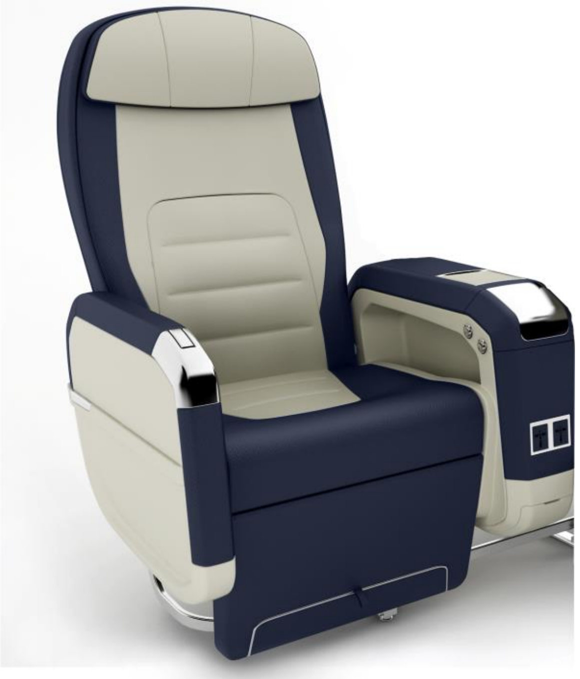
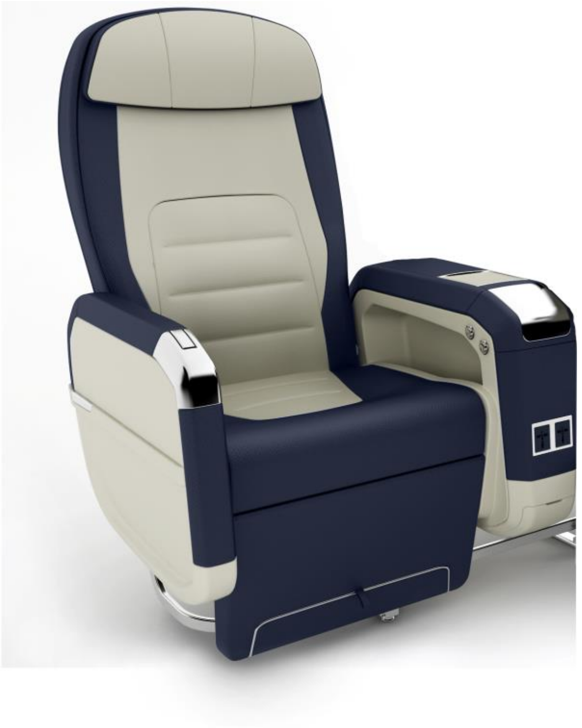
Clairvoyant Mechanisms for Online Auctions

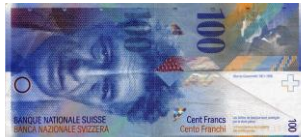
Philipp Brandes, Zengfeng Huang, Hsin-Hao Su and Roger Wattenhofer

