Are Financial Networks Highly Complex?

Roger Wattenhofer
Are financial markets too complex?

The global credit market has plummeted and the collapse of various hedge funds has raised fears that the credit market conflagration could become a true economic crisis.

Richard Bookstaber, a former academic, who went on to head risk management for Morgan Stanley, and now runs a large hedge fund at FrontPoint Partners, argues in his book A Demon of Our Own Making that the financial system is divided into two parts that are driven by two different sets of incentives and rules...
Financial Networks
Financial Network
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Assets, Liabilities, Default, and Recovery Rate
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Assets, Liabilities, Default, and Recovery Rate

\[ a_u \geq l_u \Rightarrow r_u = 1 \]

\[ a_v < l_v \Rightarrow r_v = \frac{a_v}{l_v} = \frac{2}{4} = \frac{1}{2} \]
IT'S THE ECONOMY STUPID!
economic crisis (2008) = many companies involved = network
Bailouts on a Budget (of 3)
Bailouts are NP hard.*

* in some models.
Complexity
Complexity

David Hilbert

Kurt Gödel
Background
Sorting
Sorting vs. Partition

<0.01s  22 Players  <0.01s
<table>
<thead>
<tr>
<th>Sorting</th>
<th>vs.</th>
<th>Partition</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.01s</td>
<td>22 Players</td>
<td>&lt;0.01s</td>
</tr>
<tr>
<td>&lt;0.01s</td>
<td>222 Players</td>
<td>...</td>
</tr>
</tbody>
</table>
Sorting

Partition

Operations vs. Elements graph showing different time complexities for sorting algorithms.
Bailouts are NP hard.*

*in some models.
Conditional Debt
Debt = “Long” Position (Positive)

Conditional Debt = “Short” (Negative)
Short Positions

ABS: Asset-Backed Securities
CDO: Collateralized Dept Obligations
CDS: Credit Default Swaps
CLS: Collateralized Loan Obligations
MBS: Mortgage-Backed Securities

...”Financial Weapons of Mass Destruction” (Warren Buffet)
Conditional Debt Contracts

\[ 2 \cdot (1 - r_u) \]

\[ u \]

\[ v \]

\[ w \]
Conditional Debt Contracts

\[ u = 2 \cdot (1 - r_u) \]
Improve Situation
Can Bank $\nu$ Improve?

\[ a_\nu = \frac{1}{2} \]

\[ 2 \cdot (1 - r_w) \]

\[ 2 \cdot (1 - r_u) \]
Can Bank $\nu$ Improve?

$av = 2$

$2 \cdot (1 - ru)$

$2 \cdot (1 - rw)$
Can Bank $v$ Improve?

$$a_v = 2$$

$$2 \cdot (1 - r_u)$$

$$2 \cdot (1 - r_w)$$
Can Bank $v$ Improve?

$2 \cdot (1 - r_u)$

$2 \cdot (1 - r_w)$
A Loss Can Be a Win.
How to Make Money for Nothing Like Wall Street

Credit default swaps might not be financial WMDs anymore, but Wall Street can still game them to make guaranteed profits.
Optimization
Optimize What?

Smallest amount of total debt unpaid

Best for a specific bank

Smallest percentage of liabilities in the system unpaid

Smallest number of defaults

Preferred by highest number of banks

Most representative of the solution space

Worst for a specific bank

Most balanced for two alliances of banks
All these (and more) are NP hard.
Building Circuits: NOT Gate

\[ u \quad r_u \in \{0,1\} \]

\[ r_v = \text{NOT} \quad r_u \]
Building Circuits: OR Gate

\[ r_w = r_u \ OR \ r_v \]
Financial Networks are Computers.
Sequential Defaults
An Example

2 \cdot (1-r_u)

2 \cdot (1-r_v)

\infty

u_0

1

v_0

1

u_0

1

v_0
An Example

$2 \cdot (1-r_u)$

$2 \cdot (1-r_v)$

$r_u = 0, r_v = 1$

$r_u = 1, r_v = 0$

$r_u = \frac{2}{3}, r_v = \frac{2}{3}$
An Example

\[ 2 \cdot (1 - r_v) \]

\[ 2 \cdot (1 - r_u) \]

\[ ru = 0, r_v = 1 \]

\[ ru = 1, r_v = 0 \]

\[ ru = \frac{2}{3}, r_v = \frac{2}{3} \]
Resurrection!
Basic Properties

Defaults are reversible

Some equilibria not reachable

Might not stabilize

Depends on order of updates
Basic Properties

Defaults are reversible

Some equilibria not reachable

Might not stabilize $\rightarrow$ stabilization time?

Depends on order of updates $\rightarrow$ best order?
Basic Properties

Defaults are reversible

Some equilibria not reachable

Might not stabilize $\rightarrow$ stabilization time? $\rightarrow$ $\theta(2^n)$

Depends on order of updates $\rightarrow$ best order?
Basic Properties

Defaults are reversible

Some equilibria not reachable

Might not stabilize \(\rightarrow\) stabilization time? \(\rightarrow\) \(\theta(2^n)\)

Depends on order of updates \(\rightarrow\) best order? \(\rightarrow\) NP-hard
Reduction from MAXSAT

variables

\[ 1 - r_u \]
\[ 1 - r_v \]
\[ x_0 \]
\[ \overline{x}_0 \]

clauses

\[ x_1 \]
\[ x_3 \]
\[ x_4 \]
Best Order from Specific Point of View?
Reducing Risk
Hard → Easy?
<table>
<thead>
<tr>
<th></th>
<th>no default costs</th>
<th>default costs</th>
</tr>
</thead>
</table>
| **long positions only** | 1 maximal solution  
Easy to find                          | 1 maximal solution  
Easy to find                          |
| **+ covered shorts**    | 1 maximal solution  
Easy to find                          | 1 maximal solution  
Easy to find                          |
| **+ any shorts**         | Many Pareto-optimal solutions  
Any/best one hard to find  
*with money or cycle restrictions:*  
best one still hard to find  
*with money+cycle restrictions:*  
unique solution, easy to find | Many Pareto-optimal solutions  
Any/best one hard to find  
*with money restrictions:*  
Possibly no solution  
*with cycle restrictions:*  
a solution always exists  
*with money+cycle restrictions:*  
best one still hard to find |
In Short, What Regulators Should Do

No Uncovered Short Positions
Thank You!

Questions & Comments?