Coloring Unstructured Radio Networks

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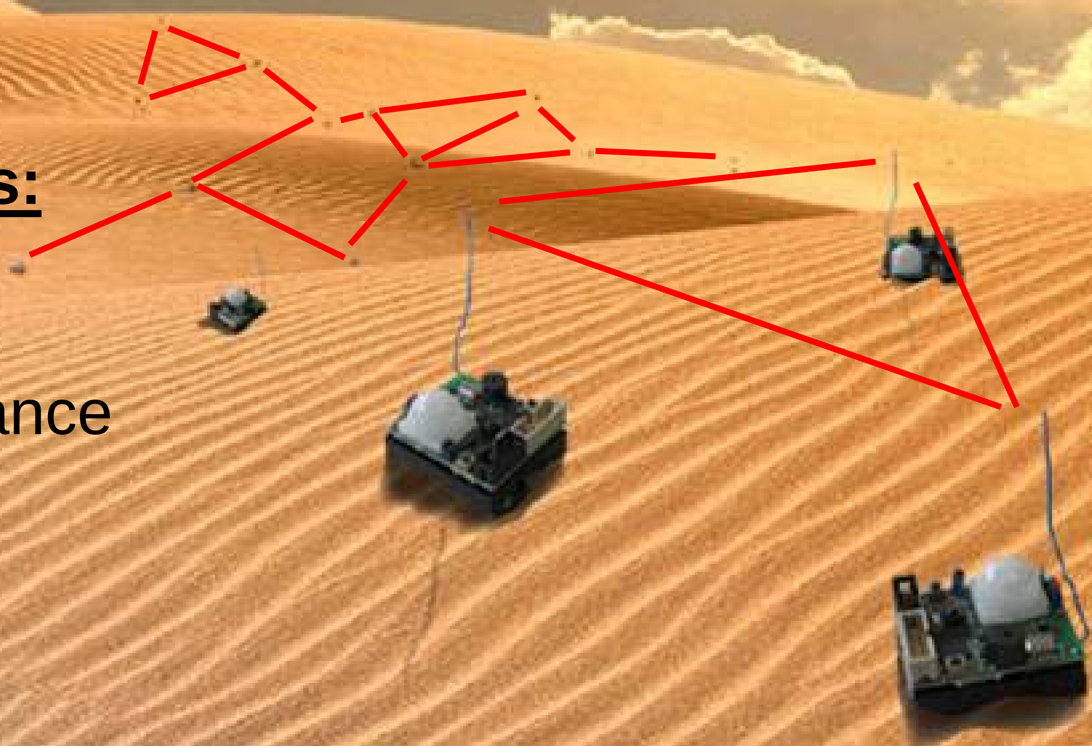
Wireless Ad Hoc and Sensor Networks

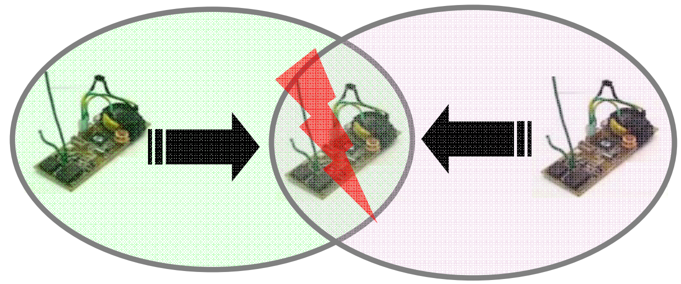
Application Scenarios:

- Data Gathering
- Monitoring, Surveillance
- Disaster Relief
- Many others....

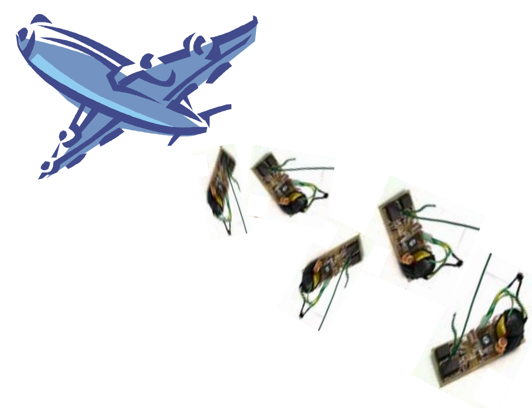


Challenges: Differences to Wired Networks

- No built-in infrastructure
  - Nodes need to set up their own infrastructure (Initially, no available MAC layer)