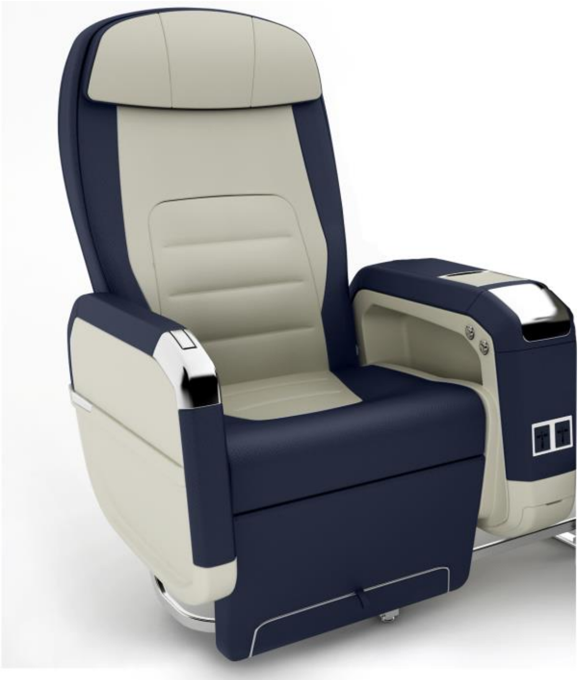
Online Auctions

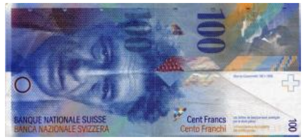


Valuation




100
BANQUE NATIONALE SUISSE
BANCA NAZIONALE SVIZZERA
Cent Francs
Cento Franchi
100

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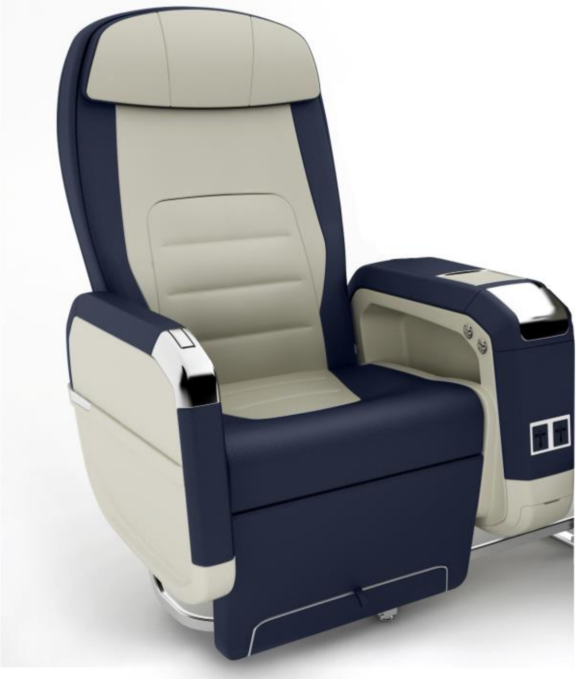
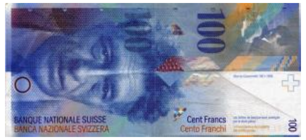
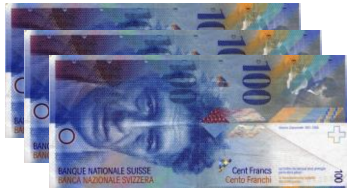
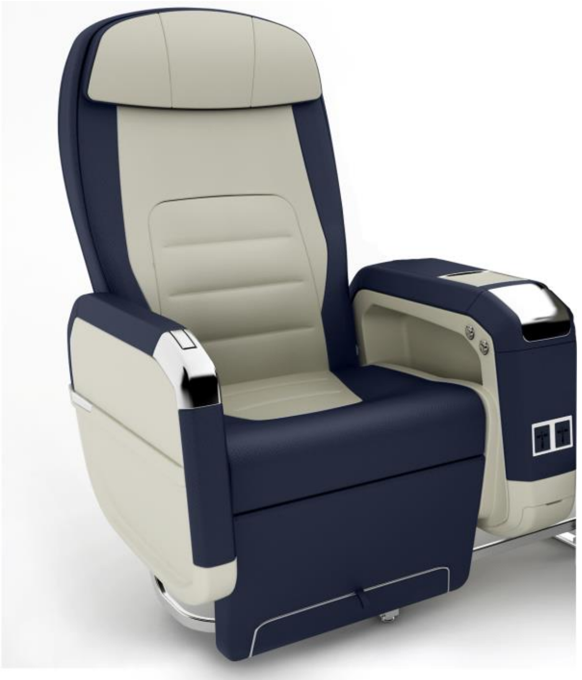
LIBERTAS
AEQUITAS
VERITAS
2009
IN CRYPTOGRAPHY WE TRUST

