# Validity in Network-Agnostic Byzantine Agreement


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**Marc Dufay**
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Roger Wattenhofer






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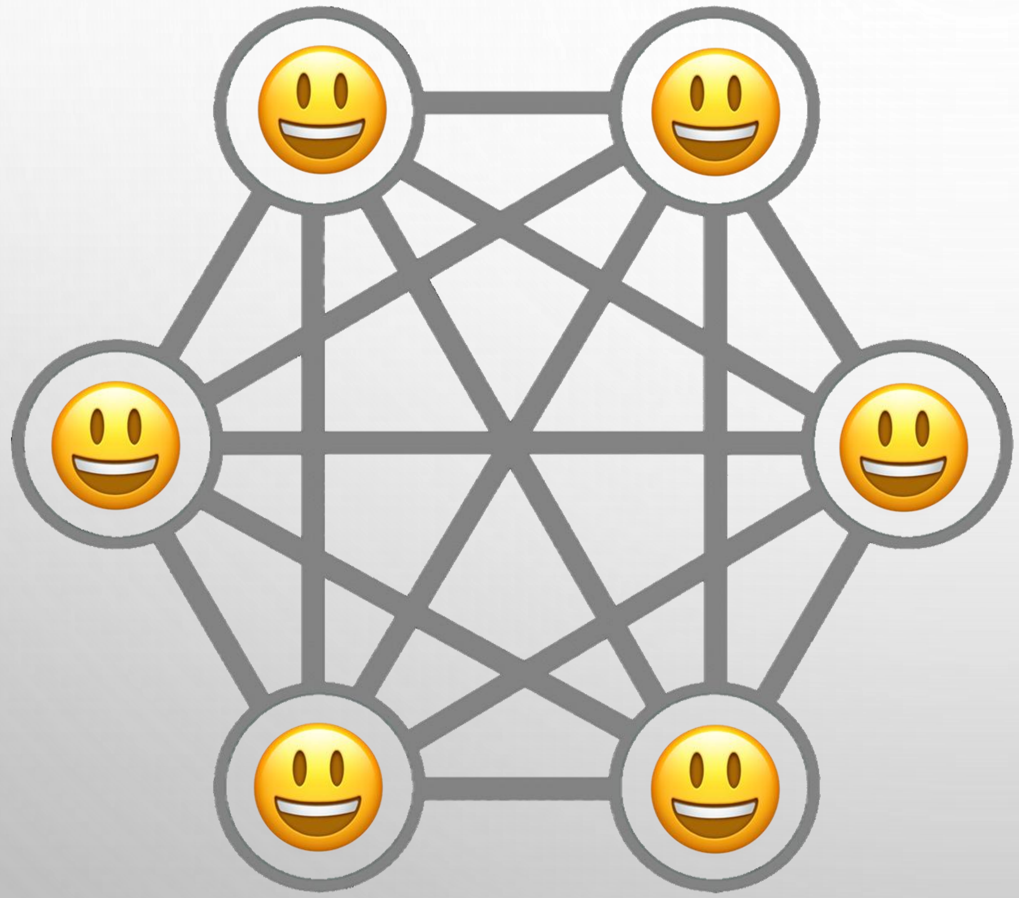
ETH zürich


**DISC 2025**

# MODEL



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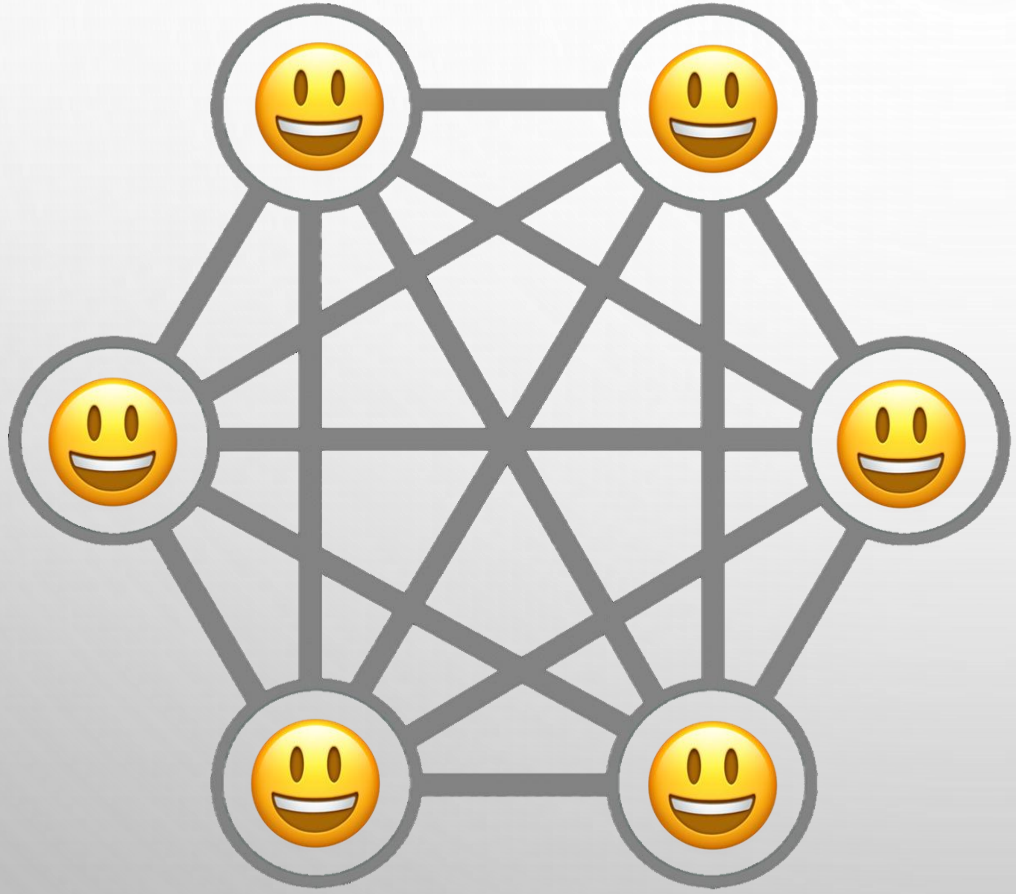