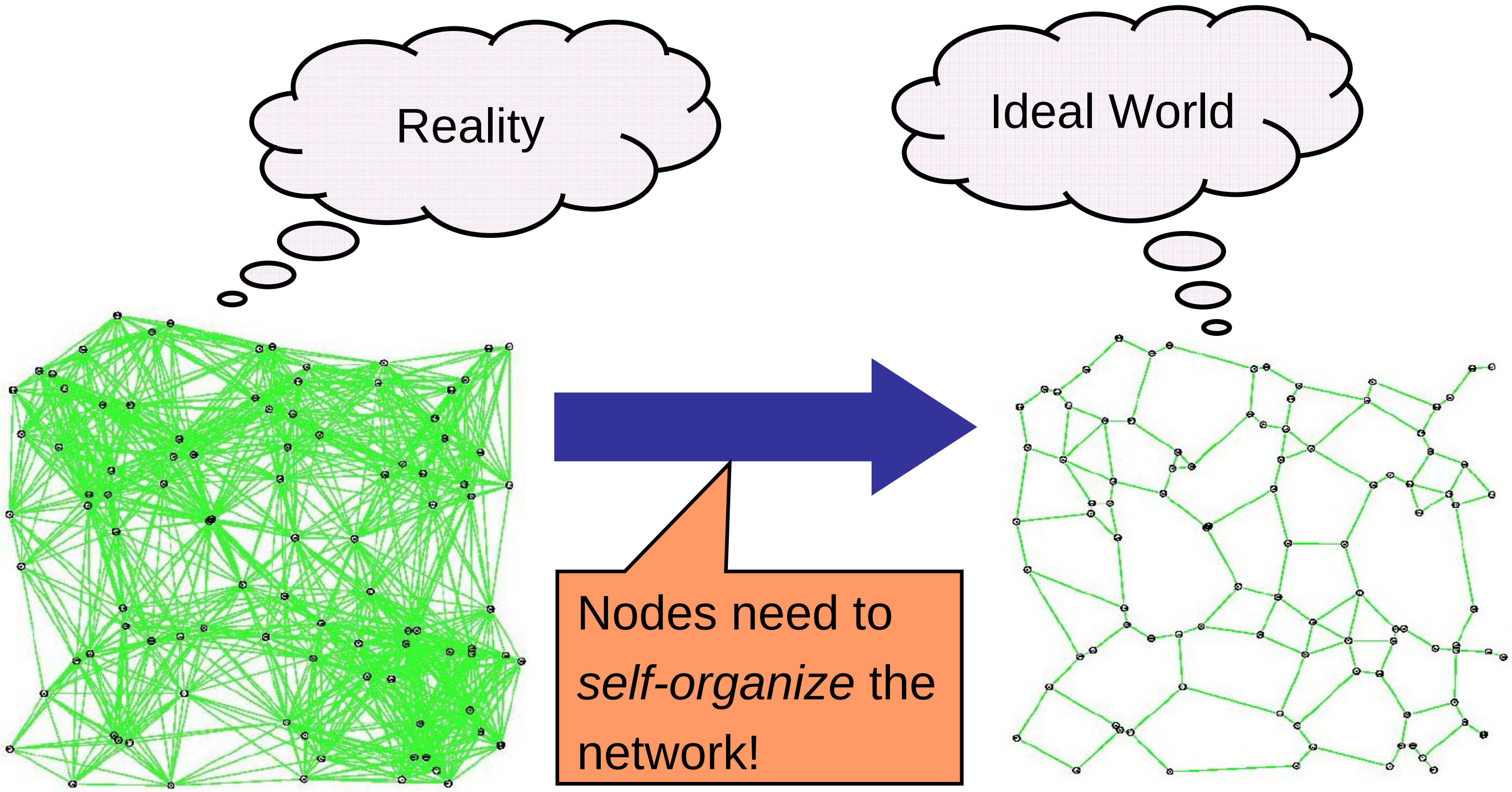


- Communication on shared medium
  - Collisions, Interference,...
- Absence of a-priori knowledge
  - Nodes do not know network topology
  - Nodes do not even know neighbors!



- Energy and memory are scarce
- Mobility, node failures,
- Nodes may be deployed at different times...

Challenges: Differences to Wired Networks

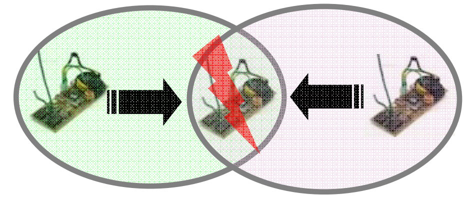


→ Nodes need to bring structure into the network.

→ Nodes must set up a MAC layer!

Distributed Vertex Coloring

- A particularly useful structure is a vertex coloring!
- We model the network as a graph G=(V,E).



Each node assigns itself a color such that,
No two neighbors have the same color

→ A coloring is a step towards a functional MAC layer!

- Frequency Division Multiple Access (FDMA) (identify each color with a frequency)
- Time Division Multiple Access (TDMA) (identify each color with a time-slot)

2-hop coloring yields a collision-free MAC layer

A good coloring should use as few colors as possible!

Distributed Vertex Coloring

- In our paper, we study 1-hop coloring!

- A 1-hop coloring is no MAC layer, but...
  - ...it avoids direct interference between nodes!
  - ...it can be turned into a 2-hop coloring by halving transmission ranges (in dense networks!)
  - ...it induces clusters

- And from a theory point of view...