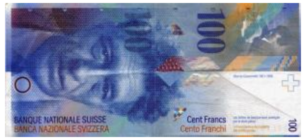






Preemption Price

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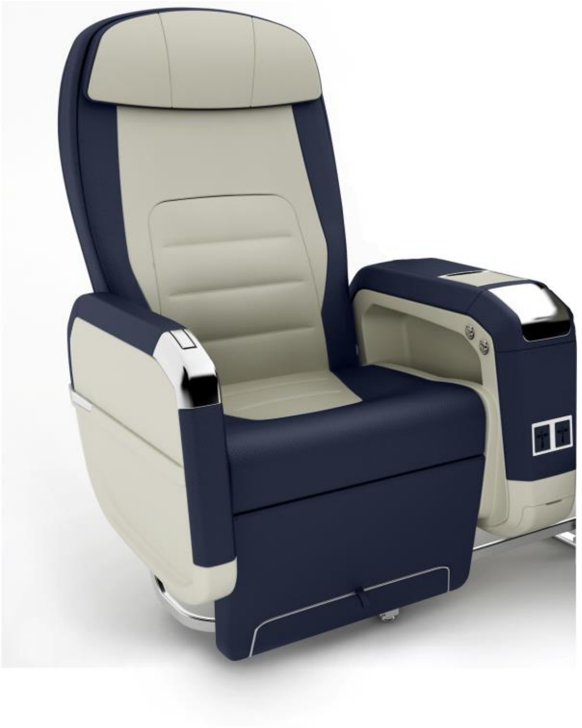

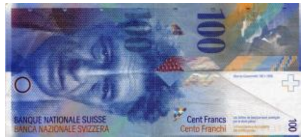
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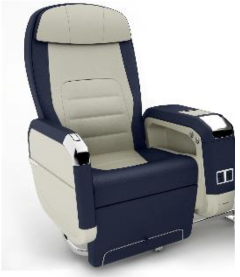
2009
IN CRYPTOGRAPHY WE TRUST

What if we knew the future?



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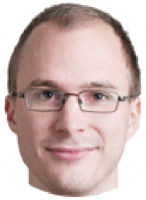
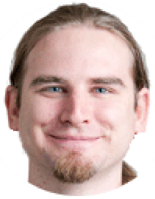
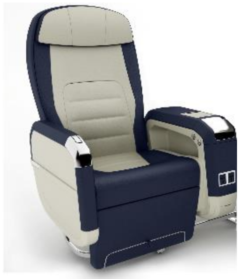
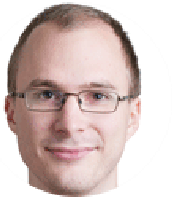
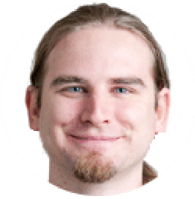
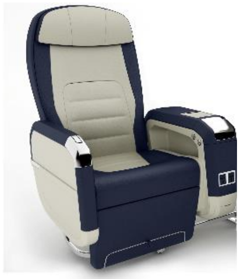
					
$10	$2000	$200	$750	$5000	$3000

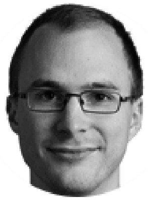
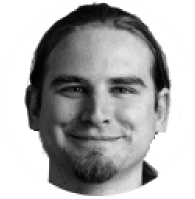
What if we knew the future?



$1	$10	$5	$200	$1500	$2500
					
$10	$2000	$200	$750	$5000	$3000

Clairvoyant Model

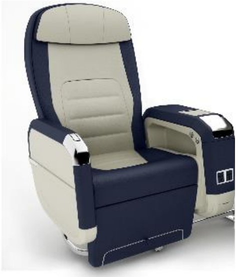


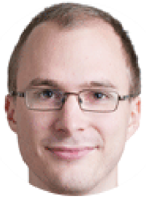
\$1	\$10	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

Calculate in every round r: $\frac{\text{opt}(r)}{\text{gain}(r)} = \frac{1}{1}$

Score for a set S of accepted bidders: $\max_r \frac{\text{opt}(r)}{\text{gain}(r)}$

Clairvoyant Model

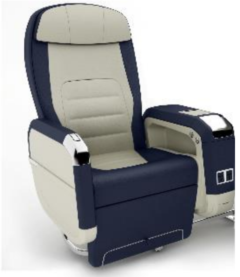


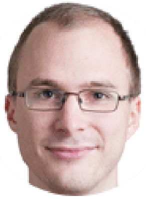
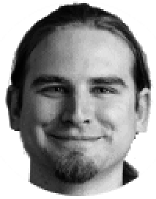
\$1	\$10	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

