Complexity of Scheduling with Analog Network Coding

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ABSTRACT

In this paper we analyze the complexity of scheduling wireless links in the physical interference model with analog network coding capability. We study two models with different definitions of network coding. In one model, we assume that a receiver is able to decode several signals simultaneously, provided that these signals differ in strength significantly. In the second model, we assume that routers are able to forward the interfering signal of a pair of nodes that wish to exchange a message, and nodes are able to decode the "collided" message by subtracting their own contribution from the interfered signal. For each network coding definition, we construct an instance of the scheduling problem in the geometric SINR model, in which nodes are distributed in the Euclidean plane. We present NP-completeness proofs for both scenarios.

Categories and Subject Descriptors

F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems—*Geometrical Problems and Computations, Sequencing and Scheduling*; H.1.1 [**Models and Principles**]: Systems and Information Theory

General Terms

Theory.

1. INTRODUCTION

The problem of scheduling link transmissions in wireless networks has received a lot of attention in the last years. A variety of interference models have been studied, ranging from graph-based, such as the protocol interference model, to more realistic representation of signal propagation, such as the physical interference model [7]. An important issue is the complexity of scheduling problems. How much computation is it needed to find a minimum length schedule for a given set of communication requests? Is this a hard problem, even in a simplified model? Recently, it was shown that this problem is NP-complete in the geometric physical interference model, where nodes live in a Euclidean space [5].

One of the key concepts on which wireless interference models rely is the definition of a successful transmission. It has typically been assumed that a receiver successfully decodes one, and only one, message at a time. In graph-based models, a typical pre-condition for a set of links to be scheduled concurrently is that they form some sort of independent set, i.e., only one link is allowed to transmit at a time in its neighborhood. In physical interference models, this requirement of spatial separation is not always necessary. In [18], for example, it was shown that, with appropriate power control, two sender-receiver pairs, being one positioned in the transmission line of the other, can be scheduled without a collision, i.e., a receiver r_i can successfully decode a message from a sender s_i in spite of another concurrently transmitting sender s_j positioned closer to r_i than s_i. For such a scenario to work, the signal-to-interference-plus-noise-ratio at r_i must exceed a certain threshold, i.e., only the strongest signal can be decoded by the receiver.

More recently, the fact that wireless interference is harmful and that a receiver can only decode the strongest signal at a time has been revised. Techniques such as cochannel separation and network coding have radically changed the definition of a successful transmission. Cochannel separation techniques allow the receiver to decode several signals simultaneously under the assumption that these signals differ significantly in their strength. Analog network coding makes it possible to simultaneously decode two signals of similar strength, under the assumption that the receiver knows one of the interfered signals by having overheard or forwarded it earlier [12].

Network coding brought a lot of revision to the building blocks of models used to study the problem of scheduling wireless links. Most of the results in network coding have typically concentrated on capacity improvements in graph-based models and on feasibility of practical protocol design, but have not addressed the fundamental issue of complexity of scheduling in the physical interference model. Does the fact that a receiver is able to decode more than one signal simultaneously make the problem easier? Does the problem remain NP-complete? Does it open possibilities for better approximation algorithms?

This work presents some initial steps into the study of these issues. We analyze two models with different definitions of analog network coding. In one model, we assume

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that a receiver is able to decode several signals simultaneously, provided that these signals differ in strength significantly. In the second model, we assume that routers are able to forward the superposition of two interfering signals of nodes that wish to exchange a message, and nodes are able to decode the "collided" message by subtracting their own contribution from the interfered signal. For each network coding definition, we construct an instance of the scheduling problem in the geometric physical interference model, in which nodes are distributed in the Euclidean plane, and present NP-completeness proofs for both scenarios.

In Section 2 we discuss some related work. In Section 3 we define two different models of analog network coding. To the first model we refer as *analog coding by filtering* and to the second model we refer as *analog coding by signal mixing*. To the scheduling problems defined in each of these models we refer as *Scheduling with Analog Coding by Filtering* (SACF) and *Scheduling with Analog Coding by Signal Mixing* (SACSM). In Sections 4 and 5 we present NP-completeness proofs for SACSM and SACF, respectively. Finally, in Section 6 we discuss some conclusions.

2. RELATED WORK

The complexity of scheduling wireless requests without network coding has been extensively studied in many models. Most of the hardness results are derived in graph-based models [11, 15] or in non-geometric physical interference models, where the values in the gain (or path-loss) matrix are chosen arbitrarily, i.e., are not constrained by triangular inequality [2, 14, 16]. More recently, scheduling has been shown to be NP-complete in the more restricted, geometric SINR model [5].

Network coding is a recent concept, which extends the traditional definition of routing by allowing routers to not just forward copies of received messages, but to mix the bits in packets before forwarding them. The main initial result states that full capacity can be achieved in a graph where one source multicasts information to other nodes in a multi-hop fashion and any node in the network is allowed to encode all its received data before passing it on [1]. In [17] it was shown that linear codes are sufficient to achieve multicast capacity. In [19] it was shown that encoding and decoding can be done in polynomial time. In [10] it was shown that multicast capacity can be achieved in a distributed manner, by using random linear codes over a sufficiently large finite field. In [22] it was shown that the minimum energy-per-bit multicast in a graph-based wireless network model can be solved as a linear program. In [13] network coding was made practical, being developed into a link layer enhancement scheme for multi-hop wireless networks. A mixing engine was introduced into the nodes, operating between the MAC and the network layer, in order to identify opportunities to make bitwise XORs of different packets and sending them in a single transmission.

Network coding in the physical layer, or analog network coding, is similar in spirit to digital network coding. However, it operates on the raw analog signal, instead of first decoding and then mixing packets in a bitwise manner. Some techniques, such as cochannel signal separation, explore differences in the characteristics of interfered signals, such as signal's strength, to decode several signals simultaneously [8, 9]. Other analog coding techniques exploit the fact that, in a wireless network, often a receiver has prior knowledge about some packets destined to other nodes, by having overheard or forwarded them earlier. In [21] the impact of such knowledge in combination with nested coding on the capacity region was analyzed. In [12, 23] pairs of nodes that wish to exchange packets through a relay node are encouraged to transmit simultaneously. The relay node, without decoding the collided signal, amplifies and forwards it. The destination nodes then extract the packet destined to them by filtering out their own contribution from the mixed signal. The algorithms in [12, 23] have focused on decoding only two signals that interfered with each other, mainly in the canonical 2-way relay topology.