The distributed complexity of coloring in unstructured radio networks.

Distributed Coloring: Related Work

- Three-coloring a ring in time $O(\log^* n)$ [Cole, Vishkin, 86]
- In time $O(\log^* n)$, rooted trees and bounded degree graphs can be colored with 3 and $\Delta+1$ colors, respectively.
  [Goldberg, Plotkin, Shannon STOC 87]
- All these results are asymptotically optimal [Linial, 92]

- Arbitrary graphs colorable with $\Delta+1$ colors in time $O(\Delta^2 + \log^* n)$ ...
  [Goldberg, Plotkin, Shannon, STOC 87]
- .. or in time $O(\Delta \log n)$ ... [Awerbuch, Goldberg, Luby, Plotkin, FOCS 89]
- Further improvements via network decomposition
  [Panconesi, Srinivasan, 96]

- Coloring for the purpose of obtaining a TDMA scheme
  [Ramanathan, Lloyd, SIGCOMM 92], [Krumke, Marathe, Ravi, 01]

Distributed Coloring: Related Work

In multi-hop radio network models...

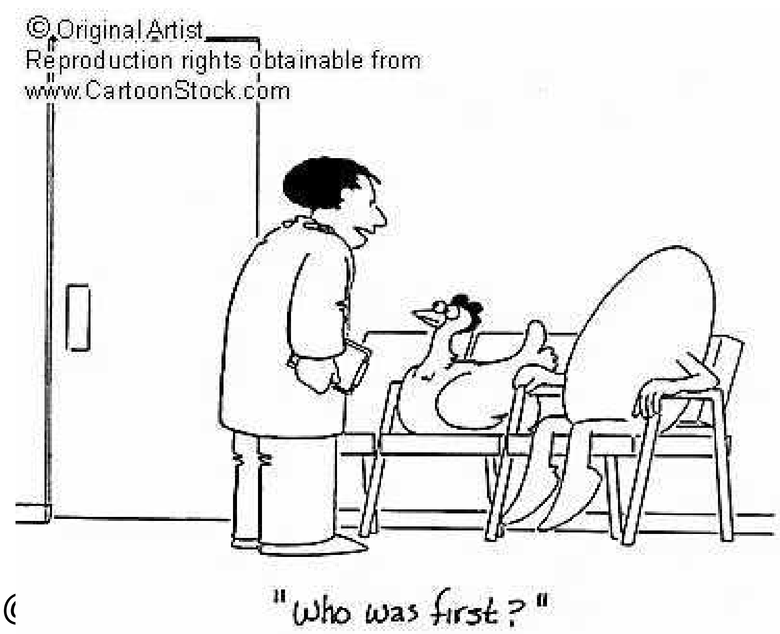
- Communication primitives such as *broadcast* or the *wake-up problem* have been thoroughly studied
- Less is known about local network coordination structures (e.g. colorings)

Distributed Coloring: Related Work

- Most existing algorithms assume...
  - ... point-to-point connections
  → Message-Passing Model
  - ... absence of interference issues
  → Collision detection mechanism
  - ... Synchronous wake-up
  - ... nodes know their neighbors, or even two hop neighbors

Chicken-and-Egg Problem:

1) Coloring algorithms are used to establish a MAC layer
2) Coloring algorithms are based on a MAC layer!

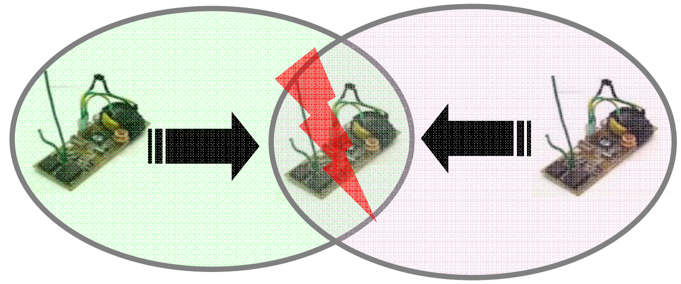


Overview

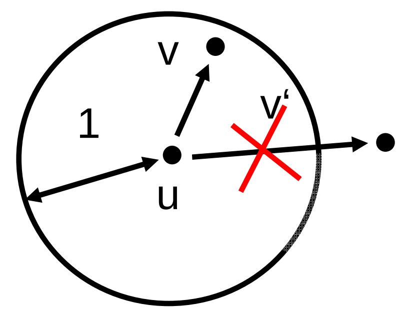
- Coloring in Ad Hoc and Sensor Networks
- Related Work
- Model
- Algorithm & Analysis
- Conclusions & Open Problems

Unstructured Radio Networks – Model (1)

- Multi-Hop
  - Hidden Terminal Problem



- No collision detection
  - Nodes cannot distinguish collisions from ambient noise
  - A sender does not know whether its transmission was correctly received!