- $n$ parties

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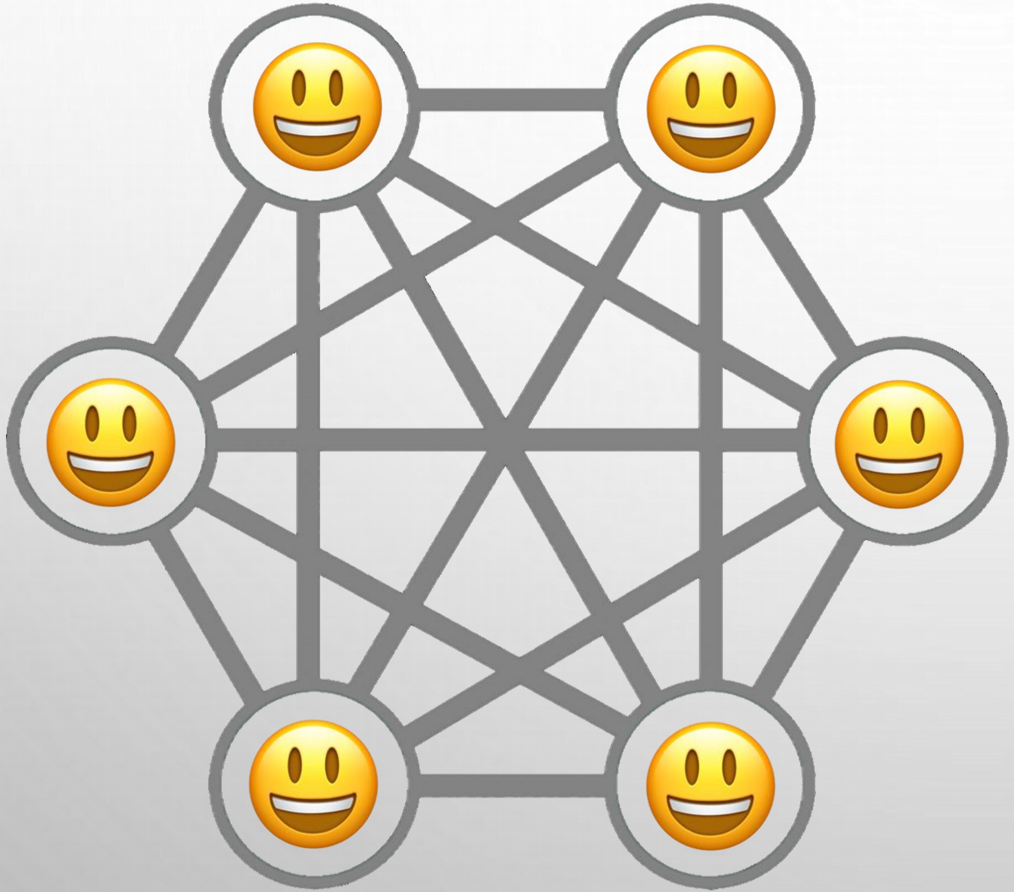