3. MODELS

In this work we study the problem of scheduling wireless requests in the physical interference model. In order to capture analog network coding capability, we work with two different definitions of a successful transmission: *analog coding by filtering* and *analog coding by signal mixing.*

In both scenarios we assume that nodes live in a Euclidean plane and that the received power $P_r(s)$ of a signal transmitted by sender s at receiver r is

$$P_r(s) \;=\; \frac{P}{d(r,s)^{\alpha}}, \tag{1}$$

where P is the transmission power and $d(r,s)^{-\alpha}$ is the propagation attenuation (link gain). The path-loss exponent $\alpha > 2$ is a constant, whose exact value depends on external conditions of the medium, such as humidity, obstacles, etc.

In the traditional physical interference model, a receiver r successfully decodes a transmission from a sender s_x if and only if

$$\begin{aligned} SINR(r) &= \frac{P_r(s_x)}{I_r + N} = \frac{P_r(s_x)}{\sum_{y \neq x} I_r(s_y) + N} \\ &= \frac{\frac{P}{d(r,s_x)^{\alpha}}}{\sum_{y \neq x} \frac{P}{d(r,s_y)^{\alpha}} + N} \;\geq\; \beta, \end{aligned} \tag{2}$$

where I_r is the interference, or power, perceived by r from all concurrent transmissions in the network, and β is the minimum signal-to-interference-plus-noise-ratio (SINR) required for a successful message decoding. Typically, it is assumed that $\beta \geq 2$.

In this work we assume that all nodes transmit with the same power level. This assumption is also referred to as *uniform power assignment scheme* [6]. This kind of power assignment has been widely adopted in practical systems and has been studied in depth in [20].

Next, we introduce the definitions of successful transmission used in this work.

3.1 Analog Coding by Filtering

In this model, we assume that a receiver r is able to decode several signals simultaneously, provided that these signals differ in strength significantly. This kind of model has been studied in the context of cochannel signal separation [8, 9].

Consider a set of k signals (sorted in decreasing order of power received at r): $\Upsilon = \{P_r(s_1), P_r(s_2), \cdots, P_r(s_k)\}$. We assume that receiver r is able to decode all k signals in Υ if

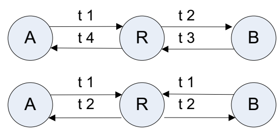

Figure 1: Analog network coding by signal mixing.

and only if the following condition holds $\forall x \in \{1, \cdots, k\}$:

$$\frac{P_r(s_x)}{\sum_{\substack{P_r(s_y)\in\Upsilon,\\ P_r(s_y)<P_r(s_x)}} P_r(s_y) + \sum_{P_r(s_z)\notin\Upsilon} P_r(s_z) + N} \geq \beta, \quad (3)$$

where the first component of the denominator is the accumulated interference caused by transmissions in Υ, which have weaker power level than $P_r(s_x)$; the second component of the denominator is the accumulated interference of all other concurrent transmissions in the network, which are not in Υ; N is the ambient noise; and β is the minimum SINR threshold.

The idea is that, one by one, each signal $P_r(s_x) \in \Upsilon$ can be "filtered out" from the accumulated interference, provided that the SINR between this signal and the remaining interference is above the threshold β. The key point here is that a receiver r is able to decode not only the strongest signal, as in the traditional physical model, but also a relatively weak signal, provided that each of the stronger signals have been filtered out. Therefore, a signal $P_r(s_x)$ can be correctly decoded if and only of all stronger signals $P_r(s_y) > P_r(s_x)$ obey the following constraints:

$$\begin{aligned} \frac{P_r(s_y)}{P_r(s_x) + \sum_{P_r(s_z)<P_r(s_y)} P_r(s_z) + N} &\geq \beta, \\ \forall P_r(s_y) > P_r(s_x), &\qquad \text{and} \qquad (4) \\ \frac{P_r(s_x)}{\sum_{P_r(s_z)<P_r(s_x)} P_r(s_z) + N} &\geq \beta. \qquad (5) \end{aligned}$$

3.2 Analog Coding by Signal Mixing

Our second definition of analog network coding was introduced in [23, 12]. This model explores the fact that in a wireless network, when two packets collide, nodes often know one of the colliding packets due to having forwarded it earlier or having overheard it. Consider a situation where two nodes A and B wish to send a message to each other (see Fig. 1). Due to the interference of concurrent transmissions or due to the ambient noise, A and B cannot communicate directly, but only through a relay node R. Instead of scheduling 4 sequential transmissions $A \to R, R \to B, B \to R, R \to A$, as in the traditional approach, by using analog network coding, A and B can transmit simultaneously, allowing their transmissions to interfere at R. The router, not being able to decode the collided packets, can simply amplify and forward the interfered signal. It has been shown in [12] that A (as well as B) is able to decode B's packet by subtracting the contribution of its own packet from the interfered signal, even if the two transmissions are not fully synchronized and the wireless channel distorts the signals. As a result, only two time slots are sufficient to schedule these requests.

In order for such a signal mixing to result in two successful transmissions, the following SINR conditions must hold in two time slots $t_i, t_j, j > i$:

$$\begin{aligned} \frac{P_R(A)}{\sum_{\substack{s_y\neq A\\ s_y\neq B}} I_R(s_y) + N} &\geq \beta, \quad \text{in } t_i && (6) \\ \frac{P_R(B)}{\sum_{\substack{s_y\neq A\\ s_y\neq B}} I_R(s_y) + N} &\geq \beta, \quad \text{in } t_i && (7) \\ \frac{P_A(R)}{\sum_{s_y\neq R} I_A(s_y) + N} &\geq \beta, \quad \text{in } t_j && (8) \\ \frac{P_B(R)}{\sum_{s_y\neq R} I_B(s_y) + N} &\geq \beta, \quad \text{in } t_j. && (9) \end{aligned}$$

This means that in order for A (and B) to be able to decode the mixed signal $(P_R(A) + P_R(B))$ amplified and forwarded by R, the signals received by R from both B and A must have, individually, an $SINR \geq \beta$. Note that the relative signal strength (SIR) of A and B must not exceed any threshold. In fact, it has been shown in [12] that even when $SIR(A, B) = 0$, i.e., when $P_R(A) = P_R(B)$, the signals can still be correctly decoded by their receivers. However, note that the mixed signal sent by R must still have $SINR \geq \beta$ at both receivers A and B.

For those transmission that occur without employing signal mixing, we define a successful transmission as in traditional physical interference model (see Eq. 2).

For the sake of simplicity, in the following analysis sections, we set $N = 0$ and ignore the influence of noise in the calculation of $SINR$. However, this has no significant effect on the results.

4. COMPLEXITY OF SACSM

In this section we prove that scheduling with analog coding by signal mixing is NP-complete in the physical interference model, where nodes live in a Euclidean space (geometric SINR model).