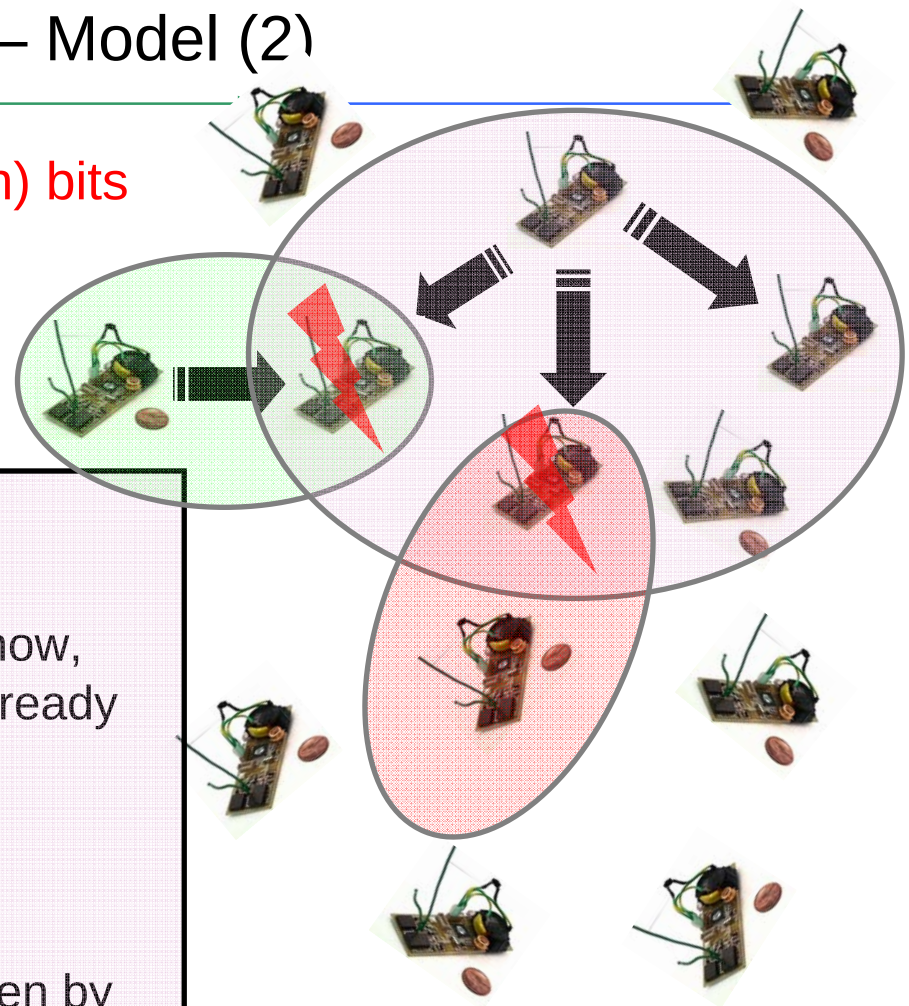
- Unit Disk Graph (UDG)
  - Two nodes can communicate iff Euclidean distance is at most 1

- No knowledge about (the number of) neighbors...

  ... except upper bounds n and Δ for number of nodes in network and the largest degree, respectively.

Unstructured Radio Networks – Model (2)

- Messages are restricted to O(log n) bits
- Nodes can wake-up at any time!
  → Asynchronous wake-up!



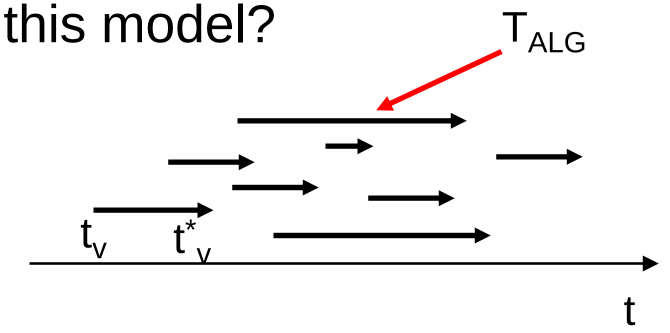
Asynchronous wake-up:

1) When waking up, a node does not know, how many neighboring nodes are already awake!
2) A node does not know when new neighbors wake up!
3) The nodes' wake-up pattern is chosen by an adversary.
4) Sleeping nodes do neither receive nor send messages

different from work on the wake-up problem or broadcast in radio networks

Unstructured Radio Networks – Model (3)

- What are the performance measures in this model?



Running Time:

- Let t_v be the time of node v's wake-up.
- Let t^*_v be the time of v's final, irrevocable decision on a color.

