Does Topology Control Reduce Interference?

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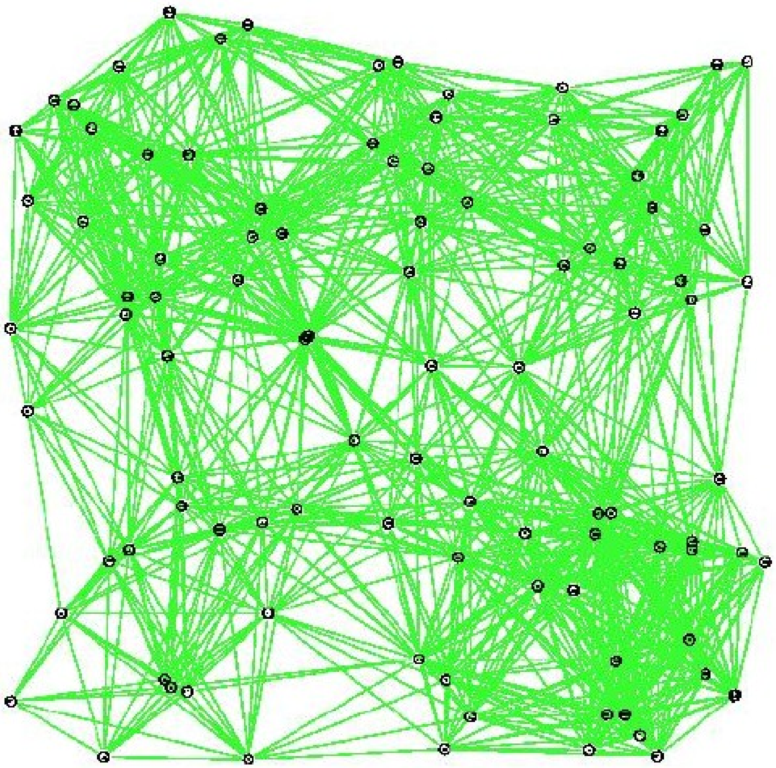
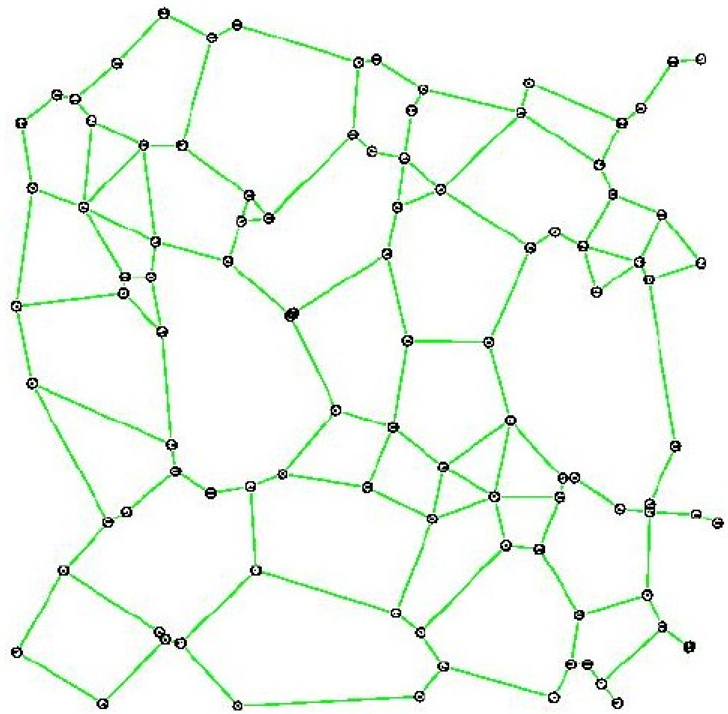
Overview

- What is Topology Control?
- Context – related work
- Explicit interference model
- Interference in known topologies
- Algorithms
  - Connectivity-preserving and spanner topologies
  - Worst case, average case
- Conclusions

Topology Control



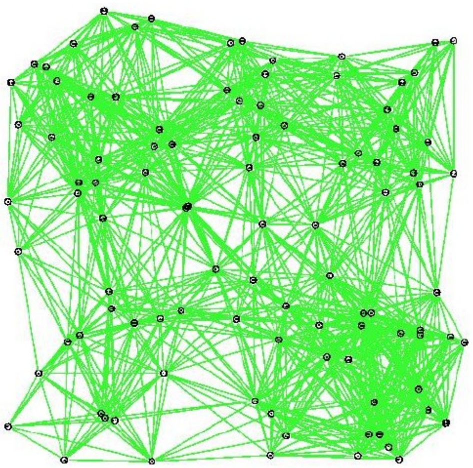


- Drop long-range neighbors: Reduces interference and energy!
- But still stay connected (or even spanner)

