preorder;
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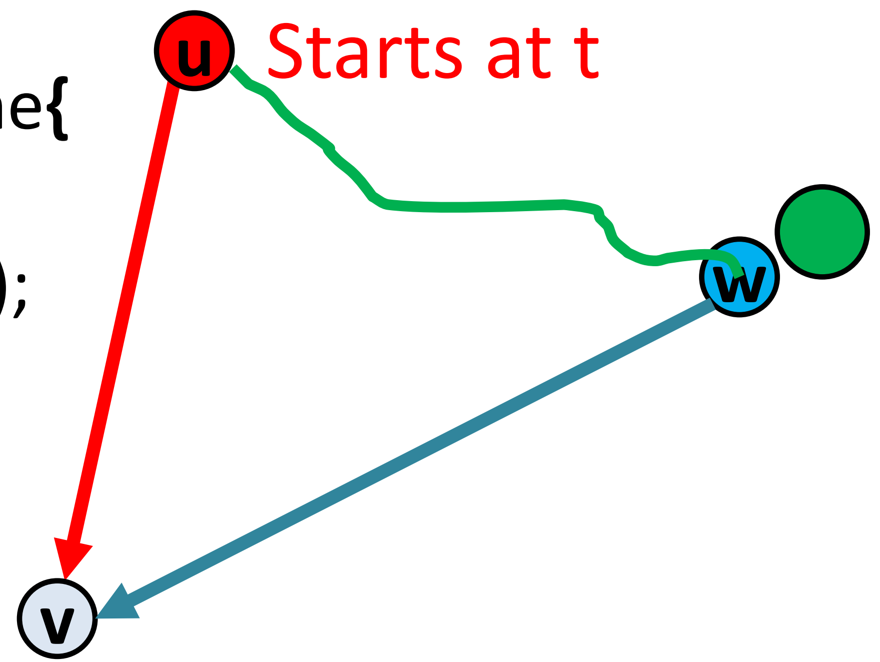
   wait 1 timeslot;
   start shortest paths(v);
   }



Arrives at $t + d(u, v)$

Arrives at $\geq t + d(u, v)$

APSP in O(n)

Compute All Pairs Shortest Paths

1. Pick a root-node r;
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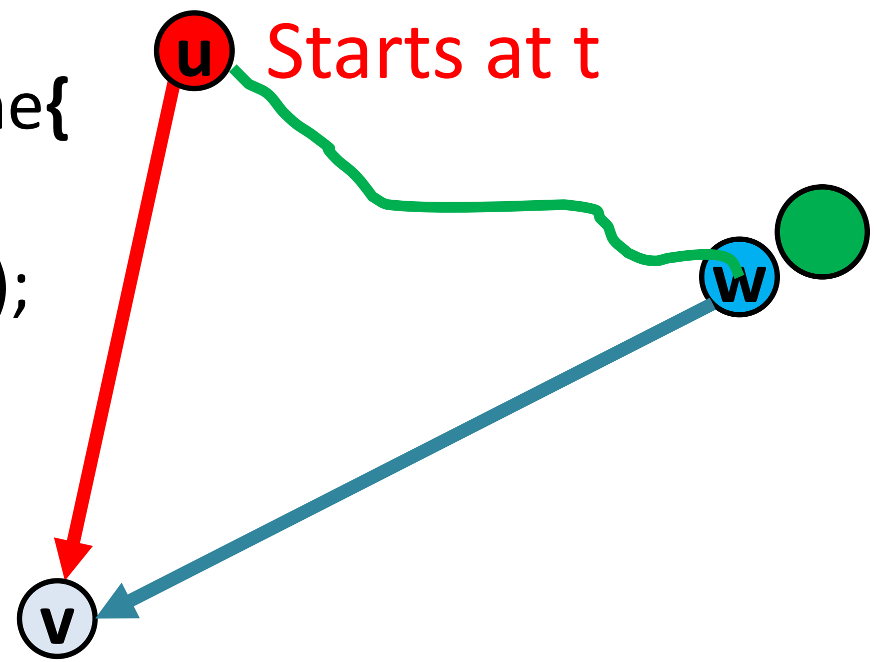

Arrives at $\quad t + d(u, v)$

Arrives at $\geq t + d(u, v)$ **+1**

APSP in O(n)

Compute All Pairs Shortest Paths

1. Pick a root-node r;
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3. Pebble **P** traverses **T** in preorder;
4. **If** P visits node v first time**{**
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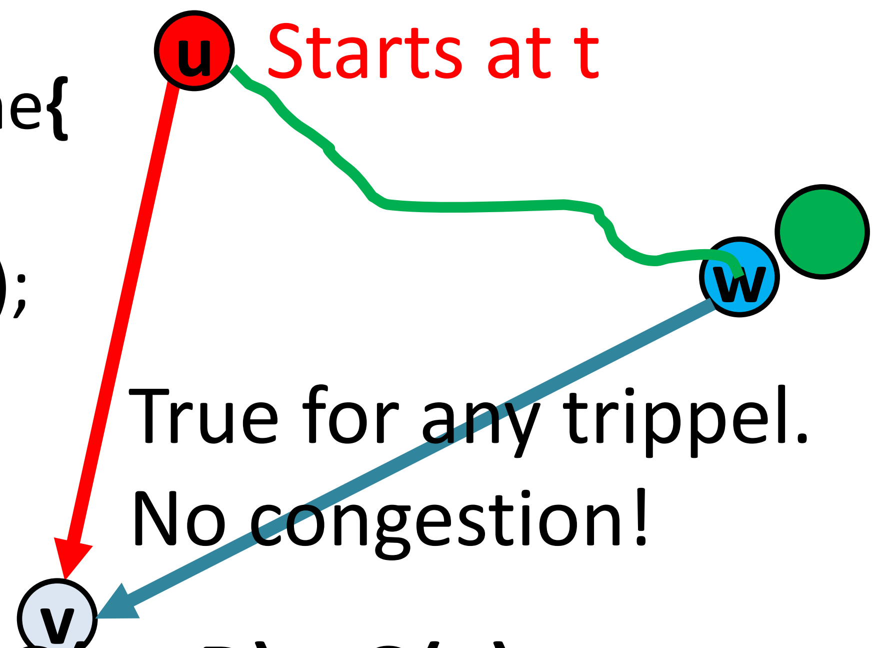


v never active for u and w at the same time!

True for any trippel.
No congestion!