To see that the decision version of the problem is in NP is straightforward. To decide whether a schedule of a given size T is feasible, we have to verify, for every transmission, whether it employed signal mixing at a relay node or not. If yes, conditions (6) through (9) must be satisfied for each triple of participating nodes A, B, R. If a transmission was scheduled without network coding, then only the condition (2) has to be verified. Since computing the $SINR$ level for each transmission in its time slot can be done in $O(n^2)$ time, a schedule is an efficiently verifiable witness for this problem.

The hardness proof is by reduction from the well known NP-complete *numerical matching with target sums* problem (NMTS) [4], which can be formulated as follows: Given 3 sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ of positive integers, is it possible to match each element $i \in \mathcal{A}$ to a distinct element $j \in \mathcal{B}$, such that their sum $(i + j)$ equals to each of the elements $k \in \mathcal{C}$? The triples (i, j, k) must form a partition in the sense that they are disjoint and cover $\mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$.

NMTS problem: Find q triples $triple_l = (i, j, k), i \in \mathcal{A} = \{i_1, \cdots, i_q\}, j \in \mathcal{B} = \{j_1, \cdots, j_q\}, k \in \mathcal{C} = \{k_1, \cdots, k_q\}$, such that:

$$\begin{aligned} triple_1 \cap triple_2 \cap \cdots \cap triple_q &= \emptyset, && (10) \\ triple_1 \cup triple_2 \cup \cdots \cup triple_q &= \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}, && (11) \\ (i + j) = k, \quad \forall (i, j, k) \in triple_l, & \qquad \forall l \in \{1, \cdots, q\} && (12) \end{aligned}$$

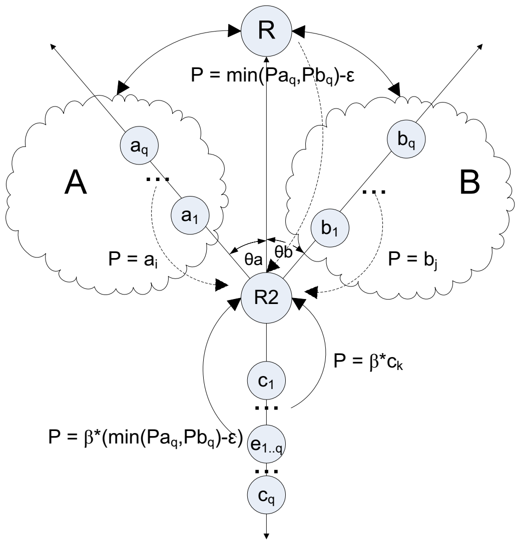

Figure 2: Reduction from NMTS: all $4q$ links can be scheduled successfully in $2q$ time slots if and only if senders $a_1, \cdots, a_q, b_1, \cdots, b_q, c_1, \cdots, c_q$, are partitioned into q triples (a_i, b_j, c_k) such that $(i + j) = k$.

Note that, for a solution to exist, we need:

$$\sum_{i \in \mathcal{A}} i + \sum_{j \in \mathcal{B}} j = \sum_{k \in \mathcal{C}} k. \tag{13}$$

THEOREM 4.1. *NMTS is reducible to SACSM in polynomial time.*

PROOF. The proof proceeds as follows. First, we define a many-to-one reduction from any instance of NMTS to an instance of SACSM. Then, we argue that the instance of SACSM cannot be scheduled in $T \leq 2q$ time slots, but can be scheduled in $T = 2q$ time slots if and only if there is a solution to the NMTS problem instance.

Consider any instance of NMTS defined by $\mathcal{A} = \{i_1, \cdots, i_q\}, \mathcal{B} = \{j_1, \cdots, j_q\}, \mathcal{C} = \{k_1, \cdots, k_q\}$. The instance of SACSM is constructed by placing $(4q + 2)$ nodes in the plane in the following way (see Figure 2). First, two nodes R and R_2 are placed at positions $(0, r(R))$ and $(0, 0)$, respectively. Thereafter, q nodes, corresponding to integers $j \in \mathcal{B}$ are placed on a straight line originating at R_2 at angle $(\pi/2 - \theta_b)$; q nodes, corresponding to integers $i \in \mathcal{A}$ are placed on a line originating at R_2 at angle $(\pi/2 + \theta_a)$; and q nodes, corresponding to integers $k \in \mathcal{C}$ are placed on a line originating at R_2 at angle $-\pi/2$. The polar coordinates of each of these $3q + 2$ nodes are:

$$r(R) = \max(a_{\max}, b_{\max}) + \epsilon, \quad \theta(R) = \pi/2, \tag{14}$$
$$r(R_2) = 0, \qquad \theta(R_2) = 0, \tag{15}$$
$$r(a_i) = \left(\frac{1}{i}\right)^{1/\alpha}, \quad \theta(a_i) = \pi/2 + \theta_a, \ \forall i \in \mathcal{A}, \tag{16}$$
$$r(b_j) = \left(\frac{1}{j}\right)^{1/\alpha}, \quad \theta(b_j) = \pi/2 - \theta_b, \ \forall j \in \mathcal{B}, \tag{17}$$
$$r(c_k) = \left(\frac{1}{\beta k}\right)^{1/\alpha}, \ \theta(c_k) = -\pi/2, \ \forall k \in \mathcal{C}, \tag{18}$$

where ϵ is a small positive constant, and angles are defined as follows:

$$\theta_a = \min(\theta_a^1, \theta_a^2), \quad \theta_b = \min(\theta_b^1, \theta_b^2), \text{ where} \tag{19}$$
$$\theta_a^1 = \arccos\left(\frac{\beta^{\frac{2}{\alpha}}(a_{\min}^2 + r(R)^2) - a_{\min}^2 - r(e_l)^2}{2\beta^{\frac{2}{\alpha}} a_{\min} r(R) + 2a_{\min} r(e_l)}\right) \tag{20}$$
$$\theta_a^2 = \arccos\left(\frac{a_{\min}^2 + r(R)^2 - \left(\frac{c_{\min} + r(R)}{\beta^{1/\alpha}}\right)^2}{2a_{\min} r(R)}\right), \tag{21}$$
$$\theta_b^1 = \arccos\left(\frac{\beta^{\frac{2}{\alpha}}(b_{\min}^2 + r(R)^2) - b_{\min}^2 - r(e_l)^2}{2\beta^{\frac{2}{\alpha}} b_{\min} r(R) + 2b_{\min} r(e_l)}\right) \tag{22}$$
$$\theta_b^2 = \arccos\left(\frac{b_{\min}^2 + r(R)^2 - \left(\frac{c_{\min} + r(R)}{\beta^{1/\alpha}}\right)^2}{2b_{\min} r(R)}\right), \tag{23}$$

where $a_{\min} = (1/i_{\max})^{1/\alpha}, i_{\max} = \max_{i \in \mathcal{A}}(i)$, $b_{\min} = (1/j_{\max})^{1/\alpha}, j_{\max} = \max_{j \in \mathcal{B}}(j)$, $c_{\min} = (1/k_{\max})^{1/\alpha}, k_{\max} = \max_{k \in \mathcal{C}}(k)$, $a_{\max} = (1/i_{\min})^{1/\alpha}, i_{\min} = \min_{i \in \mathcal{A}}(i)$, $b_{\max} = (1/j_{\min})^{1/\alpha}, j_{\min} = \min_{j \in \mathcal{B}}(j)$.