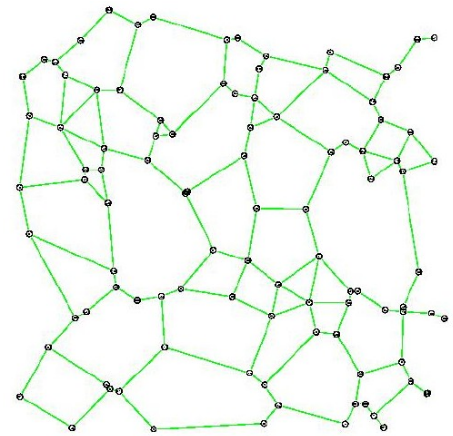
Topology Control as a Trade-Off

Sometimes also clustering, Dominating Set construction

Not in this presentation



Topology Control

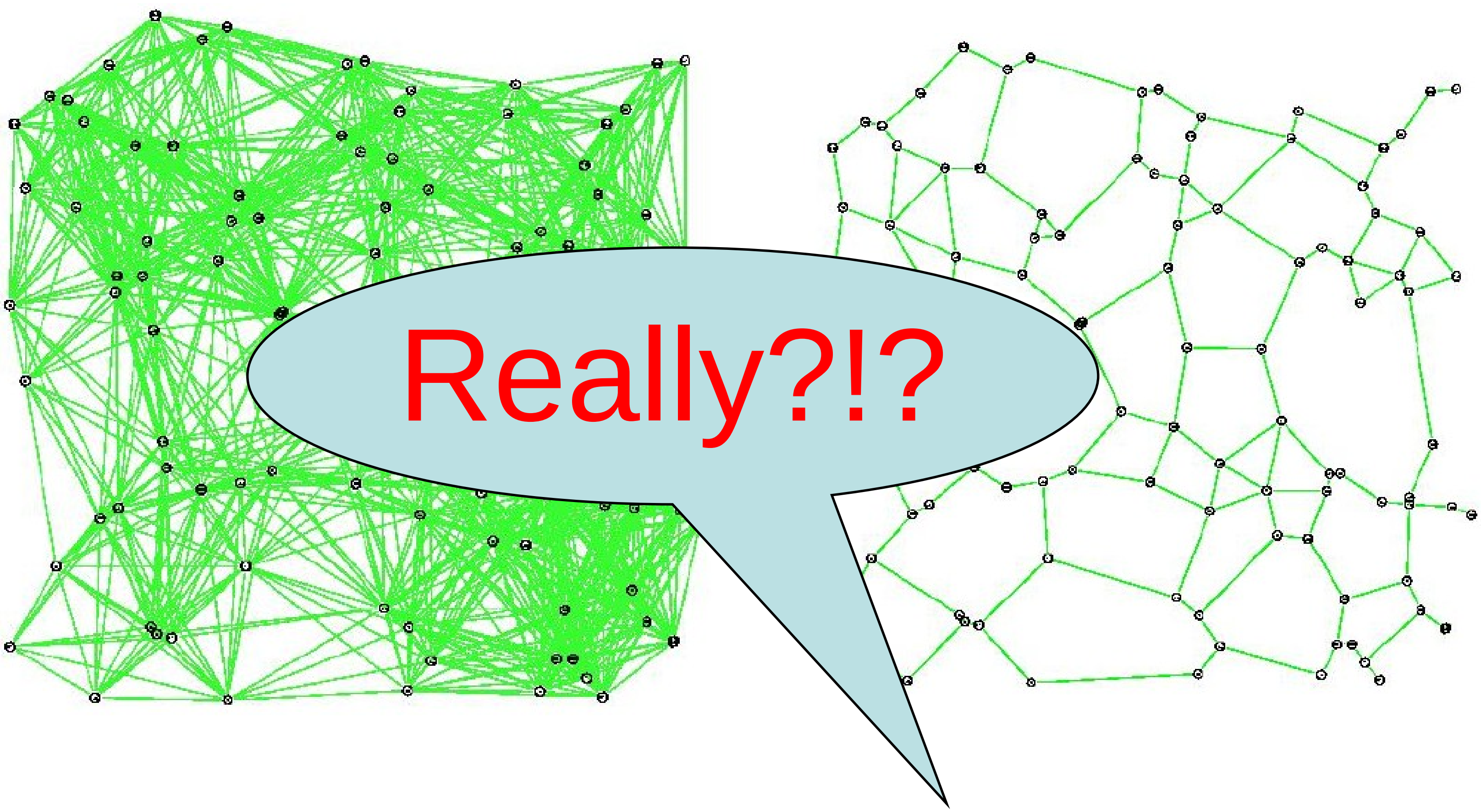


Network Connectivity
Spanner Property

$d(u,v) \cdot t \geq d_{TC}(u,v)$

Conserve Energy
Reduce Interference

Topology Control



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Context – Previous Work

- Mid-Eighties: randomly distributed nodes [Takagi & Kleinrock 1984, Hou & Li 1986]
- Second Wave: constructions from computational geometry, Delaunay Triangulation [Hu 1993], Minimum Spanning Tree [Ramanathan & Rosales-Hain INFOCOM 2000], Gabriel Graph [Rodoplu & Meng J.Sel.Ar.Com 1999]
- Cone-Based Topology Control [Wattenhofer et al. INFOCOM 2000]; explicitly prove several properties (energy spanner, sparse graph), locality
- Collecting more and more properties [Li et al. PODC 2001, Jia et al. SPAA 2003, Li et al. INFOCOM 2002] (e.g. local, planar, distance and energy spanner, constant node degree [Wang & Li DIALM-POMC 2003])