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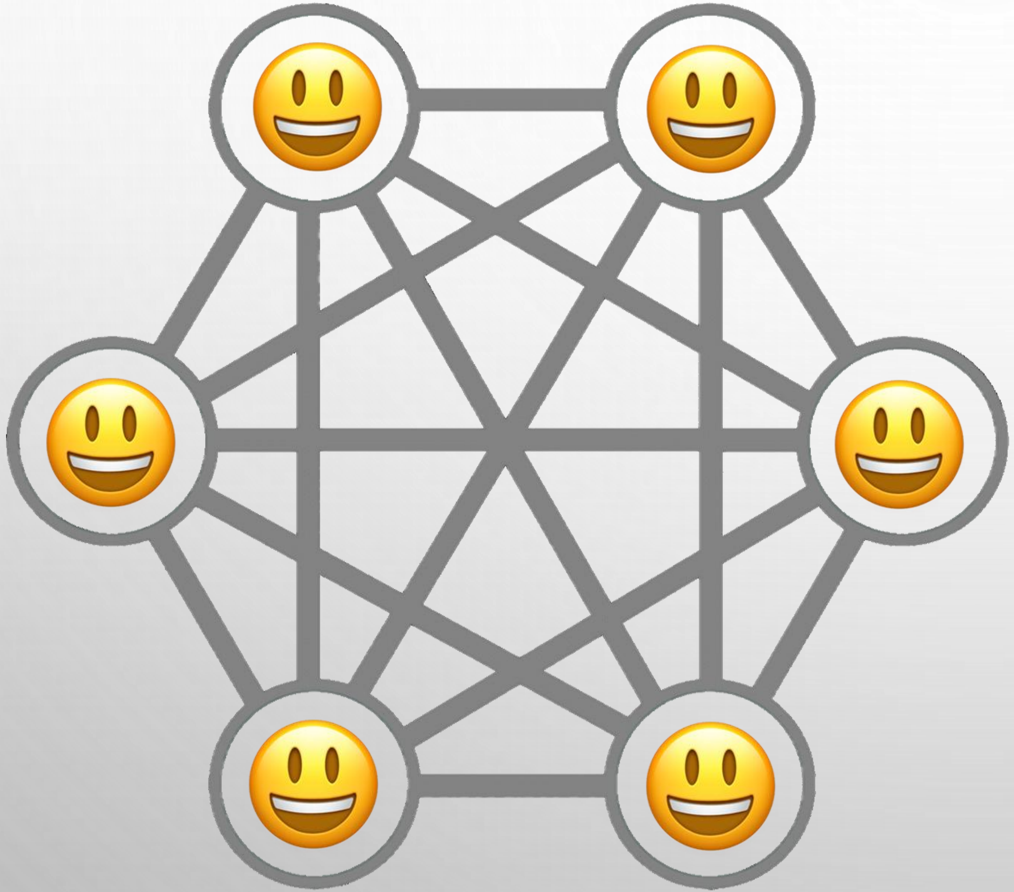
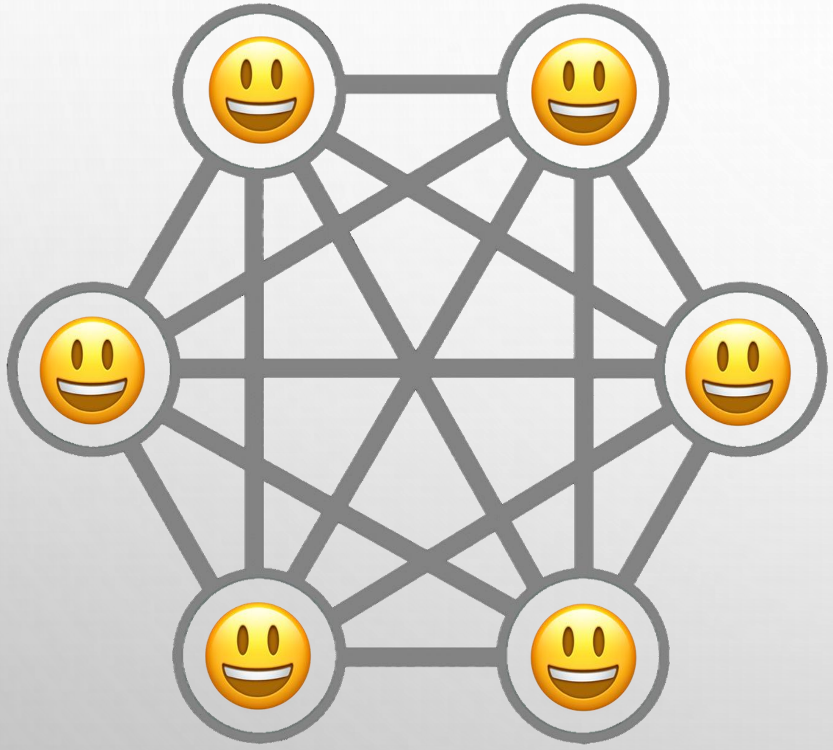