Arrives at $t + d(u,v)$

Arrives at $\geq t + d(u,v)$ +1

Runtime: O(n +D) = O(n)

APSP-Application: Compute Diameter in O(n)

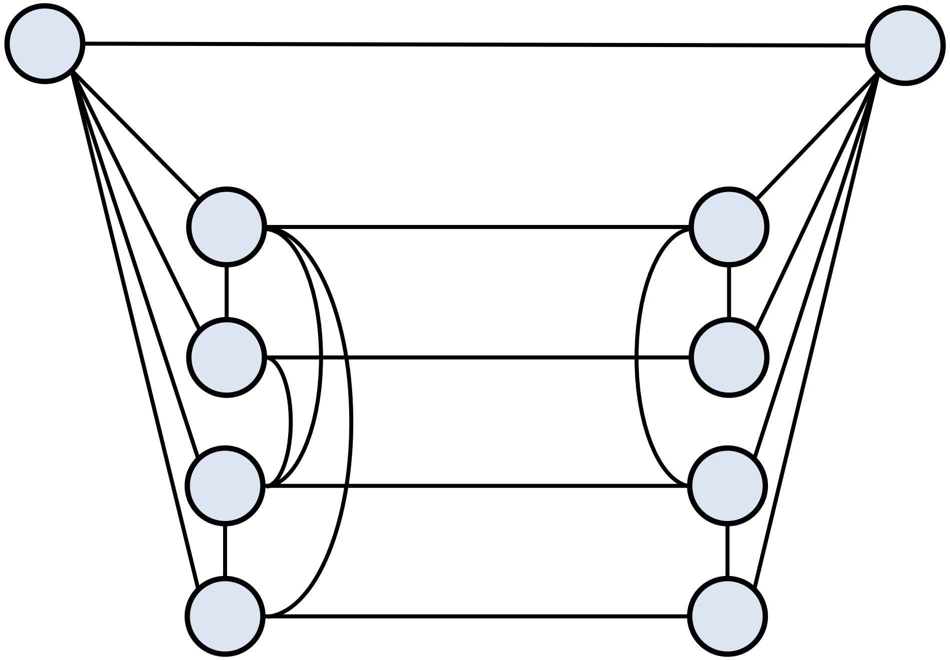
APSP-Application: Compute Diameter in O(n)

Optimal?

Diameter Lower Bound!

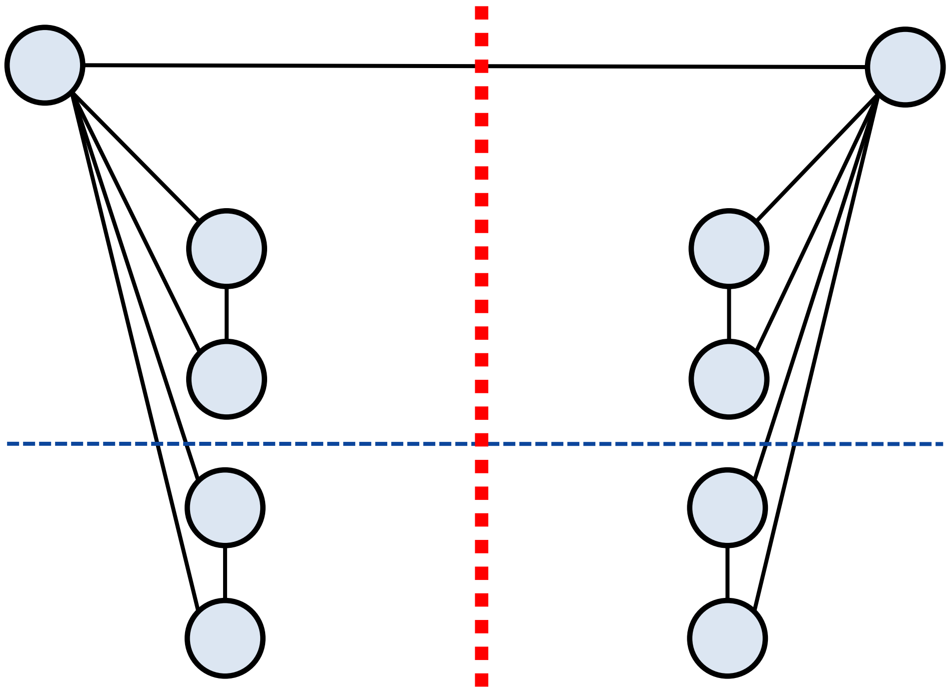
Diameter of Network?

Diameter Lower Bound!



Diameter of Network?

Diameter Lower Bound!



Diameter of Network?

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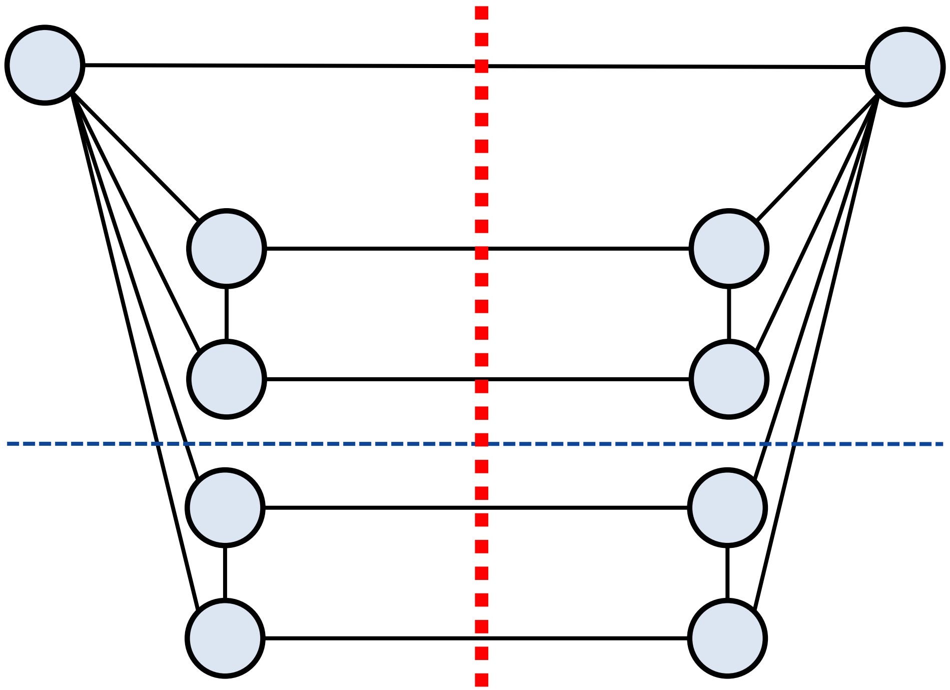
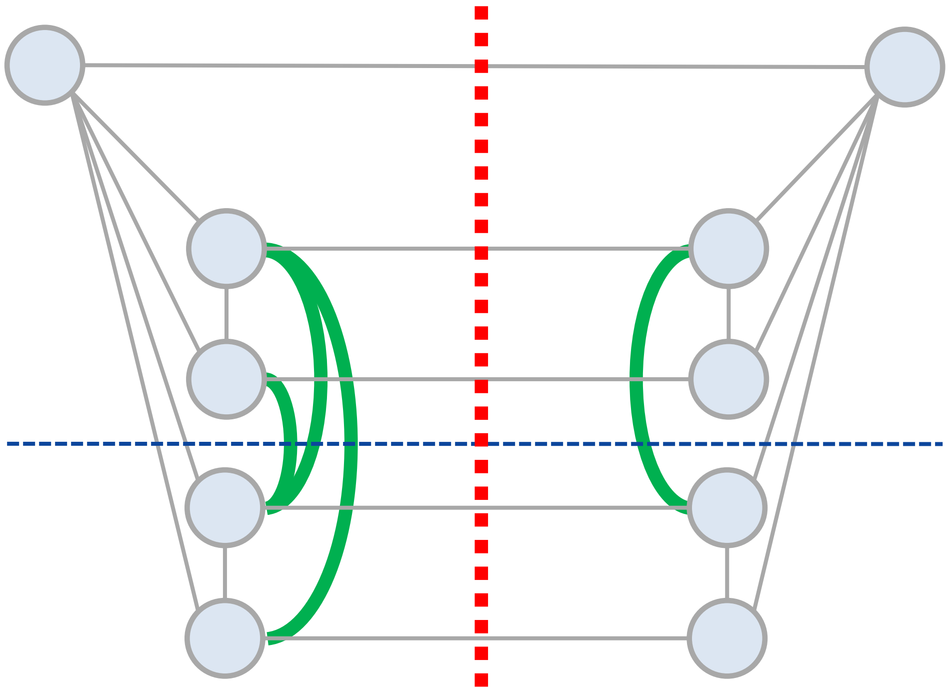