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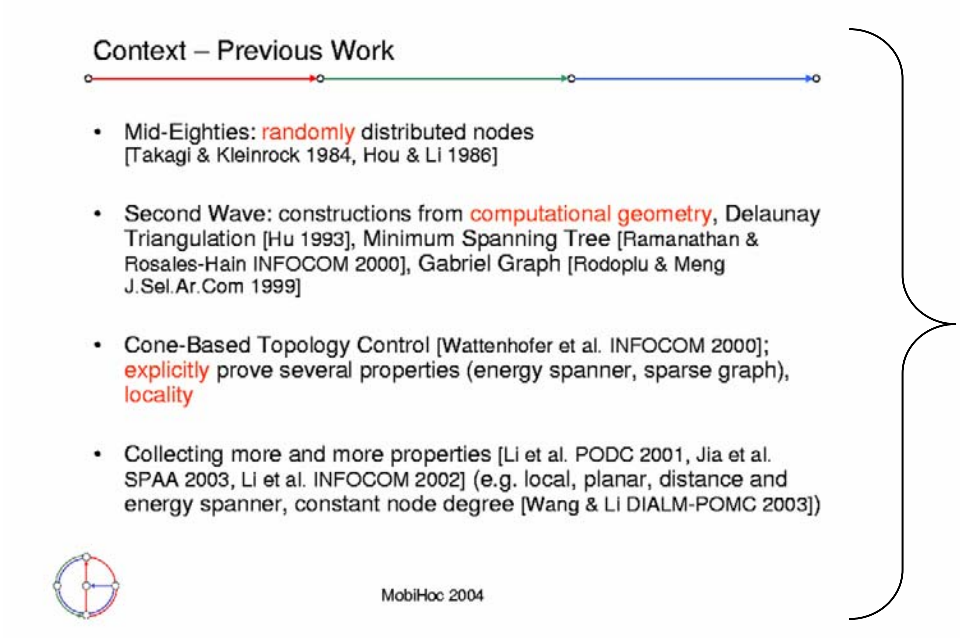
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MobiHoc 2004

Interference issue “solved” implicitly by graph sparseness or bounded degree

Explicit interference [Meyer auf der Heide et al. SPAA 2002]

– Interference between edges, time-step routing model, congestion
– Trade-offs: congestion, power consumption, dilation

– Interference model based on network traffic

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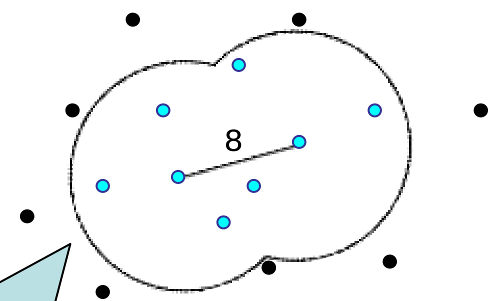
What Is Interference?

- Model
  - Transmitting edge *e* = (*u*,*v*) disturbs all nodes in vicinity
  - Interference of edge *e* =
    # Nodes covered by union of the two circles
    with center u and v, respectively, and radius |*e*|

- Problem statement
  - We want to minimize maximum interference!
  - At the same time topology must be connected or a spanner etc.



Exact size of interference range does not change the results

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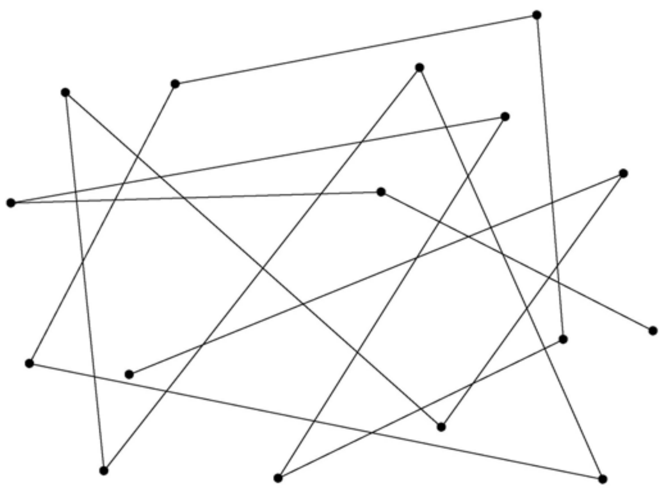
Low Node Degree Topology Control?