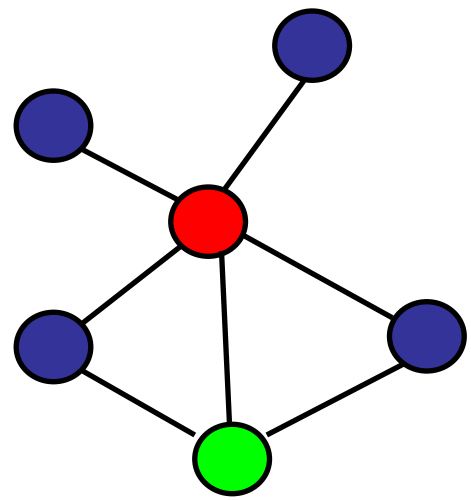
→ The running time of v is: $T_v = t^*_v - t_v$

→ The algorithm's running time is: $T_{ALG} = \max_{v \in V} T_v$

In UDG, $\Omega(\Delta)$ lower bound!

Colors:

- The *maximum color* used by the algorithm
- The *local distribution* of the colors!

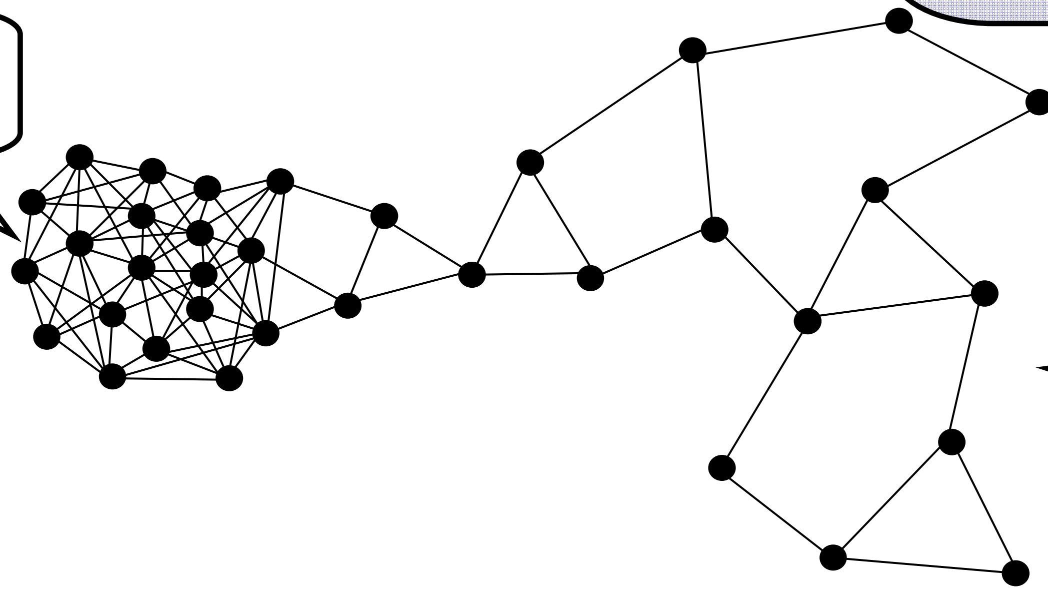


Locality in Vertex Coloring

- A good coloring should....
  - use few colors!
  - use high colors in dense areas only!

This precludes simple probabilistic algorithms In which every node chooses a random color

High colors are needed here!



Use only low colors here!

Highest color should depend on local graph properties!

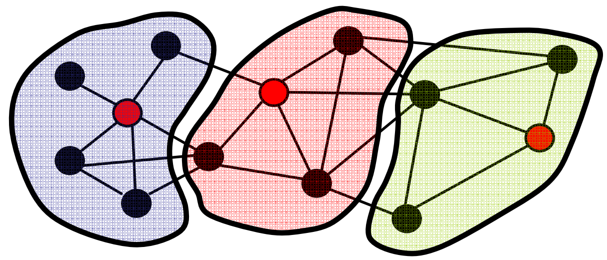
Overview

- Ad Hoc and Sensor Networks
- Clustering
- Model
- Algorithm & Analysis
- Conclusions & Open Problems

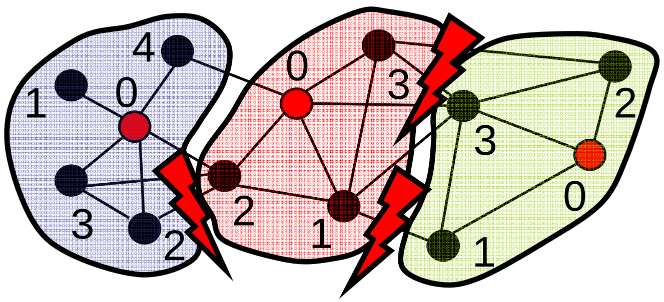
Algorithm Overview (system's view)

- Idea: Color in a two-step process!
- First, nodes select a (sparse) set of leaders among themselves