Next we position the last q nodes $\{e_1, \cdots, e_q\}$ at the following location:

$$r(e_l) = \frac{r(R)}{\beta^{\frac{1}{\alpha}}}, \quad \theta(e_l) = -\pi/2, \quad l \in \{1, \cdots, q\}. \tag{24}$$

The communication requests are defined as follows: nodes $\{c_1, \cdots, c_q, e_1, \cdots, e_q\}$ all demand to transmit to the same receiver R_2; nodes $\{a_1, \cdots, a_q\}$ and $\{b_1, \cdots, b_q\}$ are grouped into two groups A and B, respectively, and wish to transmit q messages $\{m(a_1), \cdots, m(a_q)\}$ from group A to group B and q messages $\{m(b_1), \cdots, m(b_q)\}$ from group B to group A. The exact recipient of a message $m(a_i)$ is not set, being enough to transmit successfully to any node $b_j \in B$. The same holds for a message $m(b_j)$, originated at node $b_j \in B$, which has to be transmitted to *any* node $a_i \in A$.

Having defined the geometric instance of SACSM for any instance of NMTS, we show that it cannot be scheduled in $T < 2q$ time slots using signal mixing analog coding.

It is enough to look at the $2q$ transmissions from nodes $\{c_1, \cdots, c_q, e_1, \cdots, e_q\}$ to receiver R_2. Given that signal mixing analog coding allows simultaneous decoding of two signals only when one of the signals is already known by the receiver, and at time $t = 0$ receiver R_2 does not know any of the considered $2q$ signals, it needs at least $2q$ time slots to receive and successfully decode each of them.

We proceed by showing that the problem instance defined in equations (14) through (24) can be scheduled in $T = 2q$ time slots using signal mixing analog coding if and only if there is a solution to the NMTS problem.

($\Rightarrow$) For the first part of the claim, assume we know q triples $(i, j, k), i \in \mathcal{A}, j \in \mathcal{B}, k \in \mathcal{C}$, such that conditions (10) through (13) are satisfied. To construct a $2q$-slot schedule, we assign transmissions $a_i \to R, b_j \to R, c_k \to R_2, \forall (i+j) = k$ to every odd slot $\{t_1, t_3, \cdots, t_{2q-1}\}$. Note that the relay node R receives a collided signal $(P_R(a_i) + P_R(b_j))$. To every even slot $\{t_2, t_4, \cdots, t_{2q}\}$ we assign the transmissions $R \to \{a_i, b_j\}$ and $e_l \to R_2$. In this way we schedule all $4q$ requests in $2q$ time slots. Now we prove that the obtained schedule is valid, i.e., all messages are decoded successfully.

First we look at the *odd* time slots. The SINR at receiver

R_2 is equal to:

$$\begin{aligned} SINR(R_2) &= \frac{P_{R_2}(c_k)}{P_{R_2}(a_i) + P_{R_2}(b_j)} \\ &= \frac{\frac{P}{r(c_k)^\alpha}}{\frac{P}{r(a_i)^\alpha} + \frac{P}{r(b_j)^\alpha}} = \frac{P\beta k}{P(i+j)} = \beta \quad (25) \end{aligned}$$

Now we check the conditions (6) and (7):

$$\begin{aligned} \frac{P_R(a_i)}{\sum_{\substack{s_j \neq a_i \\ s_j \neq b_j}} I_R(s_j)} = \frac{P_R(a_i)}{P_R(c_k)} = \frac{d(c_k, R)^\alpha}{d(a_i, R)^\alpha} &= \\ \frac{(r(R) + r(c_k))^\alpha}{(r(a_i)^2 + r(R)^2 - 2r(a_i)r(R)\cos\theta_a)^{\frac{\alpha}{2}}} &\geq \\ \frac{(r(R) + c_{\min})^\alpha}{(a_{\min}^2 + r(R)^2 - 2a_{\min}r(R)\cos\theta_a)^{\frac{\alpha}{2}}} &= \beta. \quad (26) \end{aligned}$$

The last inequality holds by plugging in the value of θ_a, defined in (19) (here we assume that θ_a is acute enough, s.t. $d(R, a_{\min}) > d(R, a_{\max})$.). Condition (7) is proved as in (26), using b_i instead of a_i and θ_b instead of θ_a.

Now we look at the *even* time slots. The SINR at receiver R_2 is equal to:

$$SINR(R_2) = \frac{P_{R_2}(e_l)}{P_{R_2}(R)} = \frac{\frac{P}{r(e_l)^\alpha}}{\frac{P}{r(R)^\alpha}} = \beta \quad (27)$$

And finally we check the conditions (8) and (9):

$$\begin{aligned} \frac{P_{a_i}(R)}{\sum_{s_j \neq R} I_{a_i}(s_j)} = \frac{P_{a_i}(R)}{P_{a_i}(e_l)} = \frac{d(e_l, a_i)^\alpha}{d(R, a_i)^\alpha} &= \\ \frac{(r(a_i)^2 + r(e_l)^2 - 2r(a_i)r(e_l)\cos(\pi - \theta_a))^{\frac{\alpha}{2}}}{(r(a_i)^2 + r(R)^2 - 2r(a_i)r(R)\cos\theta_a)^{\frac{\alpha}{2}}} &\geq \\ \frac{(a_{\min}^2 + r(e_l)^2 + 2a_{\min}r(e_l)\cos\theta_a)^{\frac{\alpha}{2}}}{(a_{\min}^2 + r(R)^2 - 2a_{\min}r(R)\cos\theta_a)^{\frac{\alpha}{2}}} &= \beta (28) \end{aligned}$$

Condition (9) is proved in the same way, only using b_i instead of a_i and θ_b instead of θ_a.

To sum up, we showed that in every odd time slot, conditions (6) and (7) hold for every relay node R participating in signal mixing; in every even time slot, conditions (8) and (9) hold for every sender a_i and b_j participating in signal network coding; every mixed packet forwarded by the relay node R can be decoded by at least one node in each group A and B, since exactly one node in every group is the sender of one of the mixed packets; and condition (2) holds for every transmission $\{c1, \cdots, c_q, e_1, \cdots, e_q\} \to R$ not employing network coding. This proves that our schedule guarantees successful decoding for all transmissions scheduled in each time slot $t \in \{t_1, \cdots, t_{2q}\}$.