- $n$ parties
- Fully connected network
- Authenticated channels

# AGREEMENT



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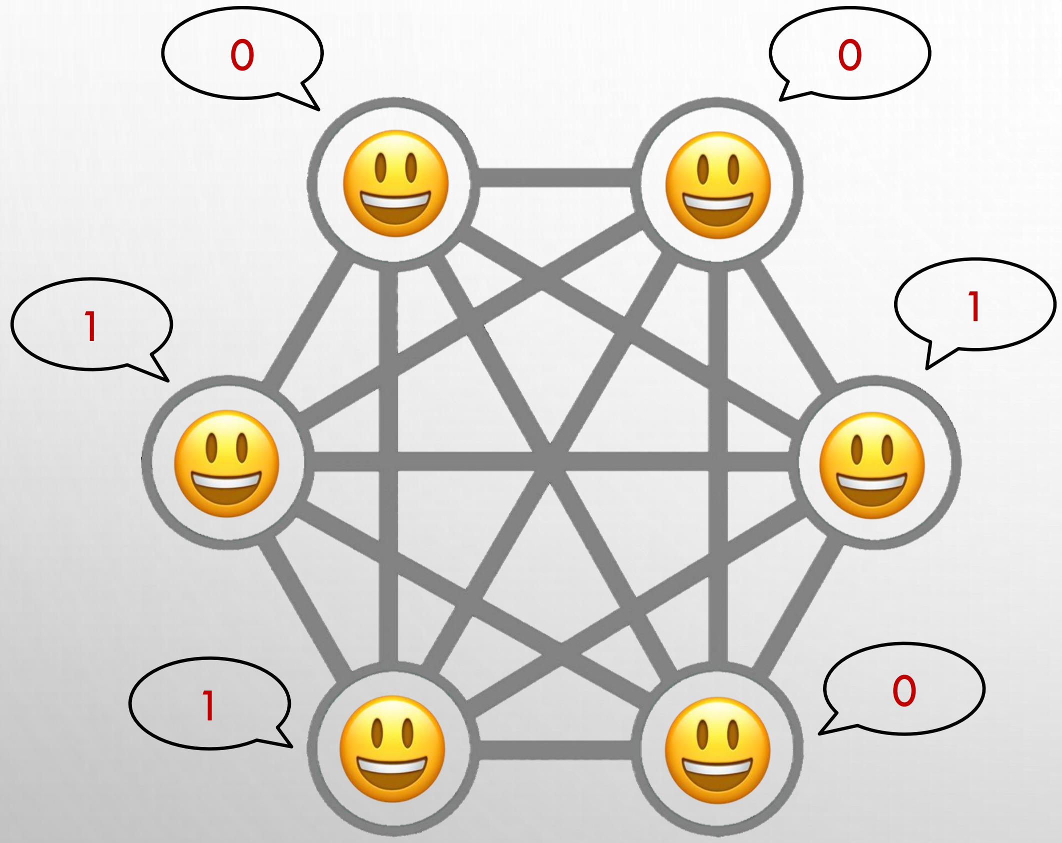


$$V_{IN} = \{0,1\}$$

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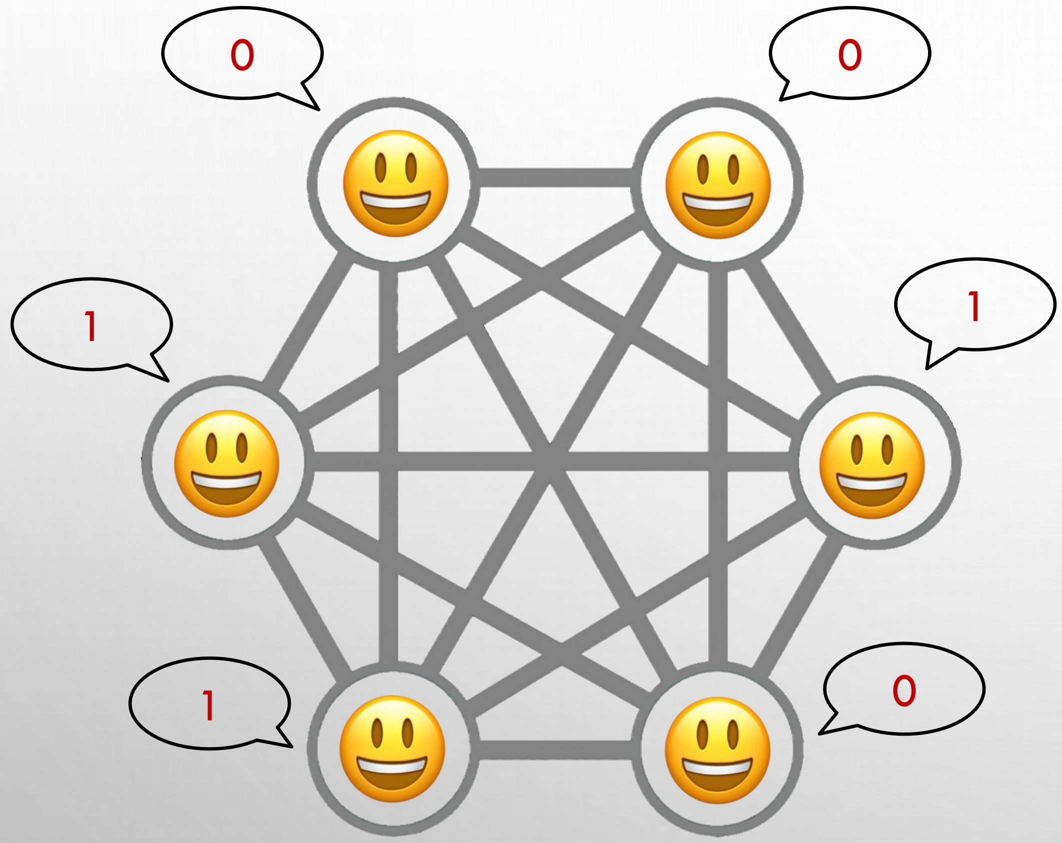