  → induces a clustering



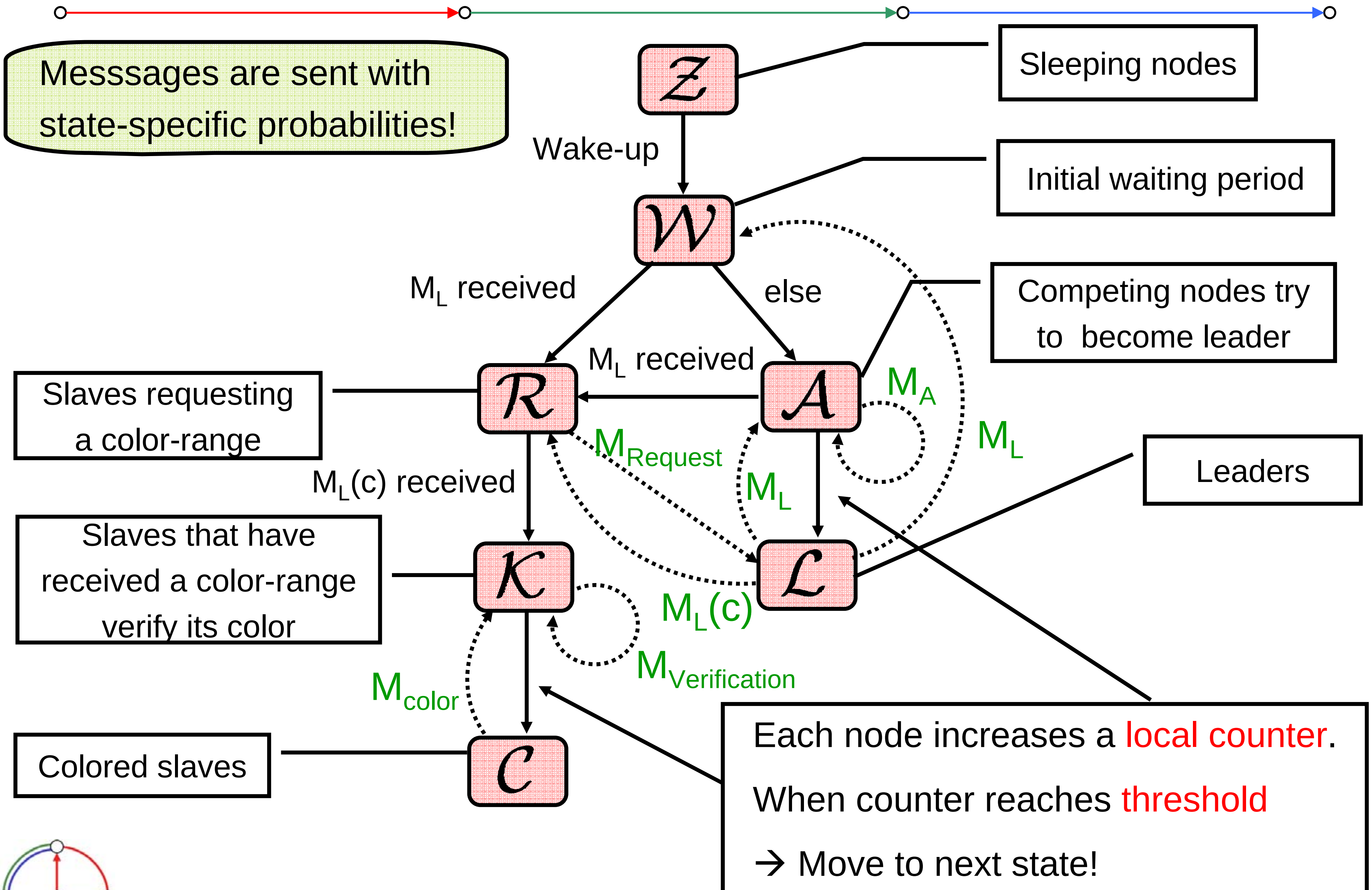
- Leaders assign initial coloring that is correct within the cluster
- Problem: Nodes in different clusters may be neighbors!



Interpret initial color as a color-range!

- In a final verification phase, nodes select final (conflict-free) color from color-range!

Algorithm Overview (a node's view)



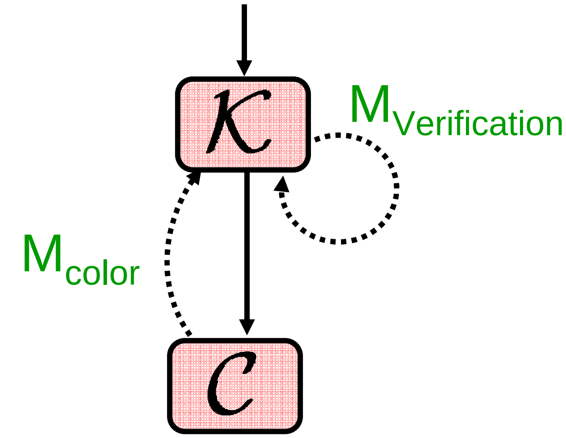
Algorithm Overview (Challenges)

- Problems:
  - → Everything happens concurrently!
  - → Nodes do not know in which state neighbors are (they do not even know whether there are any neighbors!)
  - → Messages may be lost due to collisions
  - → New nodes may join in at any time...
- Correctness!
  - → No two neighbors must choose the same color.
- No starvation!
  - → Every node must be able to choose a color within time $O(\Delta \log n)$ after its wake-up.