Calculate in every round r: $\frac{\mathrm{opt}(r)}{\mathrm{gain}(r)} = \frac{10}{1}$

Score for a set S of accepted bidders: $\max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(r)}$

Clairvoyant Model

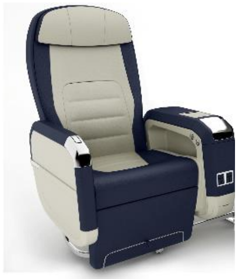


$1	$10	$5	$200	$1500	$2500
					
$10	$2000	$200	$750	$5000	$3000

Calculate in every round r: $\frac{\mathrm{opt}(r)}{\mathrm{gain}(r)} = \frac{10}{-5}$

Score for a set S of accepted bidders: $\max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(r)}$

Clairvoyant Model



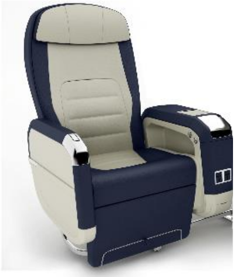
\$1	\$10	\$5	\$200	\$1500	\$2500

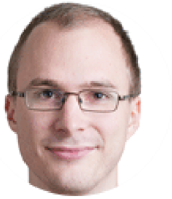
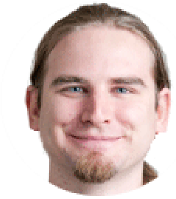
\$10	\$2000	\$200	\$750	\$5000	\$3000

Calculate in every round r: $\frac{\text{opt}(r)}{\text{gain}(r)} = \frac{200}{1}$

Score for a set S of accepted bidders: $\max_r \frac{\text{opt}(r)}{\text{gain}(r)}$

Clairvoyant Model

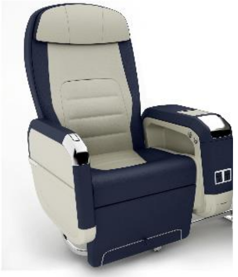


\$1	\$10	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

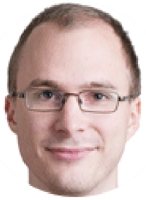
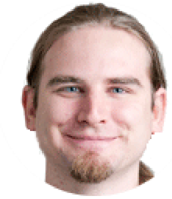
Calculate in every round r: $\frac{\mathrm{opt}(r)}{\mathrm{gain}(r)} = \frac{1500}{1500-10}$

Score for a set S of accepted bidders: $\max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(r)}$

Clairvoyant Model



$1 $10 $5 $200 $1500 $2500

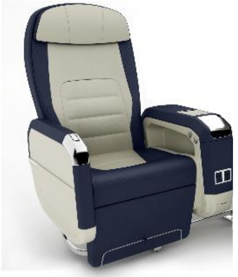
     

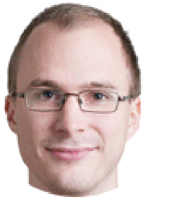
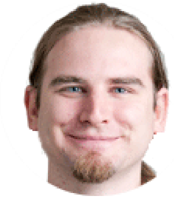
$10 $2000 $200 $750 $5000 $3000

Calculate in every round r: $\frac{\mathrm{opt}(r)}{\mathrm{gain}(r)} = \frac{2500}{1500-10}$

Score for a set S of accepted bidders: $\max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(r)}$

Clairvoyant Model

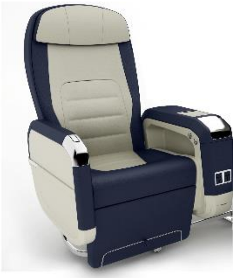


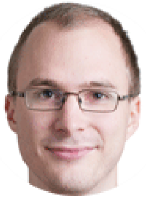
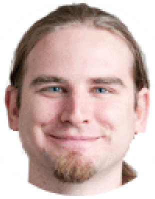
\$1	\$10	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

Calculate in every round r: $\frac{\mathrm{opt}(r)}{\mathrm{gain}(r)}$

Score for a set S of accepted bidders: $\max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(r)}$