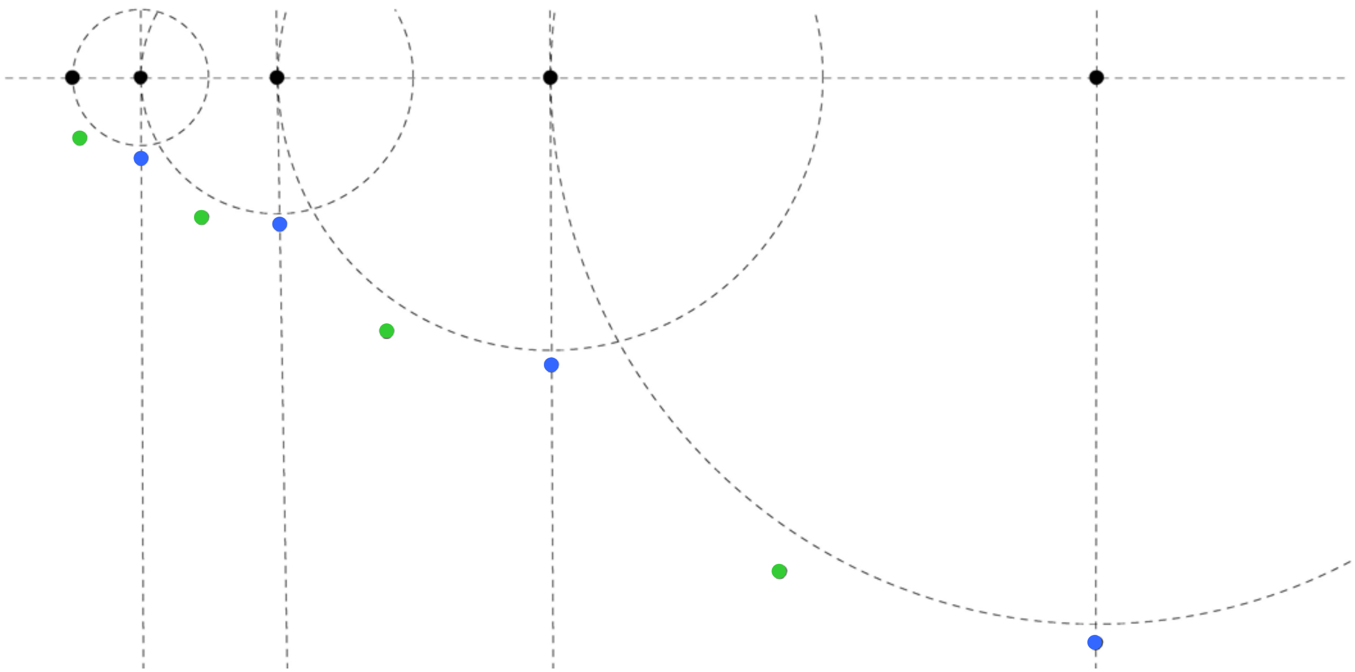
Low node degree does not necessarily imply low interference:



Very low node degree but huge interference

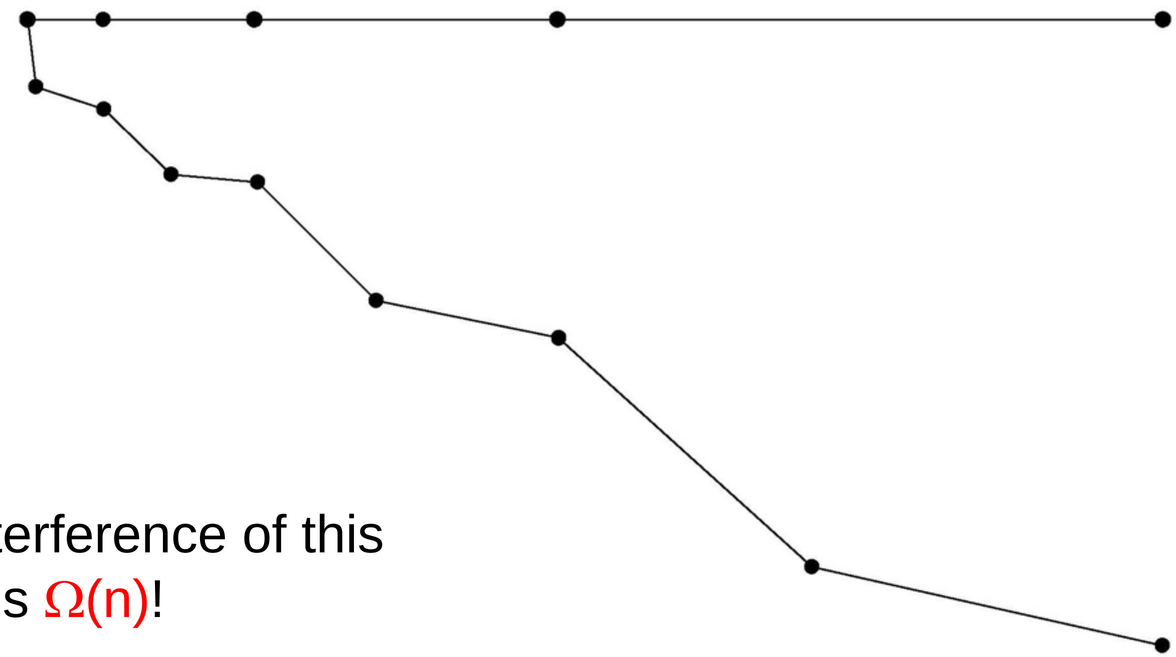
Let’s Study the Following Topology!