How to achieve both?

Avoid Starvation - Idea

- Use counters and appropriate thresholds
- Example: Consider state $\mathcal{K}$, node v verifies c



0) When receiving M_{color}(c) verify c+1

1) When entering state $\mathcal{K}$, set counter to 0.

2) In each time-slot, increase counter by 1.

3) When reaching $\sigma\Delta$log n, choose color and move to state $\mathcal{C}$

4) With probability p_K, transmit $M_{Verification}$(counter,c) and set counter to

$$counter := \max\{counter, \gamma\Delta \log n\} + 1$$

Cascading resets..?

5) When receiving $M_{Verification}$(counter*,c) from another node:

If counters are within $\gamma\Delta \log n$ of one another → Reset counter!

This method achieves both correctness and quick progress (in every region of the graph)!

Avoid Starvation - Idea

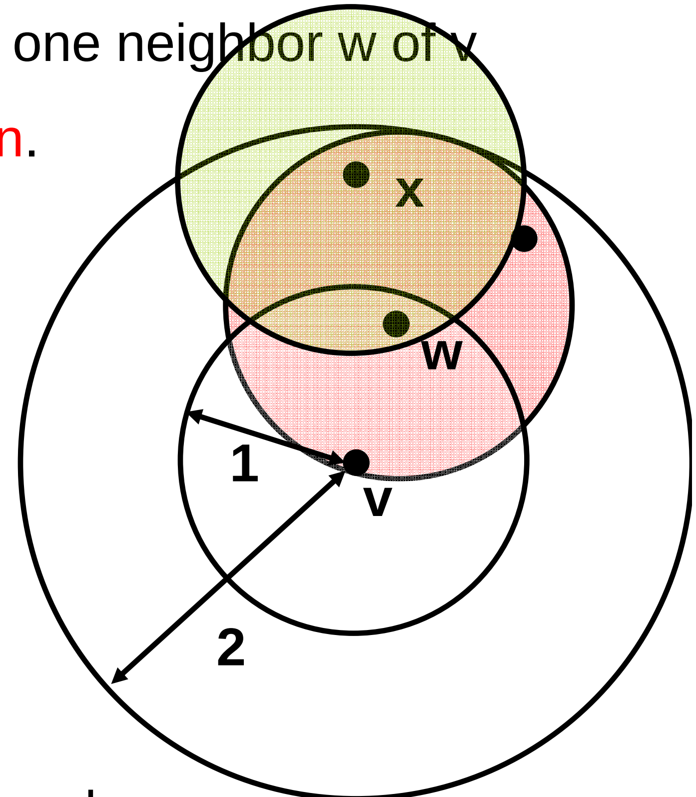
- Consider a node v entering state $\mathcal{K}$ at time t_v and verifying color c
- We show that by time $t_v+O(\Delta \log n)$, at least one neighbor w of v has transmitted (broadcast!) without collision.
- w has counter at least $\gamma\Delta \log n+1$
- All neighbors of w verifying c

  - either reset their counter

  - or have a counter that is at least $\gamma\Delta \log n$ away from w's counter.

→ w cannot be reset anymore by nodes in $\mathcal{K}$!

→ w may get M_{color} from a node $x \in \mathcal{C}$ that has chosen the color c earlier!



x covers a constant fraction of the disk of radius 2!

Avoid Starvation - Idea

Each taking time O(Δ log n)

- After a constant number of repetitions, the disk will be covered

  → node v either chooses c or receives M_{color} and verifies c+1

  → The argument repeats itself for c+1

- Because the set of leaders is sparse

  → v must verify only up to color c+μ, for $\mu \in O(1)$

W.h.p, every node spends only O(Δ log n) time-slots in state $\mathcal{K}$

In the proof, we similarly avoid starvation in all states!

- Specifically, we prove that: $T_{\mathcal{W}}, T_{\mathcal{A}}, T_{\mathcal{R}}, T_{\mathcal{K}} \in O(\Delta \log n)$

  Hence, $T = T_{\mathcal{W}} + T_{\mathcal{A}} + T_{\mathcal{R}} + T_{\mathcal{K}} \in O(\Delta \log n)$

Results

With high probability, the distributed coloring algorithm ...