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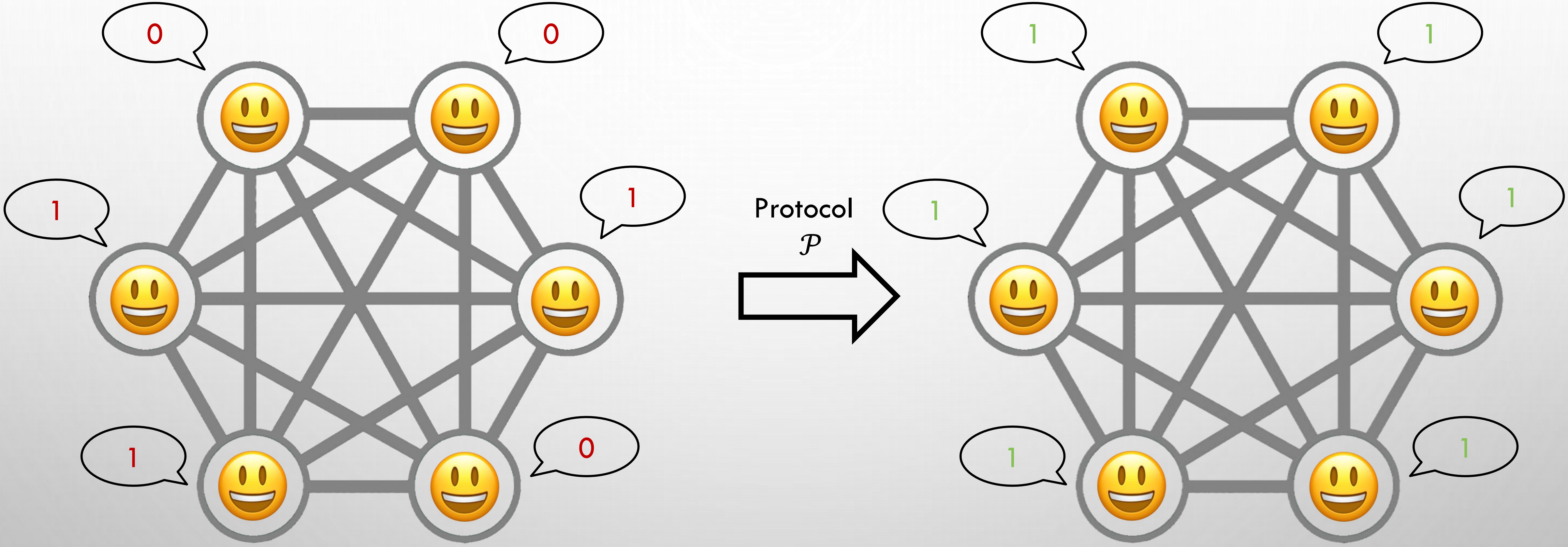


$V_{IN} = \{0,1\}$

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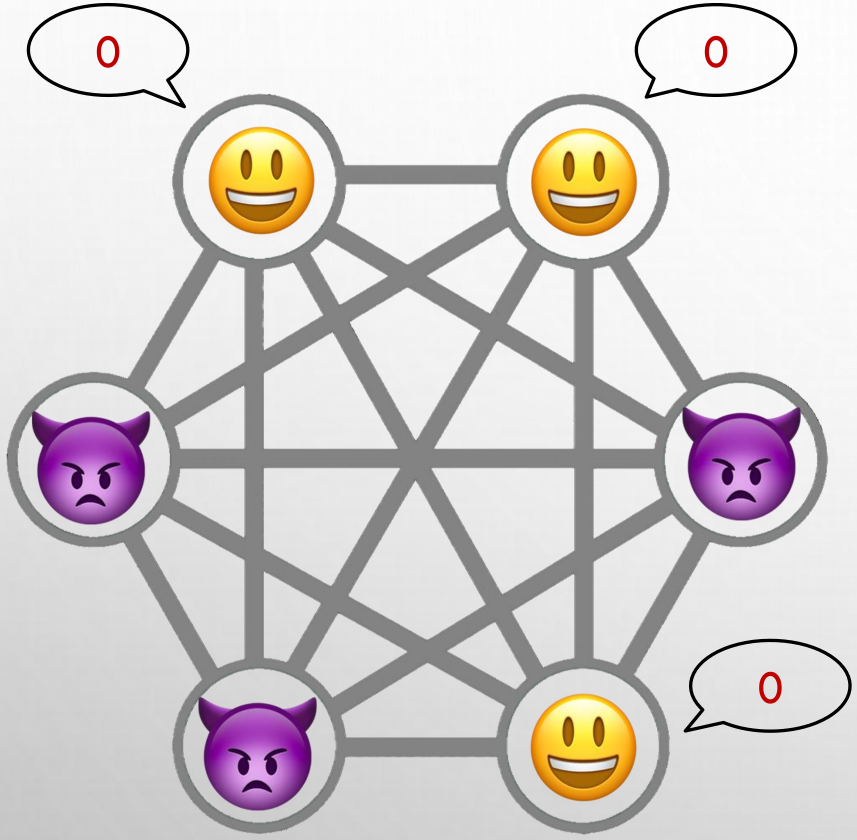
# Byzantine Agreement