…from a worst-case perspective



Topology Control Algorithms Produce…

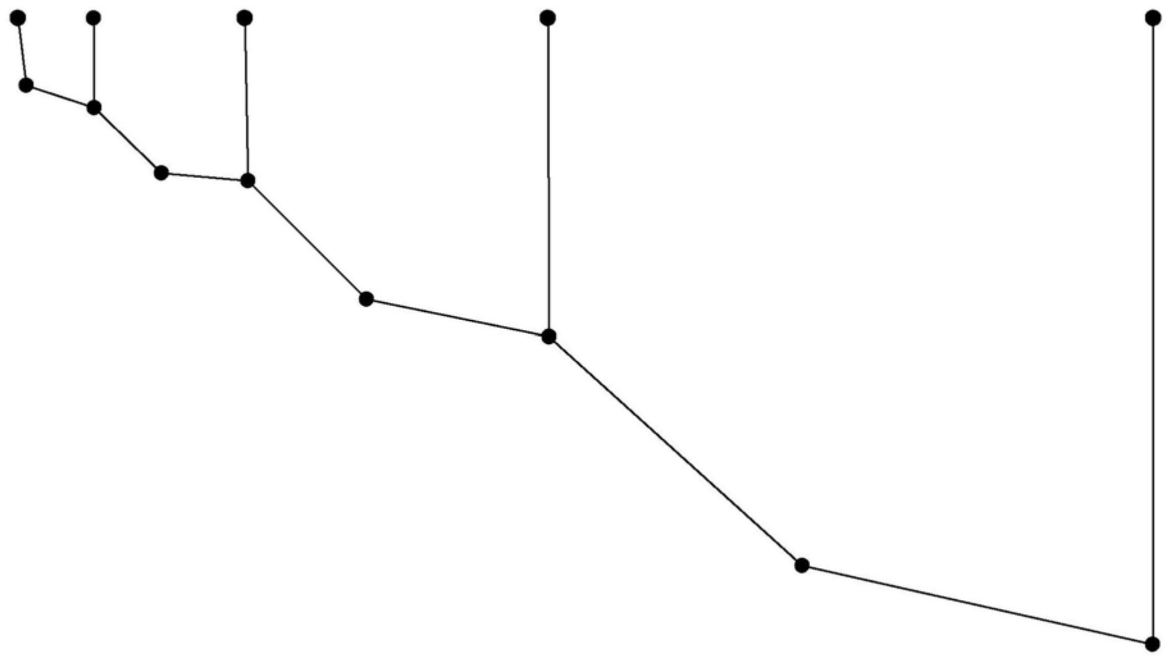
- All known topology control algorithms (with symmetric edges) include the nearest neighbor forest as a subgraph and produce something like this:



- The interference of this graph is $\Omega(n)$!