($\Leftarrow$) For the second part of the claim, we need to show that if no solution to the NMTS problem exists, we cannot find a $2q$-slot schedule for the SACSM instance. No solution to NMTS implies that for at least one triple $(i, j, k), i \in \mathcal{A}, j \in \mathcal{B}, k \in \mathcal{C}$, it holds that $(i + j) > k$. Assume we could still find a *valid* schedule with only $2q$ slots. As we have already pointed out, transmissions from nodes $\{c_1, \cdots, c_q, e_1, \cdots, e_q\}$ to receiver R_2 have to be scheduled sequentially. So let's assume we have q time slots, in which senders $\{c_1, \cdots, c_q\}$ are scheduled, and another q time slots, in which senders $\{e_1, \cdots, e_q\}$ are scheduled. We will show that there is no way to schedule the remaining senders $\{a_1, \cdots, a_q, b_1, \cdots, b_q\}$ in parallel. First we look at time slots with an assigned sender e_l. Assume that at least one sender a_i or b_j transmits simultaneously. The SINR at R_2 would be:

$$\begin{aligned} SINR(R_2) &= \frac{P_{R_2}(e_l)}{P_{R_2}(a_i/b_i)} = \frac{\frac{P}{r(e_l)^\alpha}}{\frac{P}{r(a_i/b_i)^\alpha}} \\ &\leq \frac{\frac{P}{\left(\frac{\max(a_{\max}, b_{\max}) + \epsilon}{\beta^{\frac{1}{\alpha}}}\right)^\alpha}}{\frac{P}{a/b_{\max}^\alpha}} < \beta, \quad (29) \end{aligned}$$

where a_i/b_i and a/b is an abuse of notation, meaning "a_i or b_i" and "$a_{\max}$ or $b_{\max}$", respectively.

This means that all $2q$ senders $\{a_1, \cdots, a_q, b_1, \cdots, b_q\}$ have to be scheduled in the remaining q time slots, together with senders $\{c_1, \cdots, c_q\}$. Since signal mixing analog coding only applies to 2 simultaneous transmissions, one from set A and another from set B, exactly 2 senders $\{a_i, b_j\}$ have to be scheduled in each of these q time-slots. Now consider the time slot t, correspondent to triple $(a_i, b_j, c_k)|(i + j > k)$. The SINR at receiver R_2 is:

$$\begin{aligned} SINR(R_2) &= \frac{P_{R_2}(c_k)}{P_{R_2}(a_i) + P_{R_2}(b_j)} \\ &= \frac{\frac{P}{r(c_k)^\alpha}}{\frac{P}{r(a_i)^\alpha} + \frac{P}{r(b_j)^\alpha}} \\ &= \frac{P\beta k}{P(i+j)} < \beta, \quad (30) \end{aligned}$$

i.e., at least one transmission $c_k \to R_2$ cannot be decoded correctly within $2q$ time slots if there is no solution to the NMTS problem. This completes the proof. $\square$

5. COMPLEXITY OF SACF

In this section we prove that scheduling with analog coding by filtering is also NP-complete in the geometric SINR model.

To see that the decision version of the problem is in NP is straightforward. To decide whether a schedule of a given size T is feasible, we have to verify, for every transmission, whether there is a time slot assigned to it and if the conditions (4) and (5) are satisfied, considering the concurrently scheduled transmissions in each time-slot. Since computing the $SINR$ level for each transmission in its time slot can be done in $O(n^2)$ time, a schedule is an efficiently verifiable witness for this problem.

We proceed by presenting a polynomial-time reduction from *3-partition*, a problem closely related to the *subset sum* problem. 3-partition was proved to be NP-complete by Garey and Johnson in 1975 [3] and can be formulated as follows: Given a set $\mathcal{I}$ of integers, is it possible to partition this set into m subsets $\mathcal{I}_1, \cdots, \mathcal{I}_m$, such that the sum of the numbers in each subset is equal? The subsets $\mathcal{I}_1, \cdots, \mathcal{I}_m$ must form a partition in the sense that they are disjoint and they cover $\mathcal{I}$. Let σ denote the (desired) sum of each subset $\mathcal{I}_i$, or equivalently, let the total sum of the numbers in $\mathcal{I}$ be $m\sigma$. The 3-partition problem remains NP-complete when every integer in $\mathcal{I}$ is strictly between $\sigma/4$ and $\sigma/2$, in which case, each subset $\mathcal{I}_i$ is forced to consist of exactly three elements [3].

3-partition problem: Find $\mathcal{I}_1, \cdots, \mathcal{I}_m \subset \mathcal{I} = \{i_1, \ldots, i_n\}$

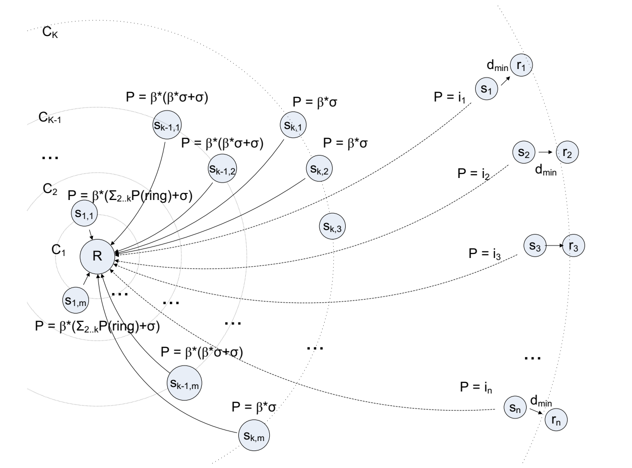

Figure 3: Reduction from 3-partition: all $(K \cdot m+n)$ links can be scheduled successfully in m time slots if and only if the senders $s_1, \cdots, s_n$, corresponding to the integers $i_1, \cdots, i_n$, are partitioned into m subsets, each summing up to exactly σ.

s.t.:

$$\begin{aligned}
\mathcal{I}_1 \cap \mathcal{I}_2 \cap \cdots \cap \mathcal{I}_m &= \emptyset, \\
\mathcal{I}_1 \cup \mathcal{I}_2 \cup \cdots \cup \mathcal{I}_m &= \mathcal{I}, \quad \text{and} \\
\sum_{i_j \in \mathcal{I}_1} i_j = \sum_{i_j \in \mathcal{I}_2} i_j = \cdots = \sum_{i_j \in \mathcal{I}_m} i_j &= \frac{1}{m} \sum_{i_j \in \mathcal{I}} i_j.
\end{aligned}$$

THEOREM 5.1. *3-partition is reducible to SACF in polynomial time.*

PROOF. The proof proceeds as follows. First, we define a many-to-one reduction from any instance of 3-partition to a geometric (Euclidean) instance of SACF. Then, we argue that the instance of SACF cannot be scheduled in $T \leq m$ time slots, but can be scheduled in $T = m$ time slots if and only if the instance of 3-partition is solved.