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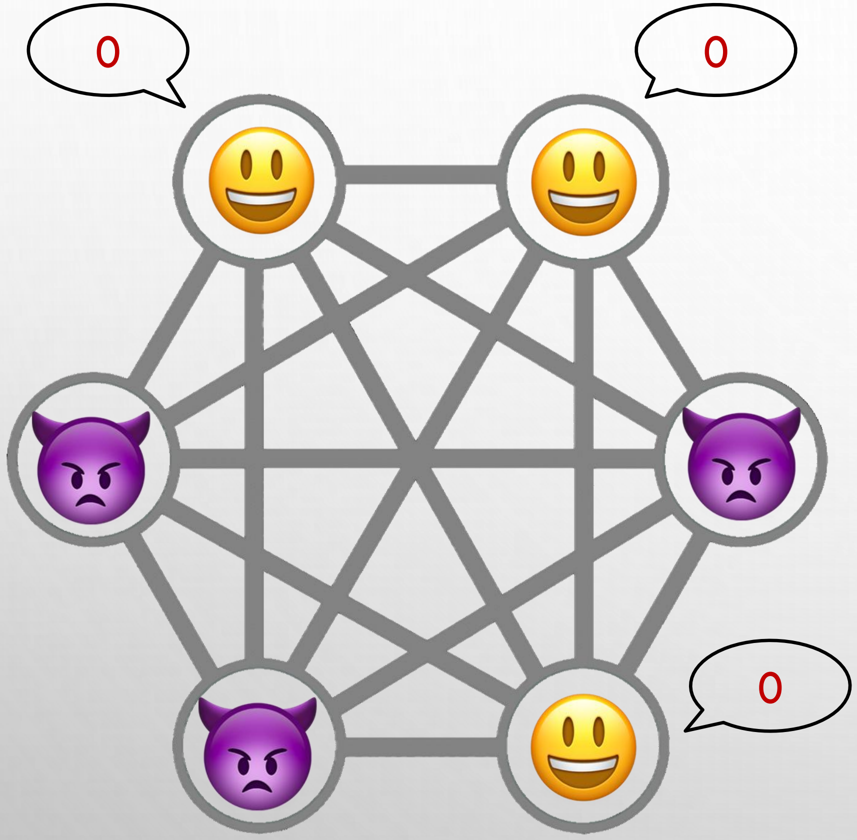


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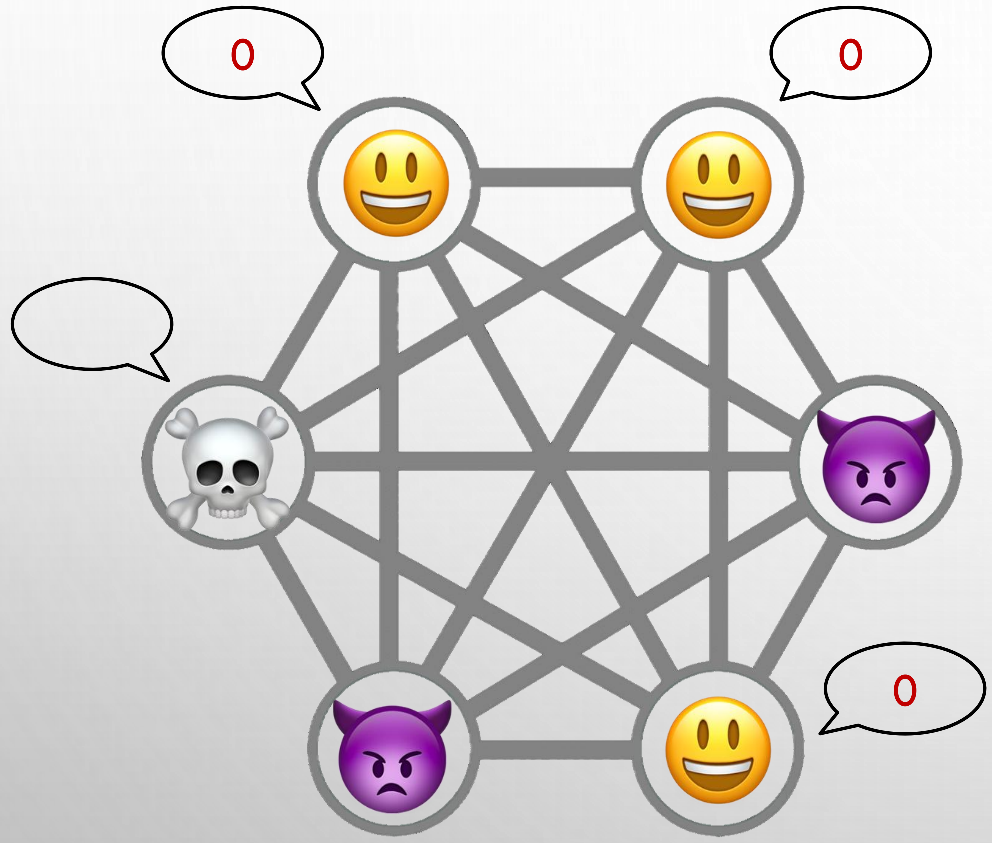