But Interference…

- Interference does not need to be high…

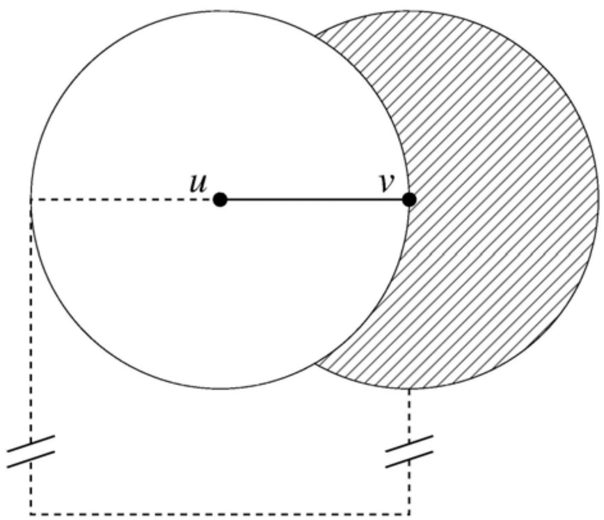


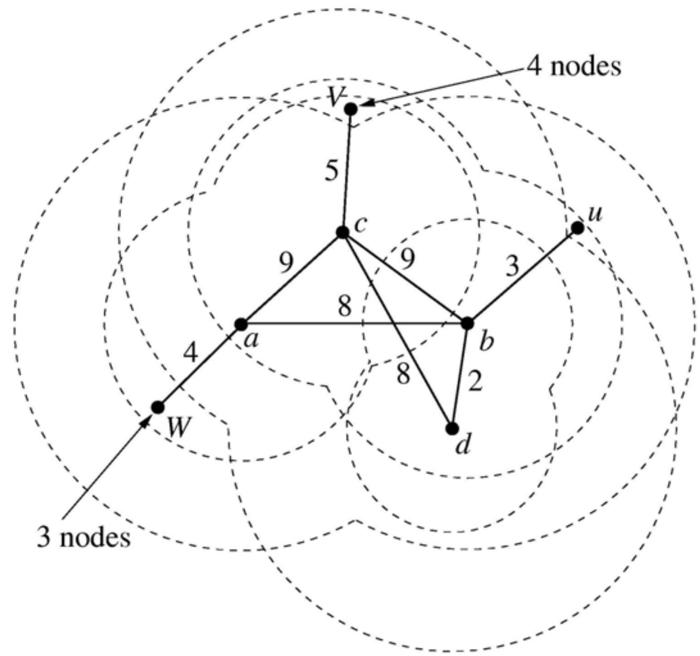
- This topology has interference O(1)!!

Interference-Optimal Topology

There is no local algorithm that can find a good interference topology

The optimal topology will not be planar





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Algorithms – Requirement: Retain Graph Connectivity

- LIFE (Low Interference Forest Establisher)
- Attribute interference values as weights to edges
- Compute minimum spanning tree/forest (Kruskal's algorithm)

Low Interference Forest Establisher (LIFE)

Input: a set of nodes V, each $v \in V$ having attributed a maximum transmission radius r_v^{max}

```
E = all eligible edges (u,v) (r_u^max ≥ |u,v| and r_v^max ≥ |u,v|) (* unprocessed edges *)
E_LIFE = ∅
G_LIFE = (V, E_LIFE)
while E ≠ ∅ do
    e = (u,v) ∈ E with minimum coverage
    if u, v are not connected in G_LIFE then
        E_LIFE = E_LIFE ∪ {e}
    end if
    E = E \ {e}
end while
```

Output: Graph G_{LIFE}

> Theorem: LIFE constructs a Minimum Interference Forest

Proof:

- Algorithm computes forest
- MST also minimizes maximum interference value

Algorithms – Requirement: Construct Spanner

- LISE (Low Interference Spanner Establisher)
- Add edges with increasing interference until spanner property fulfilled

Low Interference Spanner Establisher (LISE)

Input: a set of nodes V, each $v \in V$ having attributed a maximum transmission radius r_v^{max}

E = all eligible edges (u,v) ($r_u^{max} \geq |u,v|$ and $r_v^{max} \geq |u,v|$) (∗ unprocessed edges ∗)

$E_{LISE} = \emptyset$

$G_{LISE} = (V, E_{LISE})$

while $E \neq \emptyset$ **do**

  $e = (u,v) \in E$ with maximum coverage

  while $|p^*(u,v)$ in $G_{LISE}| > t\,|u,v|$ **do**

    f = edge $\in E$ with minimum coverage

    move all edges $\in E$ with coverage $Cov(f)$ to E_{LISE}

  end while

  $E = E \setminus \{e\}$

end while

Output: Graph G_{LISE}

Theorem: LISE constructs a Minimum Interference t-Spanner

Proof:

- Algorithm computes t-spanner
- Algorithm inserts edges with increasing coverage only "as long as necessary"

Algorithms – Requirement: Construct Spanner Locally

- LLISE
- Local algorithm: scalable
- Nodes collect (t/2)-neighborhood
- Locally compute interference-minimal paths guaranteeing spanner property
- Only request that path to stay in the resulting topology

LLISE

collect $(\frac{t}{2})$-neighborhood $G_N = (V_N, E_N)$ of $G = (V, E)$

$E' = \emptyset$

$G' = (V_N, E')$

repeat

    f = edge $\in E_N$ with minimum coverage

    move all edges $\in E_N$ with coverage $Cov(f)$ to E'

    p = shortestPath$(u - v)$ in G'