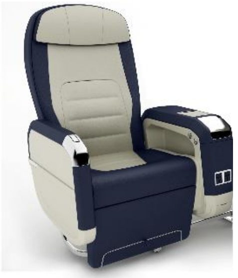
Clairvoyant Model



\$1	\$10	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

Difficulty $\Delta = \min_S \max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(S,r)}$

Δ Online Mechanisms

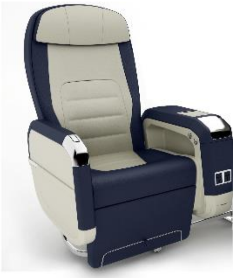


$1

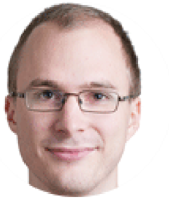
$10

Difficulty $\Delta = \min_{S} \max_{r} \frac{\text{opt}(r)}{\text{gain}(S,r)}$

Δ Online Mechanisms



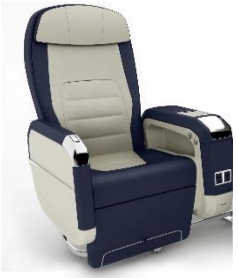
$1 $10

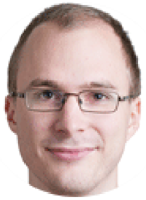
$10 $2000

Difficulty $\Delta = \min_{S} \max_{r} \frac{\text{opt}(r)}{\text{gain}(S,r)}$

Δ Online Mechanisms



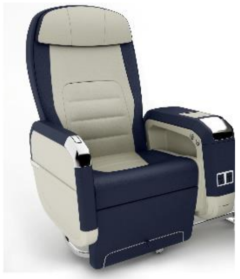
$1 $10 $5

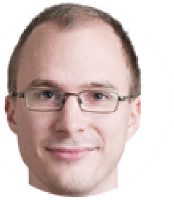

$10 $2000 $200

Difficulty $\Delta = \min_{S} \max_{r} \frac{\mathrm{opt}(r)}{\mathrm{gain}(S,r)}$

Δ Online Mechanisms



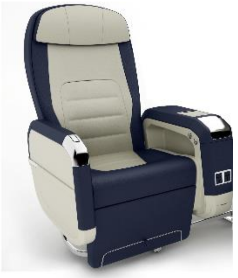
$1 $10 $5 $200

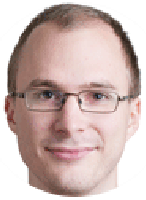
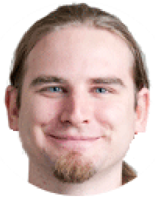
   

$10 $2000 $200 $750

Difficulty $\Delta = \min_{S} \max_{r} \frac{\text{opt}(r)}{\text{gain}(S,r)}$

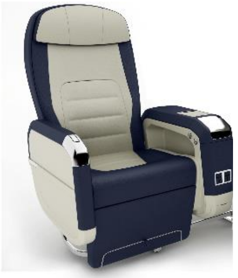
Δ Online Mechanisms

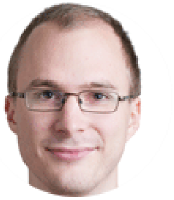
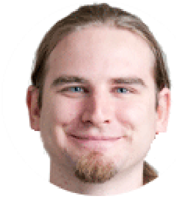


$1	$10	$5	$200	$1500
				
$10	$2000	$200	$750	$5000

Difficulty $\Delta = \min_{S} \max_{r} \frac{\mathrm{opt}(r)}{\mathrm{gain}(S,r)}$

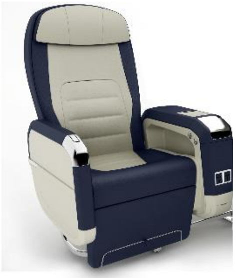
Δ Online Mechanisms



\$1	\$10	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

Difficulty $\Delta = \min_{S} \max_{r} \frac{\mathrm{opt}(r)}{\mathrm{gain}(S,r)}$