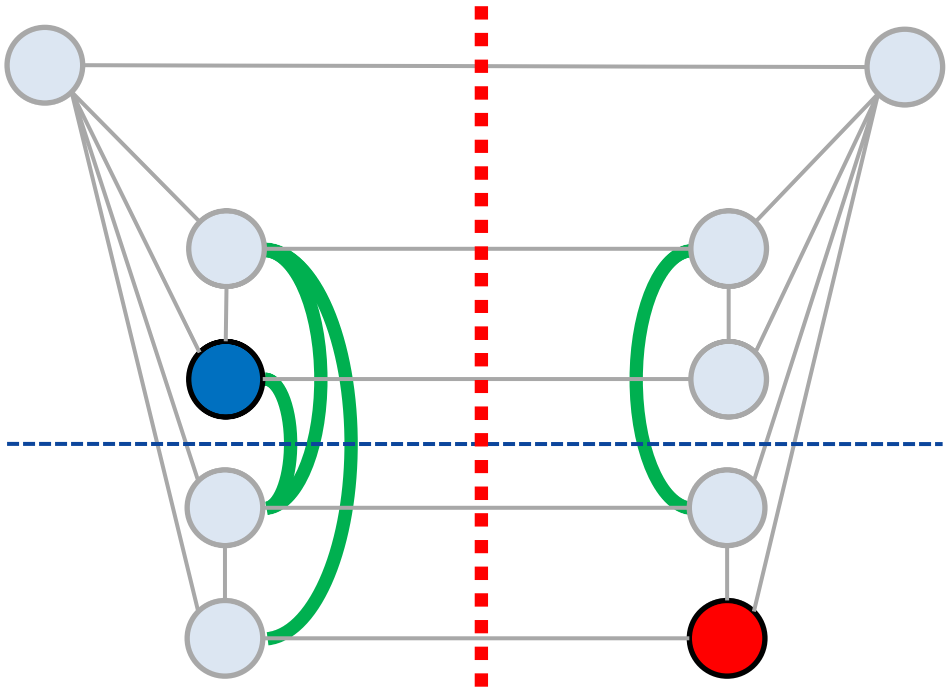
Diameter of Network?

Diameter Lower Bound!



Diameter of Network?

Diameter Lower Bound!



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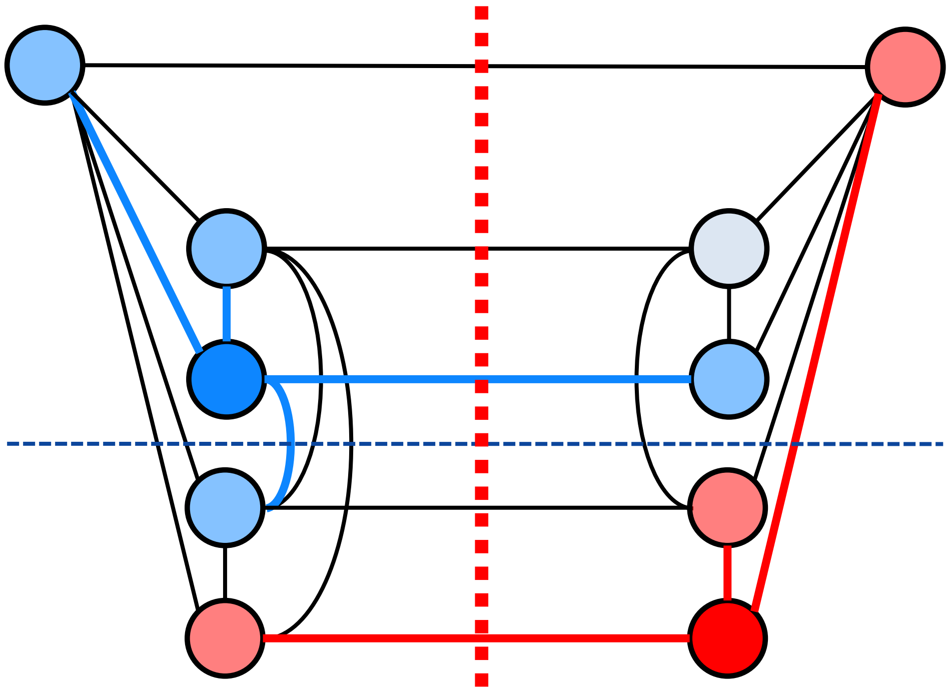