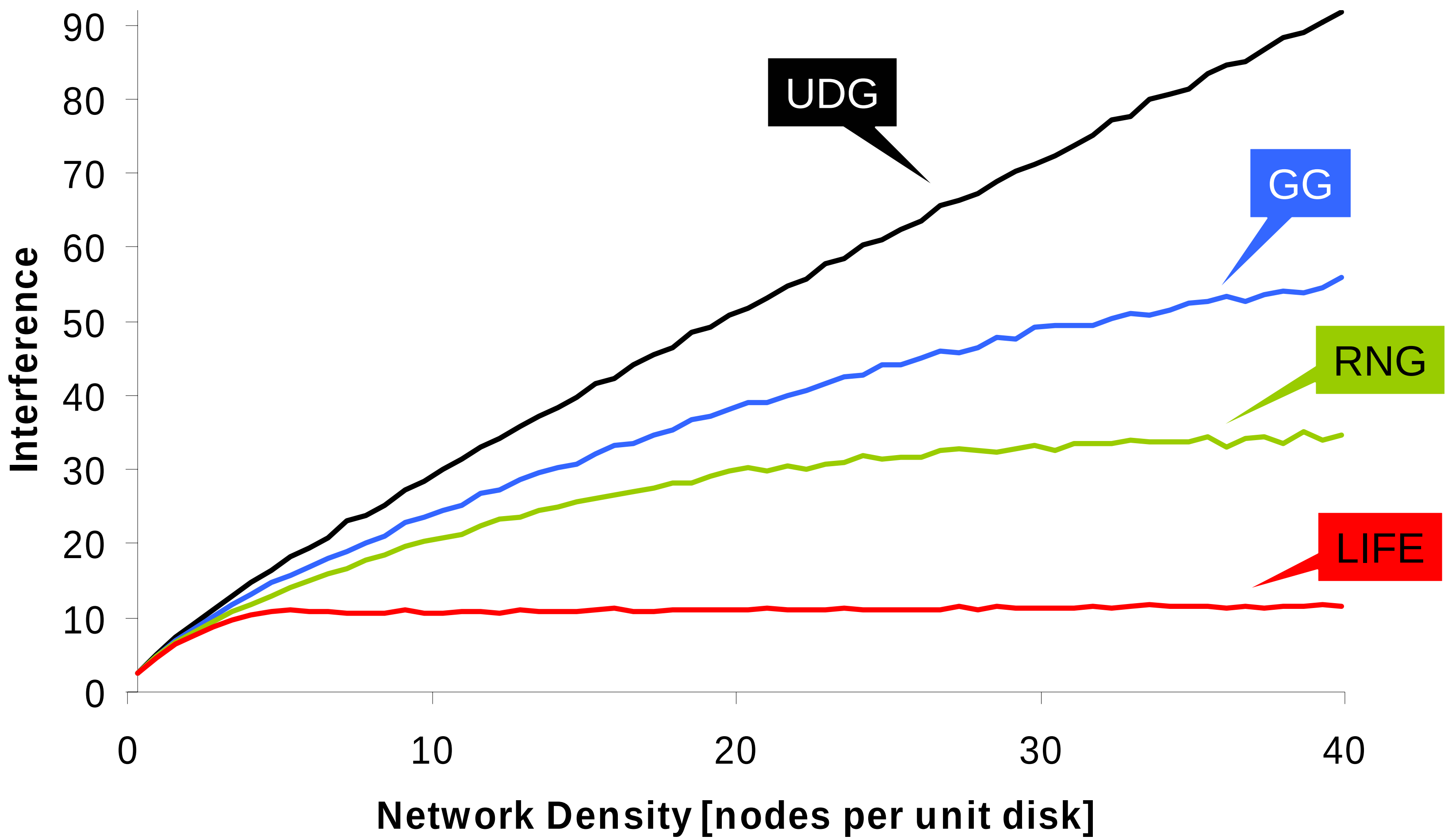
until $|p| \leq t\,|u, v|$

inform all edges on p to remain in the resulting topology.

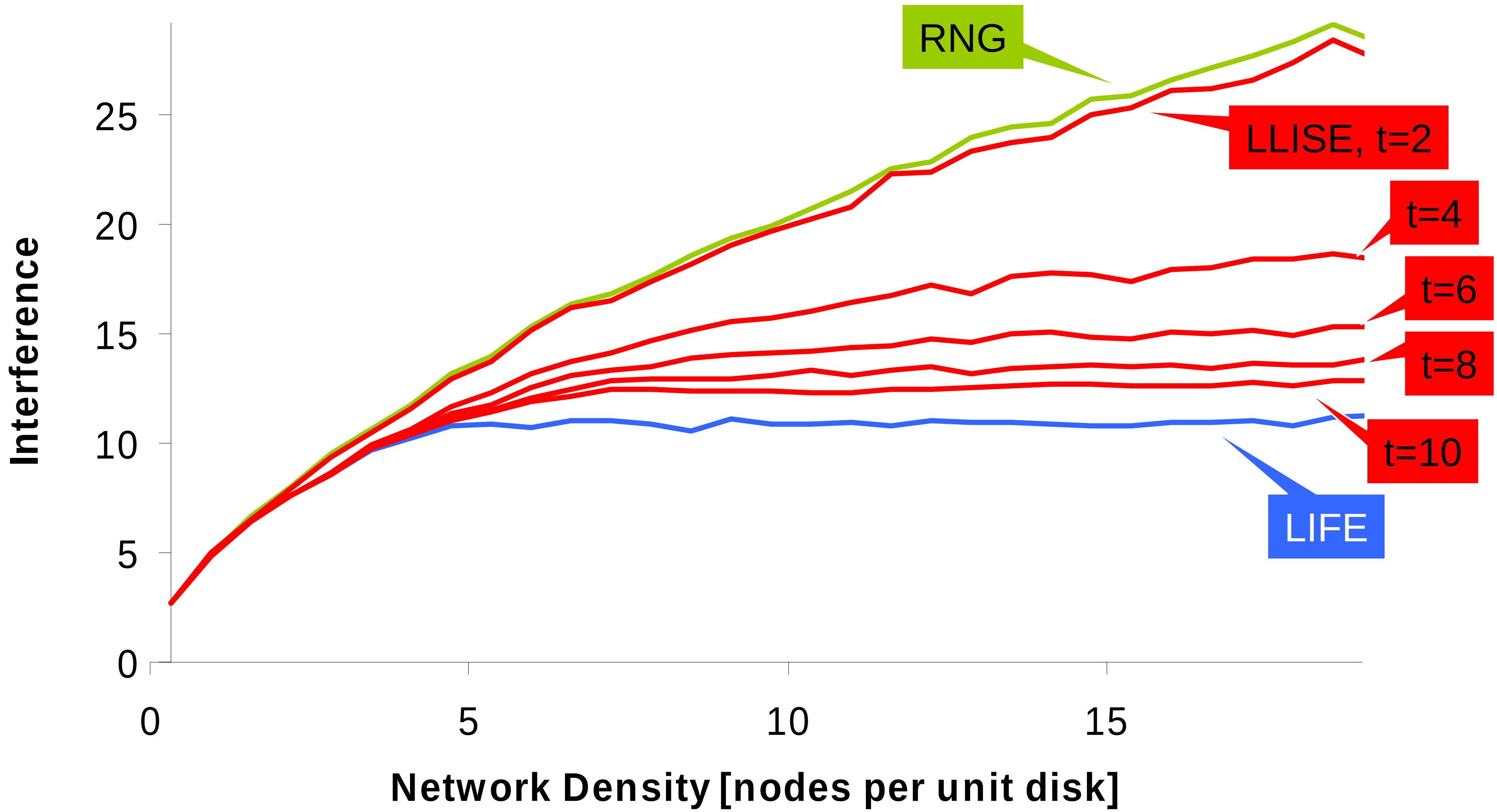
Note: $G_{LL} = (V, E_{LL})$ consists of all edges eventually informed to remain in the resulting topology.