Consider a set $\mathcal{I} = \{i_1, \ldots, i_n\}$ of positive integers, where

$$\sum_{j=1}^{n} i_j = m \cdot \sigma, \qquad i_j < \frac{\sigma}{2}, \quad \forall i_j \in \mathcal{I}. \tag{31}$$

The instance of SACF is constructed by placing $(K \cdot m+n)$ senders and $(n+1)$ receivers in the plane in the following way (see Figure 3). First, one receiver R is placed at position $(0,0)$. Thereafter, K circles are drawn around R, and m senders are placed on each circle's circumference. The outermost circle C_K has radius $(P/\beta \cdot \sigma)^{1/\alpha}$. Each inner circle's radius is recursively determined as

$$r(C_K) = \left(\frac{1}{\beta \cdot \sigma}\right)^{\frac{1}{\alpha}} \tag{32}$$

$$r(C_i) = \beta^{\frac{1}{\alpha}} \cdot \left(\sum_{j=i+1}^{k} \frac{1}{r(C_j)^{\alpha}} + \sigma\right)^{\frac{1}{\alpha}}, \quad \forall i \in \{K-1, \cdots, 1\}. \tag{33}$$

The polar coordinates of each of m senders $s_{i,1}, \cdots, s_{i,m}$ placed on circumference C_i are:

$$\begin{aligned}
r(s_{i,j}) &= r(C_i), \qquad (34)\\
&\quad \forall i \in \{1 \cdots K\}, j \in \{1 \cdots m\}, \\
\theta(s_{i,j}) &\quad \text{is free.}
\end{aligned}$$

All the positioned $m \cdot K$ senders have as intended receiver the receiver R. Now we place the remaining n senders $s_1, \cdots, s_n$ and n receivers $r_1, \cdots, r_n$.

For each integer i_j in $\mathcal{I}$, we set the radial coordinate of s_j to $(P/i_j)^{1/\alpha}$ and leave its angular coordinate free.

$$\begin{aligned}
r(s_i) &= \left(\frac{1}{i_j}\right)^{1/\alpha}, \quad \forall i_j \in \mathcal{I}, \qquad (35)\\
\theta(s_i) &\quad \text{is free.}
\end{aligned}$$

Next we position the receivers $r_i, 1 \leq i \leq n$ at distance $d_{\min}$ to their corresponding senders s_i:

$$\begin{aligned}
r(r_i) &= r(s_i) + d_{\min}, \quad \text{where} \qquad (36)\\
d_{\min} &= \frac{\frac{1}{(i_{\max}-1)^{1/\alpha}} - \frac{1}{i_{\max}^{1/\alpha}}}{1 + ((n+K-1)\beta)^{\frac{1}{\alpha}}} \qquad (37)\\
\theta(r_i) &\quad \text{is free,}
\end{aligned}$$

and $i_{\max}$ is the maximal value of the integers in set $\mathcal{I}$.

Having defined the geometric instance of SACF for any instance of 3-partition, we proceed by showing that it cannot be scheduled in $T < m$ time slots using analog coding by filtering. For that, consider any pair of senders $s_{i,x}, s_{i,y}$ positioned at the same circumference C_i. Since they are equidistant from their intended receiver R, the power perceived at R is the same:

$$\frac{P_R(s_{i,x})}{P_R(s_{i,y})} = 1 < \beta, \quad \forall s_{i,x}, s_{i,y} \in C_i, i \in \{1 \cdots K\}.$$

Given that the power levels of any pair of such transmissions do not differ, SINR conditions (4) and (5) cannot be fulfilled, and R cannot decode them simultaneously. Since this argument applies to any pair of senders belonging to the same circumference, and that there are m senders in each circumference, at least m time slots are needed to schedule any m-tuple of such requests.

To proceed with the proof, we first need Lemma 5.2, in which we show that each receiver $r_i \in \{r_1, \ldots, r_n\}$, corresponding to an integer $i \in \mathcal{I}$, is close enough to its respective sender to guarantee successful transmission, regardless of other links scheduled simultaneously. Since no two senders $s_{i,x}, s_{i,y}$ positioned at the same circumference C_i can be scheduled simultaneously, we assume that at most $(K+n)$ senders are scheduled in the same time slot as r_i, i.e., one sender in each of K circumferences, plus n senders s_i, corresponding to the n integers in $\mathcal{I}$.

LEMMA 5.2. *Consider a time slot t, in which $(n+K)$ senders are scheduled to transmit (one sender in each of K circumferences, plus n senders s_i, corresponding to the n integers in $\mathcal{I}$). It holds that for every receiver $r_i \in \{r_1, \ldots, r_n\}$, r_i decodes its message successfully, i.e., constraints (4) and (5) are satisfied.*

PROOF. We start by establishing a minimal distance between a receiver $r_i \in \{r_1, \ldots, r_n\}$ and any interfering server $s_j, j \neq i$ or $s_{x,y}, x \in \{1, \cdots, K\}, y \in \{1, \cdots, m\}$.

Since the positions of senders $s_1, \ldots, s_n$ depend on the integers $i_1, \ldots, i_n$, we can determine the minimum distance between two sender nodes s_i, s_j.

$$\begin{aligned} d(s_i, s_j) &= |d(s_i, R) - d(s_j, R)| \\ &= \left| \left(\frac{1}{i_i}\right)^{\frac{1}{\alpha}} - \left(\frac{1}{i_j}\right)^{\frac{1}{\alpha}} \right| \\ &\geq \frac{1}{(i_{\max} - 1)^{1/\alpha}} - \frac{1}{i_{\max}^{1/\alpha}} \\ &= d_{\min}\left(1 + ((n+K-1)\beta)^{\frac{1}{\alpha}}\right). \end{aligned} \tag{38}$$

We proceed by showing that any sender $s_{x,y}$ positioned on a circumference $C_x, x \in \{1, \cdots, K\}$, is even farther away:

$$\begin{aligned} d(s_i, s_{x,y}) &= |d(s_i, R) - d(s_{x,y}, R)| \\ &\geq \frac{1}{i_{\max}^{1/\alpha}} - \frac{1}{(\beta \cdot \sigma)^{1/\alpha}} \quad (39) \\ &\geq \frac{1}{(i_{\max} - 1)^{1/\alpha}} - \frac{1}{i_{\max}^{1/\alpha}} \quad (40) \\ &= \min\left(d(s_i, s_j)\right), \end{aligned}$$

where (39) and (40) hold because $\sigma > 2 \cdot i_{\max}$, $\beta > 1$, and $i_{\max} \geq 1$. (i.e., $((\sigma \cdot \beta) - i_{\max}) \geq (i_{\max} - (i_{\max} - 1))$)