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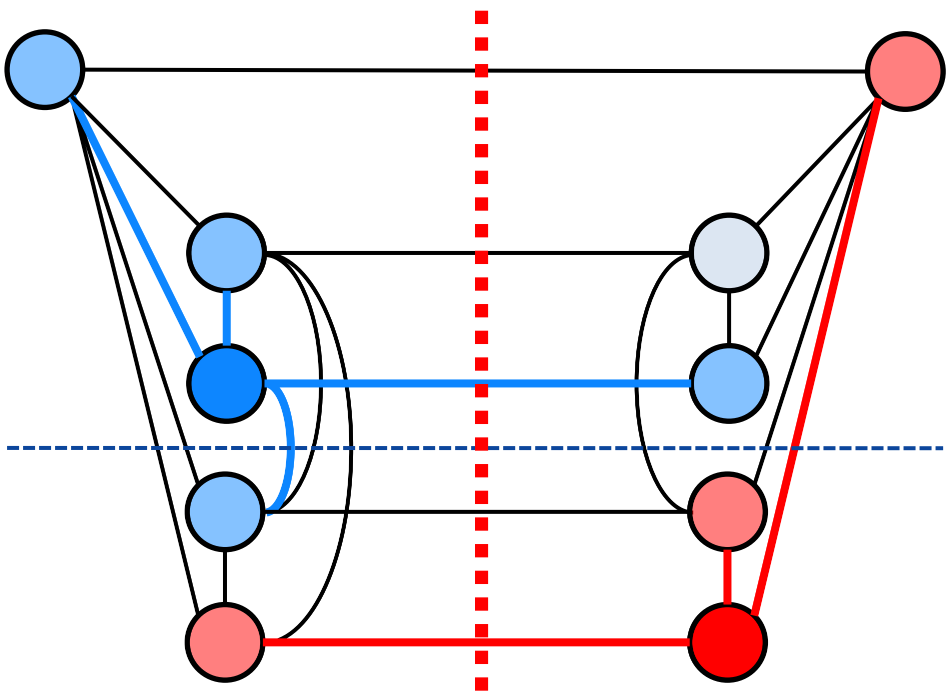
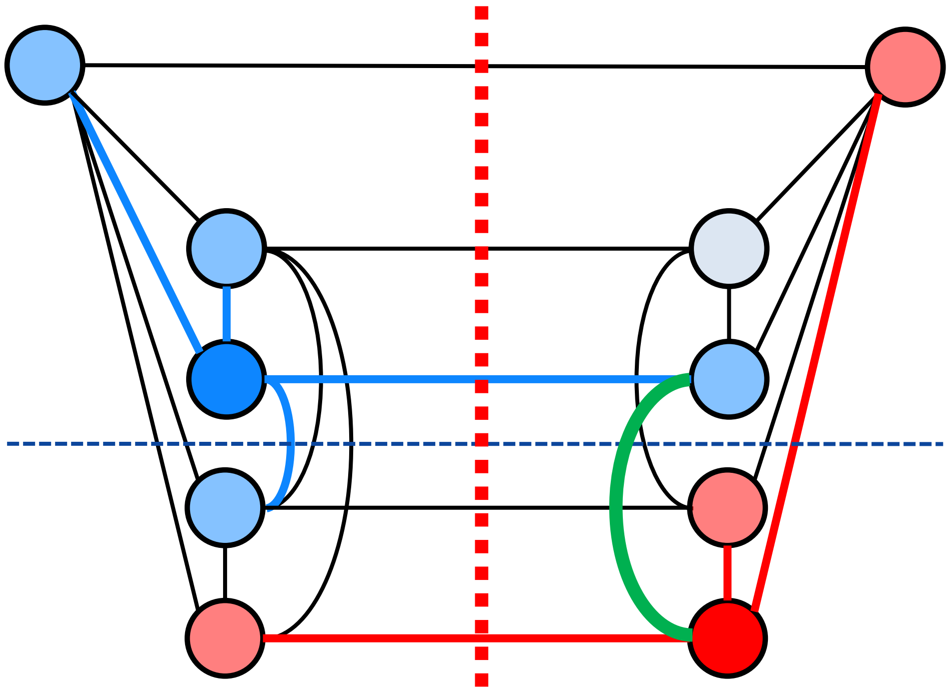


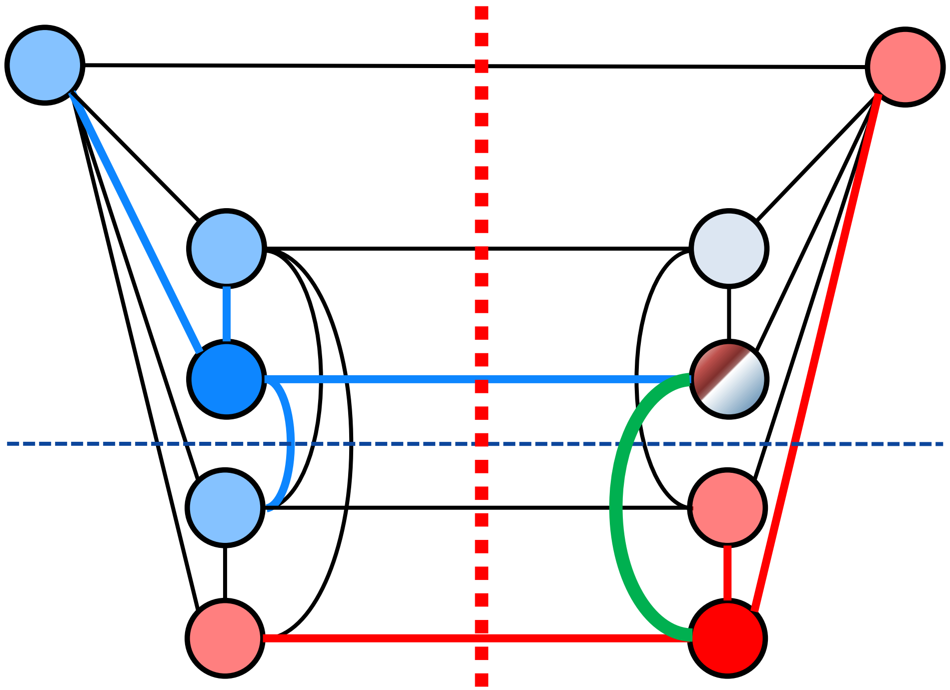
Diameter of Network?

Diameter Lower Bound!



Diameter of Network?