Theorem: LLISE constructs a Minimum Interference t-Spanner

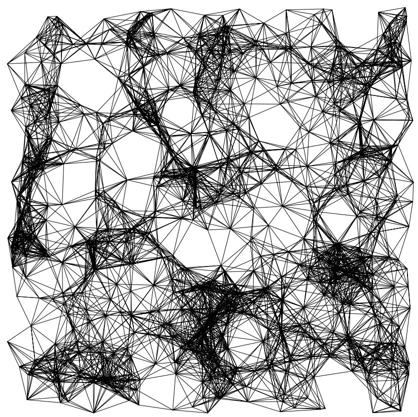
Average-Case Interference: Preserve Connectivity



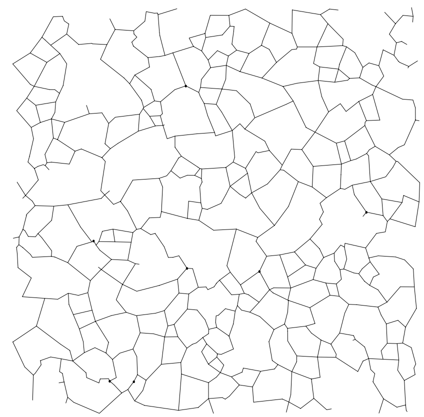
Average-Case Interference: Spanners



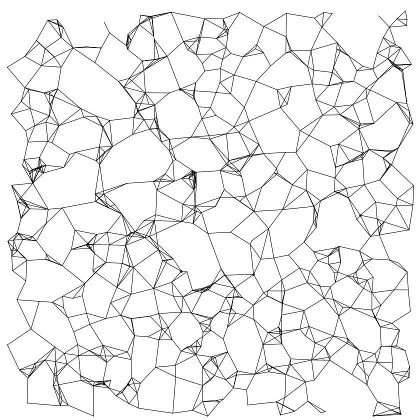
Simulation



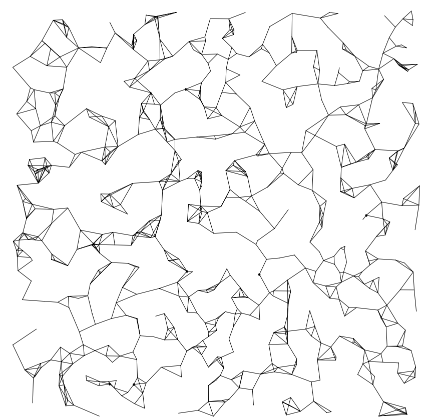
UDG, l = 50



RNG, l = 25



$LLISE_2$, l = 23



$LLISE_{10}$, l = 12

Conclusion

- Explicit interference model
- Interference produced by previously proposed topologies
- Properties of interference-optimal topology
- Algorithms
  - Interference-optimal connectivity-preserving topology
  - Local interference-optimal spanner topology

Does Topology Control reduce interference?

Yes, but only if…

Questions?
Comments?

Distributed Computing Group