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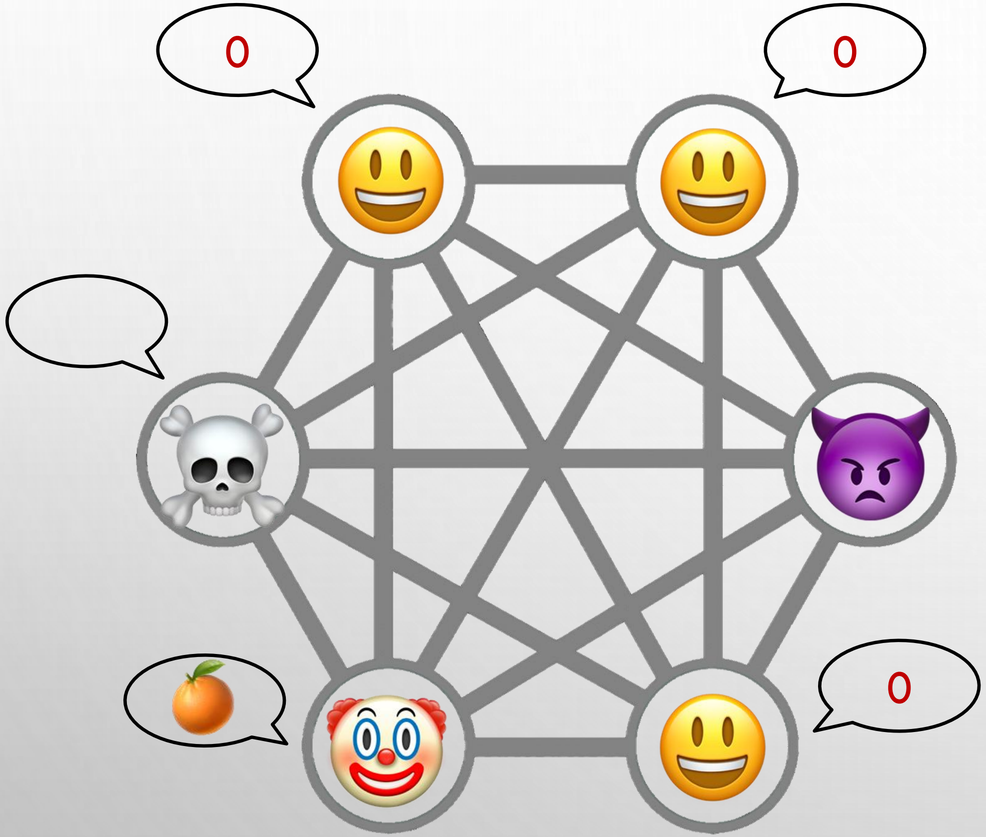


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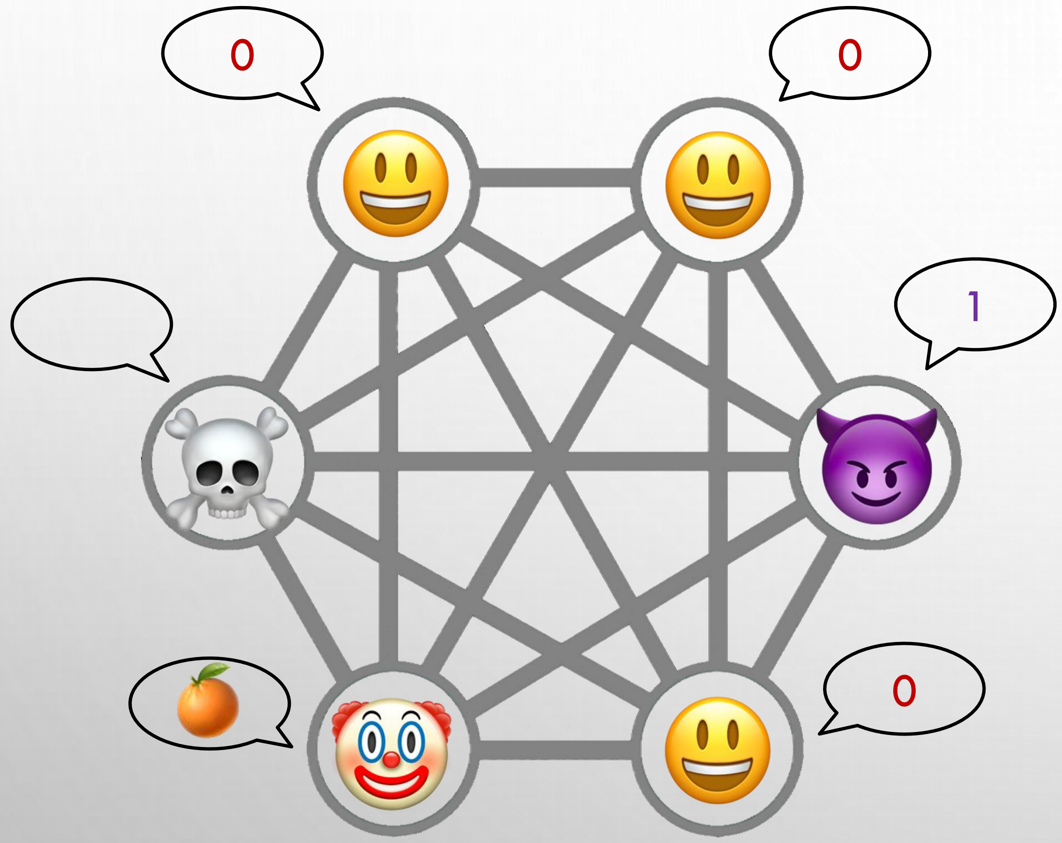