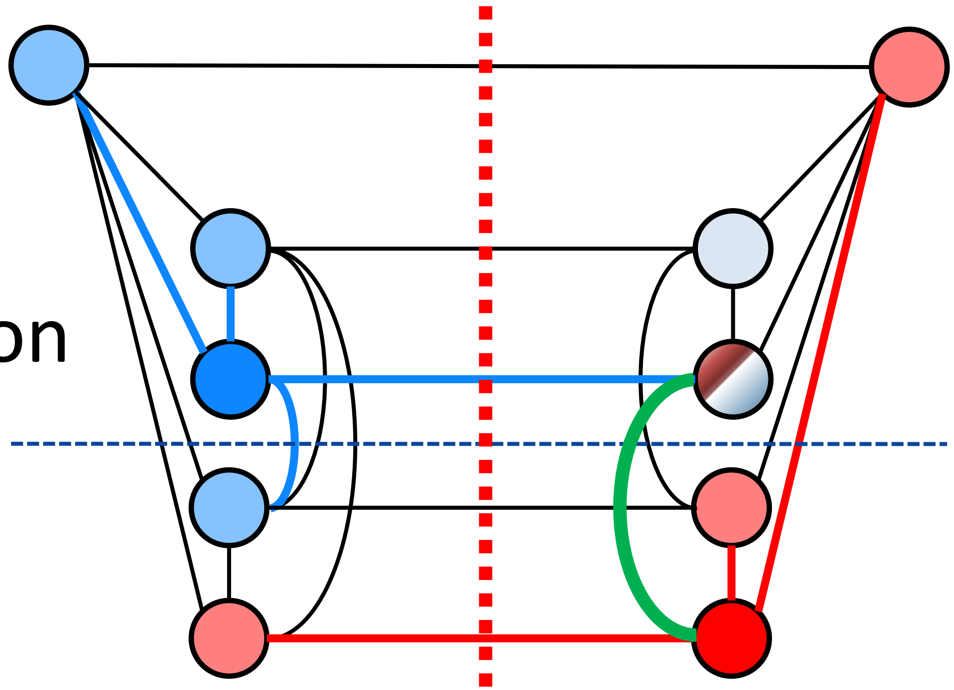
Diameter Lower Bound!



Diameter of Network?

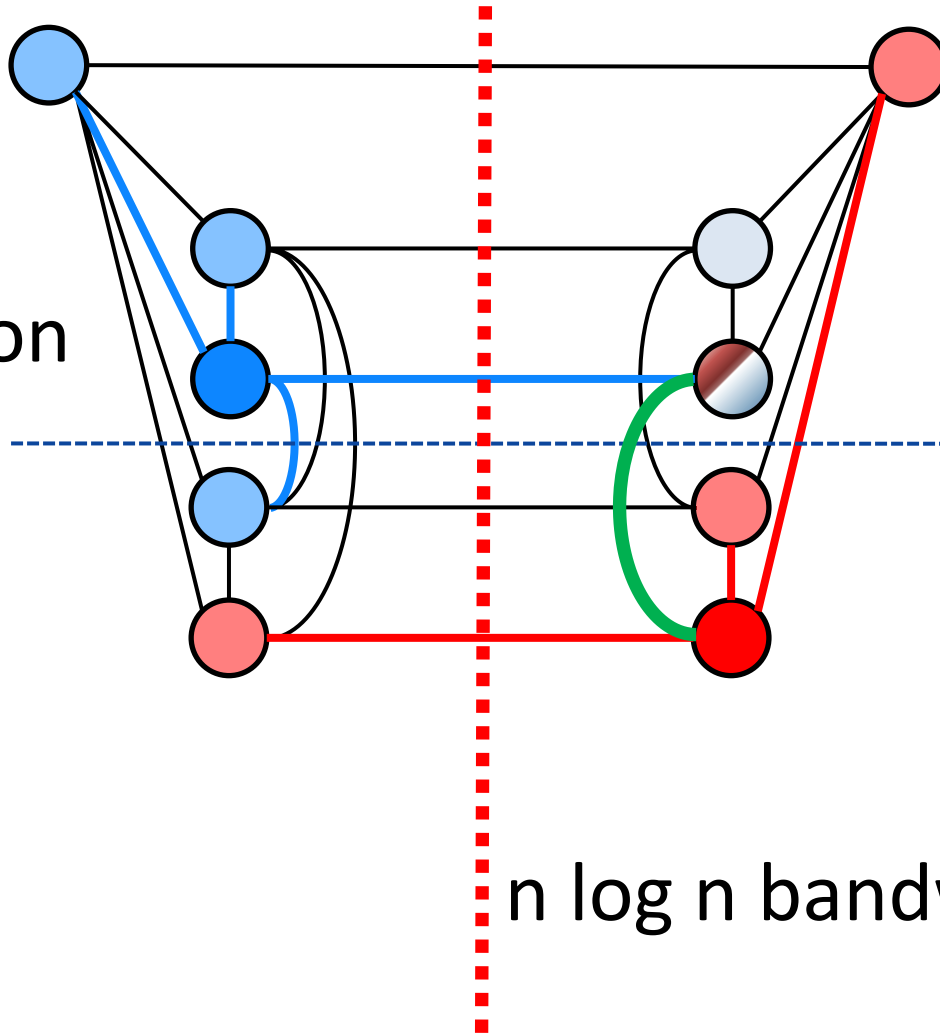
Diameter Lower Bound!

n^2 information



Diameter of Network?

Diameter Lower Bound!



Extentions

Problem	exact	$(+,1)$	$(\times,1+\varepsilon)$	$(\times,3/2-\varepsilon)$	$(\times,3/2)$	$(\times,2)$
APSP	$\tilde{\Theta}(n)$[16]	$\tilde{\Theta}(n)$[1,13]	$\tilde{\Theta}(n)$[1,13]	$\tilde{\Theta}(n)$[1,13]	–	–
eccentricity	$\tilde{\Theta}(n)$[5,11]	$\Omega(\frac{n}{D\cdot\log n}+D)$[11]	$\mathcal{O}(\frac{n}{D}+D)$[3]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[11]	–	$\Theta(D)$[18]
diameter	$\tilde{\Theta}(n)$[6,20]	$\Omega(\frac{n}{D\cdot\log n}+D)$[2]	$\mathcal{O}(\frac{n}{D}+D)$[17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[21]*	$\mathcal{O}(n^{3/4}+D)$[14]	$\Theta(D)$[18]
radius	$\mathcal{O}(n)$[8]	–	$\mathcal{O}(\frac{n}{D}+D)$[17]	–	–	$\Theta(D)$[18]
center	$\tilde{\Theta}(n)$[9,12]	$\Omega(\frac{n}{D\cdot\log n}+D)$[12]	$\mathcal{O}(\frac{n}{D}+D)$[17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[12]	–	0[19]
p. vertices	$\tilde{\Theta}(n)$[10,11]	$\Omega(\frac{n}{D\cdot\log n}+D)$[11]	$\mathcal{O}(\frac{n}{D}+D)$[17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[11]	–	0[19]
girth	$\mathcal{O}(n)$[7]	–	$\mathcal{O}\left(\min\left\{n/g+D\cdot\log\frac{D}{g}\ ,\ n\right\}\right)$[4]		–	–