By triangular inequality, we have:

$$\begin{aligned} d(s_j, r_i) &\geq d(s_i, s_j) - d_{\min} \\ &= d_{\min} \cdot ((n+K-1)\beta)^{\frac{1}{\alpha}}, \\ &\quad \forall i, j \in \mathcal{I}, i \neq j. \end{aligned} \tag{41}$$

This suffices to show that constraints (4) and (5) are satisfied for any receiver $r_i, i \in \{1, \cdots, n\}$. Since $d(s_j, r_i) > d_{\min} = d(s_i, r_i)$, the power received at r_i from s_i is stronger than from any other concurrent transmissions. Therefore, constraint (4) does not apply, and we only need to show that constraint (5) is satisfied:

$$\begin{aligned} \frac{P_{r_i}(s_i)}{\sum_{P_{r_i}(s_j) < P_{r_i}(s_i)} P_{r_i}(s_j)} &\geq \quad (42) \\ \frac{\frac{P}{d_{\min}^{\alpha}}}{(n+K-1) \cdot \frac{P}{d(s_j, r_i)^{\alpha}}} &\geq \quad (43) \\ \frac{\frac{1}{d_{\min}^{\alpha}}}{\frac{(n+K-1)}{\left(d_{\min} \cdot ((n+K-1)\beta)^{\frac{1}{\alpha}}\right)^{\alpha}}} &= \beta. \quad (44) \end{aligned}$$

□

Having proved that successful transmission is guaranteed for receivers $r_1, \ldots r_n$ under concurrent transmission of K senders $s_{x,y}$ positioned at different circumferences $C_x, x \in \{1, \cdots, K\}$ and any number of senders $s_j, j \in \{1, \cdots, n\}$ corresponding to the integers in the 3-partition instance, we now return to the proof of Theorem 5.1.

We claim that there exists a solution to the 3-partition problem if and only if there exists an m-slot schedule for the problem instance defined in equations (32) through (37).

($\Rightarrow$) For the first part of the claim, assume we know m subsets $\mathcal{I}_1, \cdots, \mathcal{I}_m \subset \mathcal{I}$, whose elements sum up to σ. To construct an m-slot schedule, $\forall i_j \in \mathcal{I}_1$, we assign the corresponding sender s_j to time slot 1, along with K senders $s_{1,1}, s_{2,1}, \cdots, s_{K,1}$. For every $i_j \in \mathcal{I}_2$, we assign the corresponding sender s_j to time slot 2, along with K senders $s_{1,2}, \cdots, s_{K,2}$. And so on until senders s_j corresponding to $i_j \in \mathcal{I}_m$ are assigned to time slot m, along with K senders $s_{1,m}, \cdots, s_{K,m}$. In this way we scheduled all $mK + n$ requests in m time slots. Now we prove that the obtained schedule is valid, i. e., all messages are decoded successfully.

Due to Lemma 5.2, we can assume that all senders $s_i, i \in \{1, \cdots, n\}$ transmit successfully and focus our analysis on the senders $s_{1,t}, \cdots, s_{K,t}, t \in \{1, \cdots, m\}$. Since in each time slot t only K senders positioned on distinct circumferences are scheduled together, the situation is the same in each t. Therefore, we only look at one time slot and show that all K transmissions are decoded successfully at receiver R.

The signal power R receives from each sender $s_{i,t}, i \in \{1, \cdots, K\}$ is equal to

$$P_R(s_{i,t}) = \frac{P}{r(s_i)^{\alpha}} = \frac{P}{\beta \cdot \left(\sum_{j=i+1}^{K} \frac{1}{r(C_j)^{\alpha}} + \sigma\right)}. \tag{45}$$

The interference R experiences from concurrently scheduled senders is

$$\begin{aligned} I_R(s_{i,t}) &= \sum_{j=i+1}^{K} P_R(s_{j,t}) + \sum_{s_j \in \mathcal{I}_t} P_R(s_j) \\ &= \sum_{j=i+1}^{K} \frac{P}{r(s_{j,t})^{\alpha}} + \sum_{s_j \in \mathcal{I}_t} P \cdot i_j \\ &= P \cdot \left(\sum_{j=i+1}^{K} \frac{1}{r(C_j)^{\alpha}} + \sigma\right), \end{aligned} \tag{46}$$

Therefore, using the notation introduced in Section 3, we show that condition (3) holds $\forall s_{x,t} \in \Upsilon = \{s_{1,t}, \cdots, s_{K,t}\}$ and, therefore, all K senders in Υ transmit successfully to receiver R in time slot t:

$$\begin{aligned} \frac{P_R(s_{x,t})}{\sum_{\substack{P_R(s_{y,t}) \in \Upsilon, \\ P_R(s_{y,t}) < P_R(s_{x,t})}} P_R(s_{y,t}) + \sum_{P_R(s_z) \notin \Upsilon} P_R(s_z)} &= \\ \frac{P_R(s_{i,t})}{I_R(s_{i,t})} = \frac{\frac{P}{\beta \cdot \left(\sum_{j=i+1}^{K} \frac{1}{r(C_j)^{\alpha}} + \sigma\right)}}{P \cdot \left(\sum_{j=i+1}^{K} \frac{1}{r(C_j)^{\alpha}} + \sigma\right)} &= \beta, \end{aligned}$$

which, in combination with Lemma 5.2, proves that our schedule guarantees successful decoding for all transmissions scheduled in each time slot $t \in \{1, \cdots, m\}$.