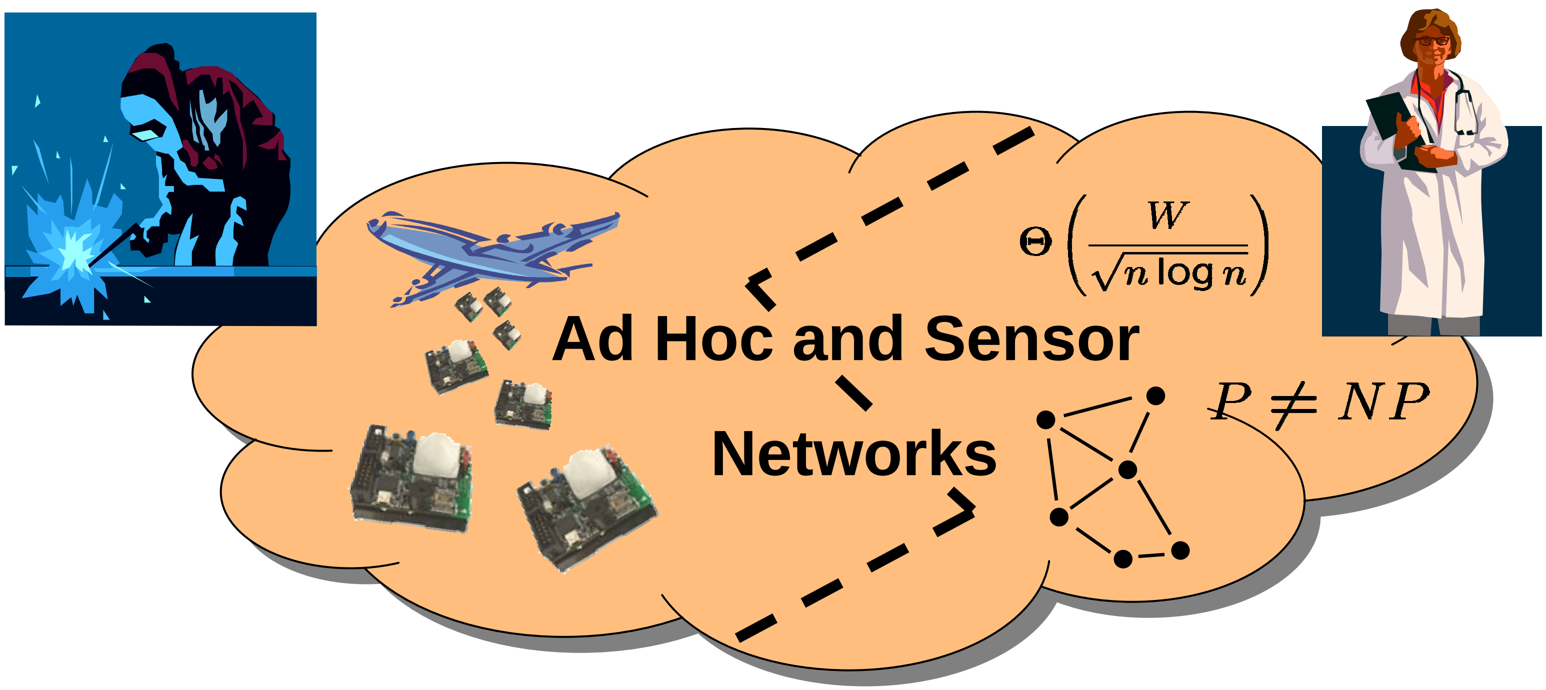
→... achieves a correct coloring using $O(\Delta)$ colors

→... every node irrevocably decides on a color within time $O(\Delta \log n)$ after its wake-up

→... the highest color depends only on the local maximum degree

Of Theory and Practice...

Practice **Theory**



There is often a big gap between theory and practice in the field of wireless ad hoc and sensor networks.

Conclusions / Open Problems

- O(Δ) coloring in harsh radio network model in time O(Δ log n) w.h.p.
  → Tight up to a factor of O(log n)
  → Color assignment according to local density

Future Directions:

- Close the remaining complexity gap
- Algorithm assumes knowledge of n and Δ
  → Remove this assumption
- 2-hop coloring ?
- Asynchronous wake-up: many open questions

Questions? Comments?

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Comments?

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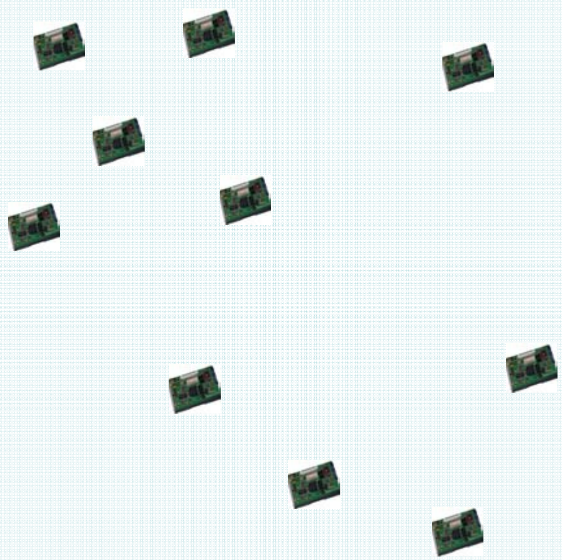
Of Theory and Practice...

Initial MAC layer



this talk → current work

Nodes know neighbors, etc.

Fair MAC layer
- Ad hoc networks

High-Throughput MAC layer
- Multimedia

Energy-Efficient MAC layer
- Long lifetime
- Sensor networks

There's more to deployment
- Time synchronization
- Topology control, etc.

Late arrivals

Mobility?

Failures?

time