For the girth, two additional ratios are of interest:

Problem	$(\times,2-\varepsilon)$	$(\times,2-1/g)$
girth	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[22]*	$\mathcal{O}\left(n^{2/3}+D\cdot\log\frac{D}{g}\right)$[15]

Extentions

Problem	exact	$(+,1)$	$(\times,1+\varepsilon)$	$(\times,3/2-\varepsilon)$	$(\times,3/2)$	$(\times,2)$
APSP	$\tilde{\Theta}(n)$[16]	$\tilde{\Theta}(n)$[1,13]	$\tilde{\Theta}(n)$[1,13]	$\tilde{\Theta}(n)$[1,13]	–	–
eccentricity	$\tilde{\Theta}(n)$[5,11]	$\Omega(\frac{n}{D\cdot\log n}+D)$[11]	$\mathcal{O}(\frac{n}{D}+D)$[3]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[11]	–	$\Theta(D)$[18]
diameter	$\tilde{\Theta}(n)$[6,20]	$\Omega(\frac{n}{D\cdot\log n}+D)$[2]	$\mathcal{O}(\frac{n}{D}+D)$[17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[21]*	$\mathcal{O}(n^{3/4}+D)$[14]	$\Theta(D)$[18]
radius	$\mathcal{O}(n)$[8]	–	$\mathcal{O}(\frac{n}{D}+D)$[17]	–	–	$\Theta(D)$[18]
center	$\tilde{\Theta}(n)$[9,12]	$\Omega(\frac{n}{D\cdot\log n}+D)$[12]	$\mathcal{O}(\frac{n}{D}+D)$[17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[12]	–	0[19]
p. vertices	$\tilde{\Theta}(n)$[10,11]	$\Omega(\frac{n}{D\cdot\log n}+D)$[11]	$\mathcal{O}(\frac{n}{D}+D)$[17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[11]	–	0[19]
girth	$\mathcal{O}(n)$[7]	–	$\mathcal{O}\left(\min\left\{n/g+D\cdot\log\frac{D}{g}\ ,\ n\right\}\right)$[4]		–	–

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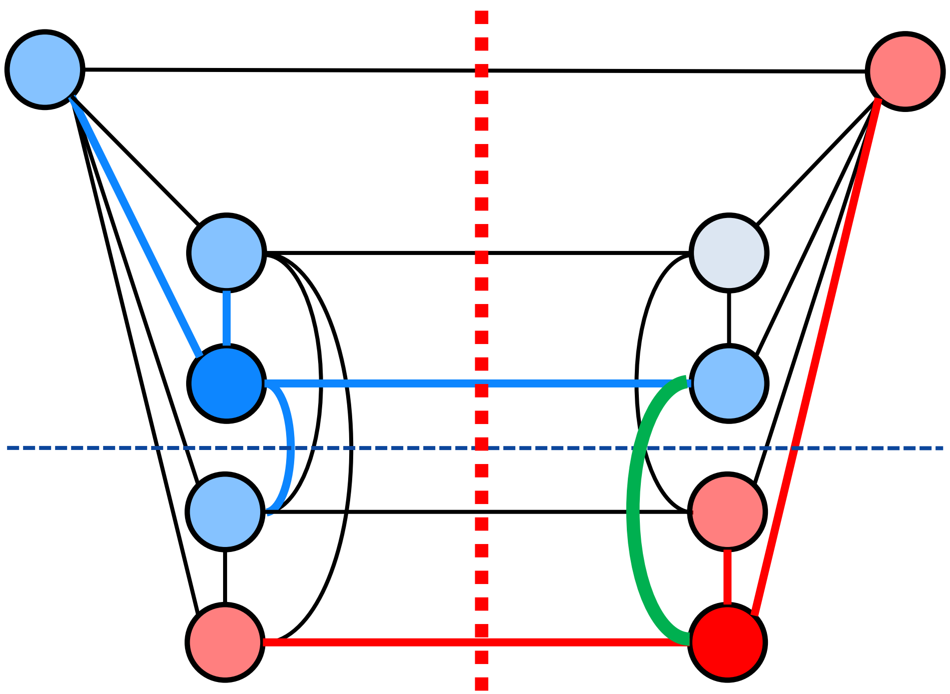
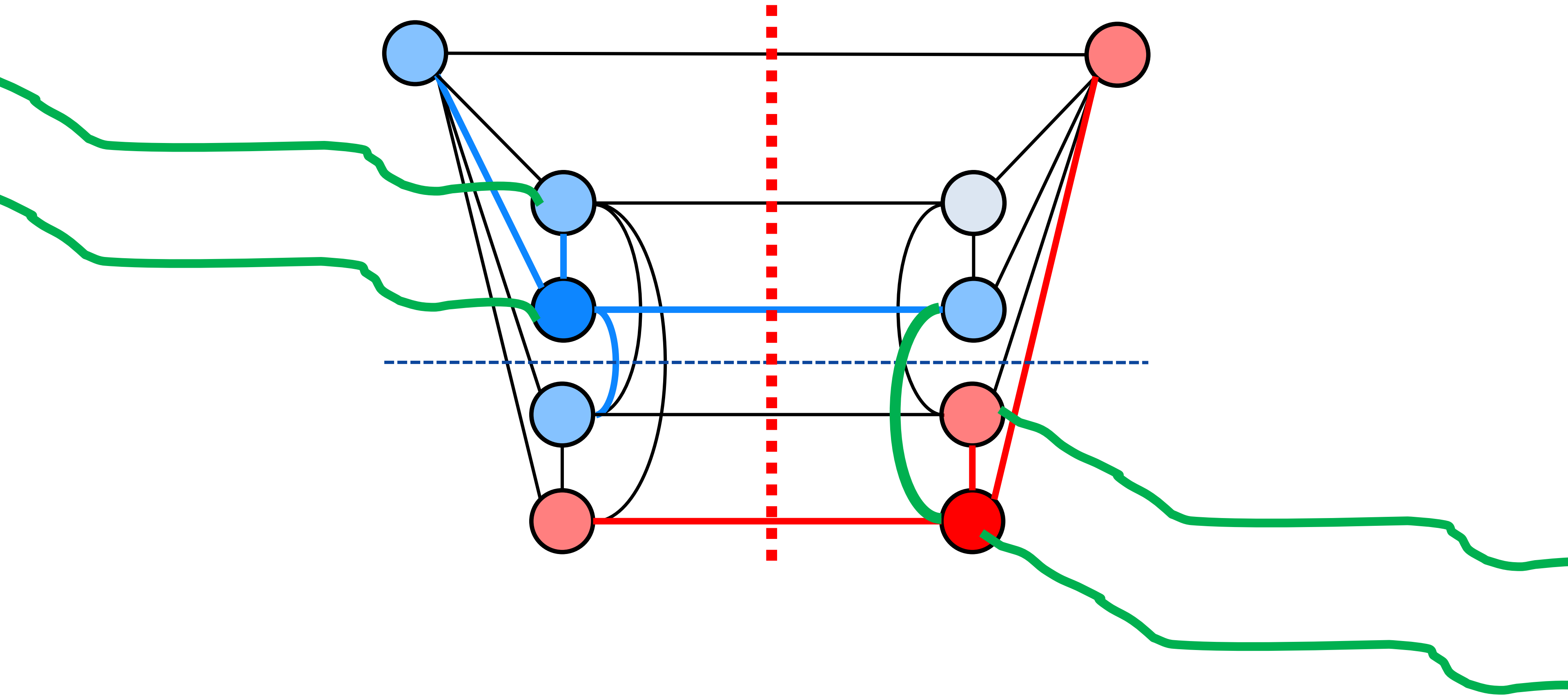
Diameter of Network?