($\Leftarrow$) For the second part of the claim, we need to show that if no solution to the 3-partition problem exists, we cannot find an m-slot schedule for our scheduling instance. No solution to 3-partition implies that for every partition of $\mathcal{I}$ into m subsets, the sum of one set $\mathcal{I}_t$ is greater than σ. Assume we could still find a schedule with only m slots. As we have already pointed out, senders positioned on the same circumference $C_i, i \in \{1, \cdots, K\}$ have to be scheduled separately. Therefore, in each time slot $t \in \{1, \cdots, m\}$, exactly one sender positioned on each circumference C_i has to be scheduled. We argue that it is not possible to schedule n senders s_j correspondent to the integers $i_j \in \{1, \cdots, n\}$ concurrently. Consider a time slot t, a sender $s_{K,t}$, positioned on the outermost circumference C_K, and a subset $\mathcal{I}_t$ of integers such that $\sum_{i_j \in \mathcal{I}_t} i_j > \sigma$. To prove that $s_{K,t}$'s transmission cannot be decoded correctly at receiver R, we can ignore the $(K-1)$ senders positioned on inner circumferences and only analyze the senders s_j correspondent to the integers $i_j \in \mathcal{I}_t$. We show that neither condition (4) nor (5) are satisfied at receiver R. To show that (4) does not hold, we observe that the ratio of the power levels of $s_{K,t}$ and s_j is always below β and, therefore, s_j's signal cannot be filtered out at receiver R, $\forall i_j \in \mathcal{I}_t$.

$$\frac{P_R(s_j)}{P_R(s_{K,t})} \leq \frac{\frac{P}{\left(\frac{1}{i_{\max}^{1/\alpha}}\right)^{\alpha}}}{\frac{P}{\left(\frac{1}{(\beta\sigma)^{1/\alpha}}\right)^{\alpha}}} = \frac{i_{\max}}{\beta\sigma} < \beta, \qquad (47)$$

where the last inequality holds since $i_{\max} < \sigma/2$.

Now we show that (5) also does not hold, since the sum of set $\mathcal{I}_t$ is greater than σ.

$$\frac{P_R(s_{K,t})}{\sum_{s_j \in \mathcal{I}_t} P_R(s_j)} = \frac{\frac{P}{\left(\frac{1}{(\beta\sigma)^{1/\alpha}}\right)^{\alpha}}}{P \sum_{i_j \in \mathcal{I}_t} \frac{1}{\left(1/i_j^{1/\alpha}\right)^{\alpha}}} < \frac{\beta\sigma}{\sigma} = \beta, \qquad (48)$$

Since neither condition (4) nor (5) are satisfied for link $s_{K,t} \rightarrow R$ when the sum of subset $\mathcal{I}_t$ is greater that σ, the transmission cannot be decoded successfully and the schedule needs more than m time slots. This completes the proof of Theorem 5.1.

6. CONCLUSION

In this work we wanted to obtain some understanding of the complexity of scheduling wireless links with analog network coding capability. Given that network coding changes the definition of a successful transmission, allowing a receiver to decode several messages simultaneously, it is interesting to analyze whether the complexity of the scheduling problem is altered. By showing that the problem remains NP-complete, we can conclude that the basic difficulties of scheduling wireless requests in a global interference model, such as geometric SINR, remain challenging even with coding capability.

The question whether network coding opens possibilities to better approximation algorithms is an interesting subject for future research. To the extent of our knowledge, there is no constant approximation algorithm to schedule wireless links in the physical interference model, assuming nodes can be arbitrarily (not uniformly at random) distributed in the Euclidean plane.

7. REFERENCES

[1] R. Ahlswede, N. Cai, S. Y. R. Li, and R. W. Yeung. Network information flow. *IEEE Trans. on Inf. Theory*, 46(4):1204–1216, 2000.

[2] M. Chatterjee, H. Lin, and S. K. Das. Rate Allocation and Admission Control for Differentiated Services in CDMA Data Networks. *IEEE Transactions on Mobile Computing*, 6(2):179–191, 2007.

[3] M. R. Garey and D. S. Johnson. Complexity results for multiprocessor scheduling under resource constraints. *SIAM Journal on Computing*, page 397, 1975.

[4] M. R. Garey and D. S. Johnson. *Computers and Intractability : A Guide to the Theory of NP-Completeness.* W. H. Freeman, 1979.

[5] O. Goussevskaia, Y. A. Oswald, and R. Wattenhofer. Complexity in geometric SINR. In *MOBIHOC*, pages 100–109, 2007.

[6] P. Gupta and P. R. Kumar. Critical Power for Asymptotic Connectivity in Wireless Networks. *Stochastic Analysis, Control, Optimization and Applications*, pages 547–566, 1998.

[7] P. Gupta and P. R. Kumar. The Capacity of Wireless Networks. *IEEE Trans. Inf. Theory*, 46(2):388–404, 2000.

[8] J. Hamkins. Joint viterbi algorithm to separate cochannel fm signals. In *IEEE Int. Conf. Acoustics, Speech, Signal Processing*, 1998.

[9] J. Hamkins. An analytic technique to separate cochannel fm signals. *IEEE Transactions on Communications*, 48(4):543–546, 2000.

[10] T. Ho, M. Medard, R. Koetter, D. R. Karger, M. Effros, J. Shi, and B. Leong. A random linear network coding approach to multicast. *IEEE Trans. on Inf. Theory*, 52(10):4413–4430, 2006.

[11] H. Hunt, M. Marathe, V. Radhakrishnan, S. Ravi, D. Rosenkrantz, and R. Stearns. NC-Approximation Schemes for NP- and PSPACE-Hard Problems for Geometric Graphs. *Journal of Algorithms*, 26, 1998.

[12] S. Katti, S. Gollakota, and D. Katabi. Embracing wireless interference: analog network coding. In *SIGCOMM*, pages 397–408. ACM Press, 2007.

[13] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft. XORs in the air: practical wireless network coding. In *SIGCOMM*, pages 243–254, 2006.

[14] I. T. L. Kozat, U.C.; Koutsopoulos. Cross-layer design for power efficiency and qos provisioning in multi-hop wireless networks. *IEEE Trans. on Wireless Communications*, 5, 2006.

[15] S. O. Krumke, M. Marathe, and S. Ravi. Models and approximation algorithms for channel assignment in radio networks. *Wireless Networks*, 6:575–584.

[16] K. Leung and L. Wang. Integrated link adaptation and power control for wireless ip networks, 2000.

[17] S. Y. R. Li, R. W. Yeung, and N. Cai. Linear network coding. *Information Theory, IEEE Transactions on*, 49(2):371–381, 2003.

[18] T. Moscibroda, R. Wattenhofer, and Y. Weber. Protocol Design Beyond Graph-based Models. In *HotNets*, 2006.

[19] P. Sanders, S. Egner, and L. Tolhuizen. Polynomial time algorithms for network information flow. In *SPAA*, pages 286–294, 2003.

[20] S. Singh and C. S. Raghavendra. PAMAS - Power Aware Multi-Access Protocol with Signalling for Ad Hoc Networks. *SIGCOMM Comput. Commun. Rev.*, 28(3):5–26, 1998.

[21] Y. Wu. Broadcasting when receivers know some messages a priori. In *IEEE Int. Symp. Inf. Theory*, 2007.

[22] Y. Wu, P. A. Chou, and S.-Y. Kung. Minimum-energy multicast in mobile ad hoc networks using network coding. *IEEE Transactions on Communications*, 53(11):1906–1918, 2005.

[23] S. Zhang, S. C. Liew, and P. P. Lam. Hot topic: physical-layer network coding. In *MOBICOM*, pages 358–365, 2006.