Δ Online Mechanisms



$1	$10	$5	$200	$1500	$2500

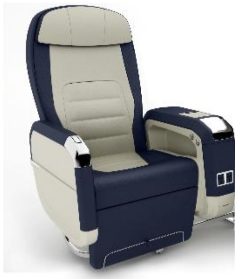
$10	$2000	$200	$750	$5000	$3000

Difficulty $\Delta = \min_S \max_r \frac{\mathrm{opt}(r)}{\mathrm{gain}(S,r)}$

Theorem

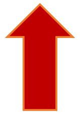
An online mechanism that knows Δ can be Δ^5 competitive

Lower Bound

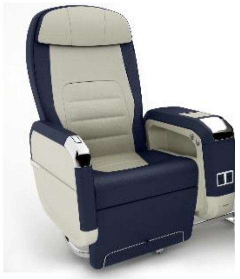


1 Δ^0 Δ^1 Δ^2 Δ^3 Δ^4 Δ^5

1 Δ^{0+2} Δ^{1+2} Δ^{2+2} Δ^{3+2} Δ^{4+2} Δ^{5+2}



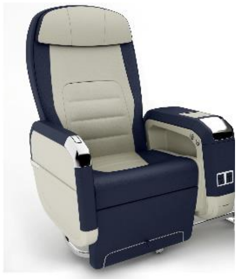
Lower Bound



v_j/Δ	v_j	$v_j\Delta^2$

$v_j\Delta$	$v_j\Delta^2$	$v_j\Delta^2$

Lower Bound



v_j/Δ

$v_j\Delta$

v_j

$v_j\Delta^2$

$v_j\Delta^2$

$v_j\Delta^2$

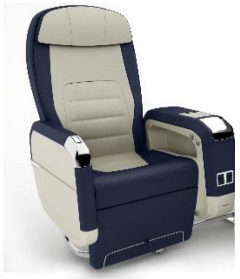
$v_j\Delta^3$

$v_j\Delta^{1000}$

$v_j\Delta^{999}$

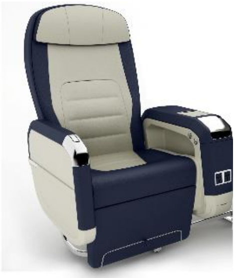
v_j

Lower Bound



v_j/Δ	v_j	$v_j\Delta^2$	$v_j\Delta^3$	$v_j\Delta^4$	
					
$v_j\Delta$	$v_j\Delta^2$	$v_j\Delta^2$	$v_j\Delta^{1000}$	$v_j\Delta^{1337}$	$v_j\Delta^{1336}$
					
					Δ^{4+2}

Lower Bound



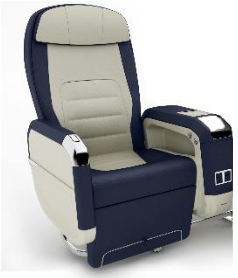
v_j/Δ v_j $v_j\Delta^2$ $v_j\Delta^3$ $v_j\Delta^4$ $v_j\Delta^5$

$v_j\Delta$ $v_j\Delta^2$ $v_j\Delta^2$ $v_j\Delta^{1000}$ $v_j\Delta^{1337}$ $v_j\Delta^{2000}$

$v_j\Delta^{1999}$

$v_j\Delta^{2000}$

Lower Bound



v_j/Δ v_j $v_j\Delta^2$ $v_j\Delta^3$ $v_j\Delta^4$ $v_j\Delta^5$

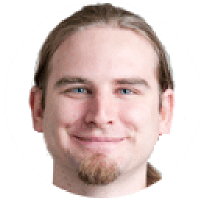
$v_j\Delta$ $v_j\Delta^2$ $v_j\Delta^2$ $v_j\Delta^{1000}$ $v_j\Delta^{1337}$ $v_j\Delta^{2000}$

$v_j\Delta^{1999}$

$v_j\Delta^{2000}$

Theorem

An online mechanism that knows Δ can be Δ^5 competitive; this is optimal.

\$10	\$100	\$5	\$200	\$1500	\$2500
					
\$10	\$2000	\$200	\$750	\$5000	\$3000

Difficulty $\Delta = \min_{S} \max_{r} \frac{\mathrm{opt}(r)}{\mathrm{gain}(S,r)}$

Theorem

An online mechanism that knows Δ can be Δ^5 competitive; this is optimal.