Diameter Lower Bound!



Diameter of Network?

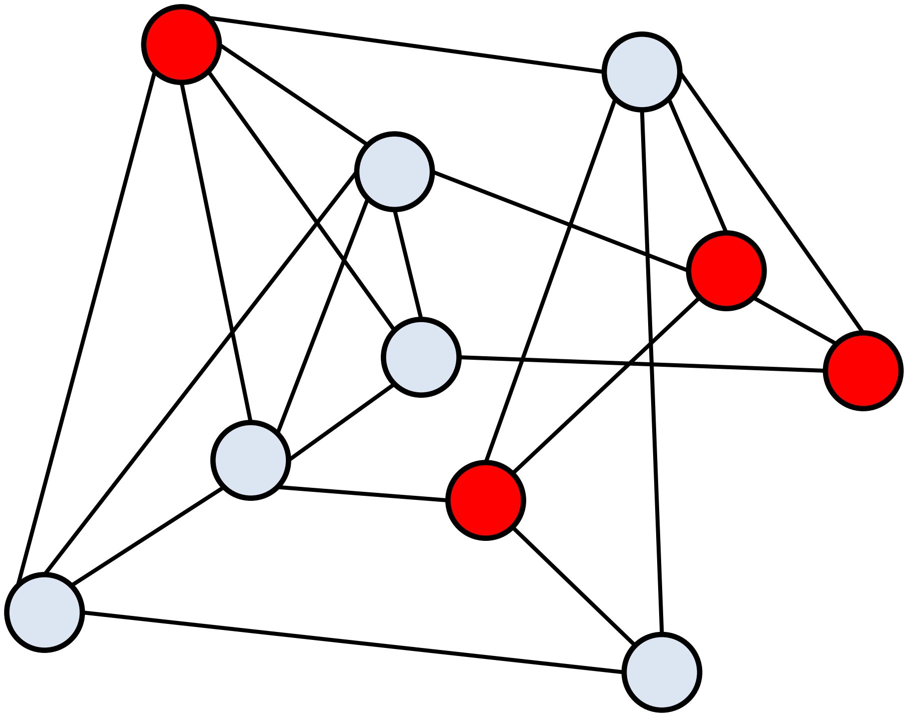
Diameter Lower Bound!



S-Shortest Path in O(|S| + D)

S-Shortest Path in O(|S| + D)

Shortest paths between S x V



S-Shortest Path in $O(|S| + D)$

(x,1+ε)-Approximating Diameter

S-Shortest Path in $O(|S| + D)$

S:= Minimum

$O(D/\varepsilon)$-Domingating Set

[Kutten, Peleg 1998]

(x,1+ε)-Approximating Diameter

S-Shortest Path in $O(|S| + D)$

S:= Minimum

$O(D/\varepsilon)$-Domingating Set

[Kutten, Peleg 1998]

Runtime: $O(n/D + D)$

(x,1+ε)-Approximating Diameter

S-Shortest Path in $O(|S| + D)$

S:= Minimum

O(D/ε)-Domingating Set

[Kutten, Peleg 1998]

Runtime: $O(n/D + D)$

Maximal Error: D/ε

(x,1+ε)-Approximating Diameter

S-Shortest Path in O(|S| + D)

S:= Minimum
O(D/ε)-Domingating Set

[Kutten, Peleg 1998]

Runtime: O(n/D + D)

Maximal Error: D/ε vs. D

Extentions

Problem	exact	$(+,1)$	$(\times,1+\varepsilon)$	$(\times,3/2-\varepsilon)$	$(\times,3/2)$	$(\times,2)$
APSP	$\tilde{\Theta}(n)^{16)}$	$\tilde{\Theta}(n)^{1,13)}$	$\tilde{\Theta}(n)^{1,13)}$	$\tilde{\Theta}(n)^{1,13)}$	–	–
eccentricity	$\tilde{\Theta}(n)^{5,11)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{11)}$	$\mathcal{O}(\frac{n}{D}+D)^{3)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{11)}$	–	$\Theta(D)^{18)}$
diameter	$\tilde{\Theta}(n)^{6,20)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{2)}$	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{21)*}$	$\mathcal{O}(n^{3/4}+D)^{14)}$	$\Theta(D)^{18)}$
radius	$\mathcal{O}(n)^{8)}$	–	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	–	–	$\Theta(D)^{18)}$
center	$\tilde{\Theta}(n)^{9,12)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{12)}$	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{12)}$	–	$0^{19)}$
p. vertices	$\tilde{\Theta}(n)^{10,11)}$	$\Omega(\frac{n}{D\cdot\log n}+D)^{11)}$	$\mathcal{O}(\frac{n}{D}+D)^{17)}$	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)^{11)}$	–	$0^{19)}$
girth	$\mathcal{O}(n)^{7)}$	–	$\mathcal{O}\left(\min\left\{n/g+D\cdot\log\frac{D}{g}, n\right\}\right)^{4)}$		–	–

For the girth, two additional ratios are of interest:

Problem	$(\times,2-\varepsilon)$	$(\times,2-1/g)$
[illegible]	[illegible] $^{22)*}$	[illegible] $^{15)}$

[Peleg, Roditty, Tal 2012]:

(x, 3/2)-approximate diameter O($\sqrt{nD}$)

Extentions

Problem	exact	$(+, 1)$	$(\times, 1+\varepsilon)$	$(\times, 3/2-\varepsilon)$	$(\times, 3/2)$	$(\times, 2)$
APSP	$\tilde{\Theta}(n)$[16)]	$\tilde{\Theta}(n)$[1,13)]	$\tilde{\Theta}(n)$[1,13)]	$\tilde{\Theta}(n)$[1,13)]	–	–
eccentricity	$\tilde{\Theta}(n)$[5,11)]	$\Omega(\frac{n}{D\cdot\log n}+D)$[11)]	$\mathcal{O}(\frac{n}{D}+D)$[3)]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[11)]	–	$\Theta(D)$[18)]
diameter	$\tilde{\Theta}(n)$[6,20)]	$\Omega(\frac{n}{D\cdot\log n}+D)$[2)]	$\mathcal{O}(\frac{n}{D}+D)$[17)]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[21)*]	$\mathcal{O}(n^{3/4}+D)$[14)]	$\Theta(D)$[18)]
radius	$\mathcal{O}(n)$[8)]	–	$\mathcal{O}(\frac{n}{D}+D)$[17)]	–	–	$\Theta(D)$[18)]
center	$\tilde{\Theta}(n)$[9,12)]	$\Omega(\frac{n}{D\cdot\log n}+D)$[12)]	$\mathcal{O}(\frac{n}{D}+D)$[17)]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[12)]	–	0[19)]
p. vertices	$\tilde{\Theta}(n)$[10,11)]	$\Omega(\frac{n}{D\cdot\log n}+D)$[11)]	$\mathcal{O}(\frac{n}{D}+D)$[17)]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$[11)]	–	0[19)]
girth	$\mathcal{O}(n)$[7)]	–	$\mathcal{O}\left(\min\left\{n/g+D\cdot\log\frac{D}{g}, n\right\}\right)$[4)]		–	–

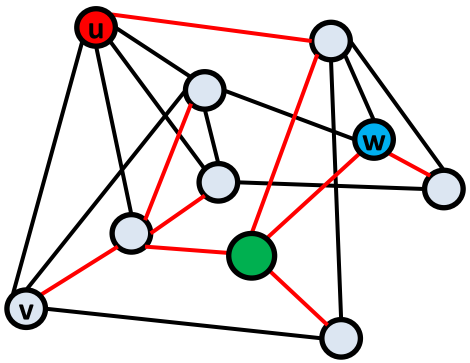
For the girth, two additional ratios are of interest:

Problem	$(\times, 2-\varepsilon)$	$(\times, 2-1/g)$
	[22)*]	[15)]

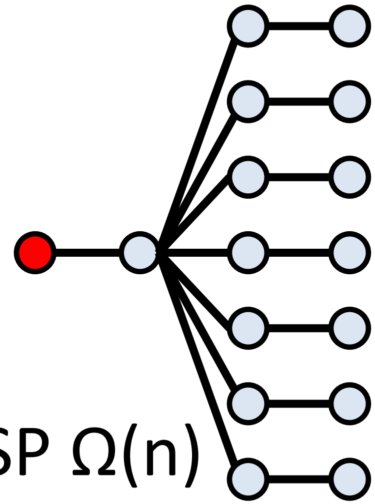
[Peleg, Roditty, Tal 2012]:

(x, 3/2)-approximate diameter $O(\sqrt{nD})$

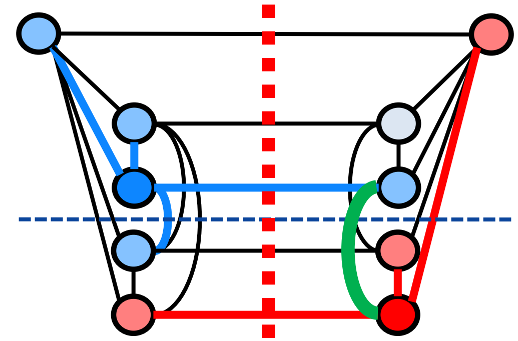
Summary



APSP O(n)



APSP Ω(n)



Diameter Θ(n)

Problem	exact	$(+,1)$	$(\times,1+\varepsilon)$	$(\times,3/2-\varepsilon)$	$(\times,3/2)$	$(\times,2)$
APSP	$\tilde\Theta(n)$ [16]	$\tilde\Theta(n)$ [1,13]	$\tilde\Theta(n)$ [1,13]	$\tilde\Theta(n)$ [1,13]	–	–
eccentricity	$\tilde\Theta(n)$ [5,11]	$\Omega(\frac{n}{D\cdot\log n}+D)$ [11]	$\mathcal{O}(\frac{n}{D}+D)$ [3]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$ [11]	–	$\Theta(D)$ [18]
diameter	$\tilde\Theta(n)$ [6,20]	$\Omega(\frac{n}{D\cdot\log n}+D)$ [2]	$\mathcal{O}(\frac{n}{D}+D)$ [17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$ [21]*	$\mathcal{O}(n^{3/4}+D)$ [14]	$\Theta(D)$ [18]
radius	$\mathcal{O}(n)$ [8]	–	$\mathcal{O}(\frac{n}{D}+D)$ [17]	–	–	$\Theta(D)$ [18]
center	$\tilde\Theta(n)$ [9,12]	$\Omega(\frac{n}{D\cdot\log n}+D)$ [12]	$\mathcal{O}(\frac{n}{D}+D)$ [17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$ [12]	–	0 [19]
p. vertices	$\tilde\Theta(n)$ [10,11]	$\Omega(\frac{n}{D\cdot\log n}+D)$ [11]	$\mathcal{O}(\frac{n}{D}+D)$ [17]	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$ [11]	–	0 [19]
girth	$\mathcal{O}(n)$ [7]	–	$\mathcal{O}\left(\min\left\{n/g+D\cdot\log\frac{D}{g},\ n\right\}\right)$ [4]		–	–

For the girth, two additional ratios are of interest:

Problem	$(\times,2-\varepsilon)$	$(\times,2-1/g)$
girth	$\Omega\left(\frac{\sqrt{n}}{\log n}+D\right)$ [22]*	$\mathcal{O}\left(n^{2/3}+D\cdot\log\frac{D}{g}\right)$ [15]

S-Shortest Paths O(|S| + D)

Thanks!