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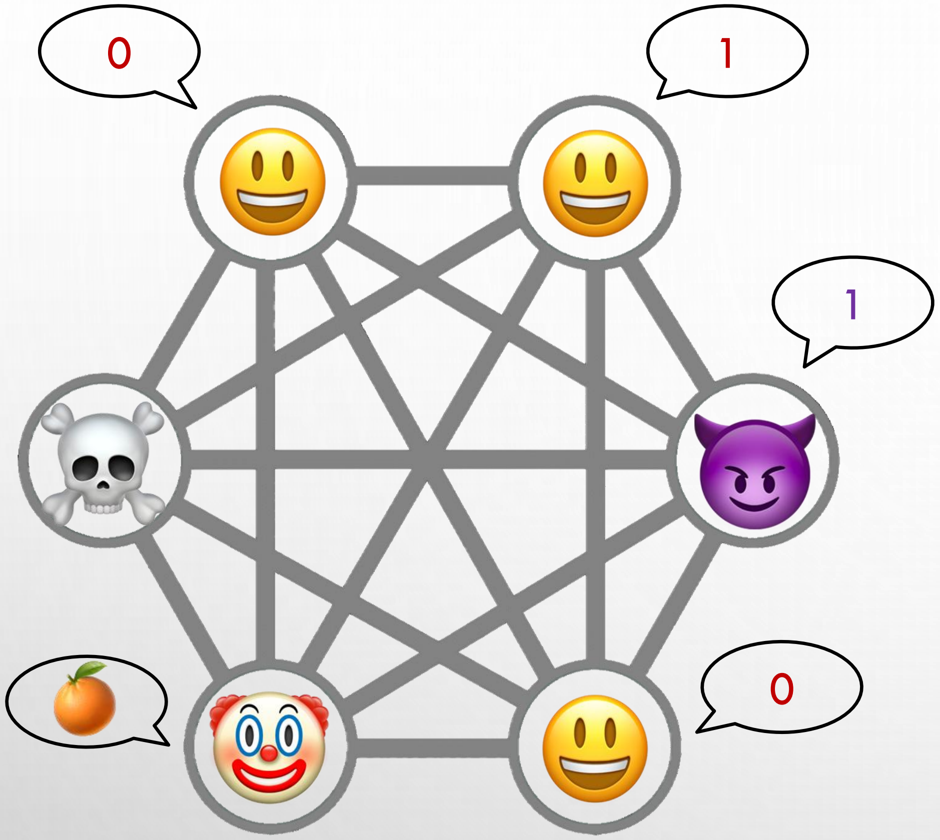
0
1
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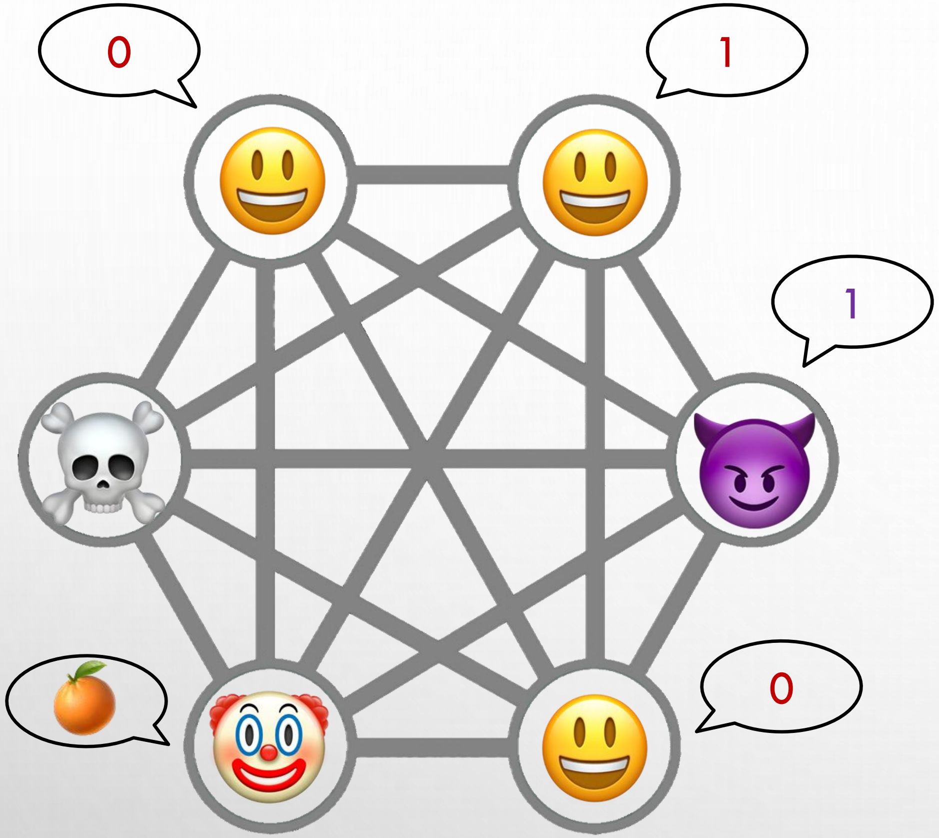


Protocol
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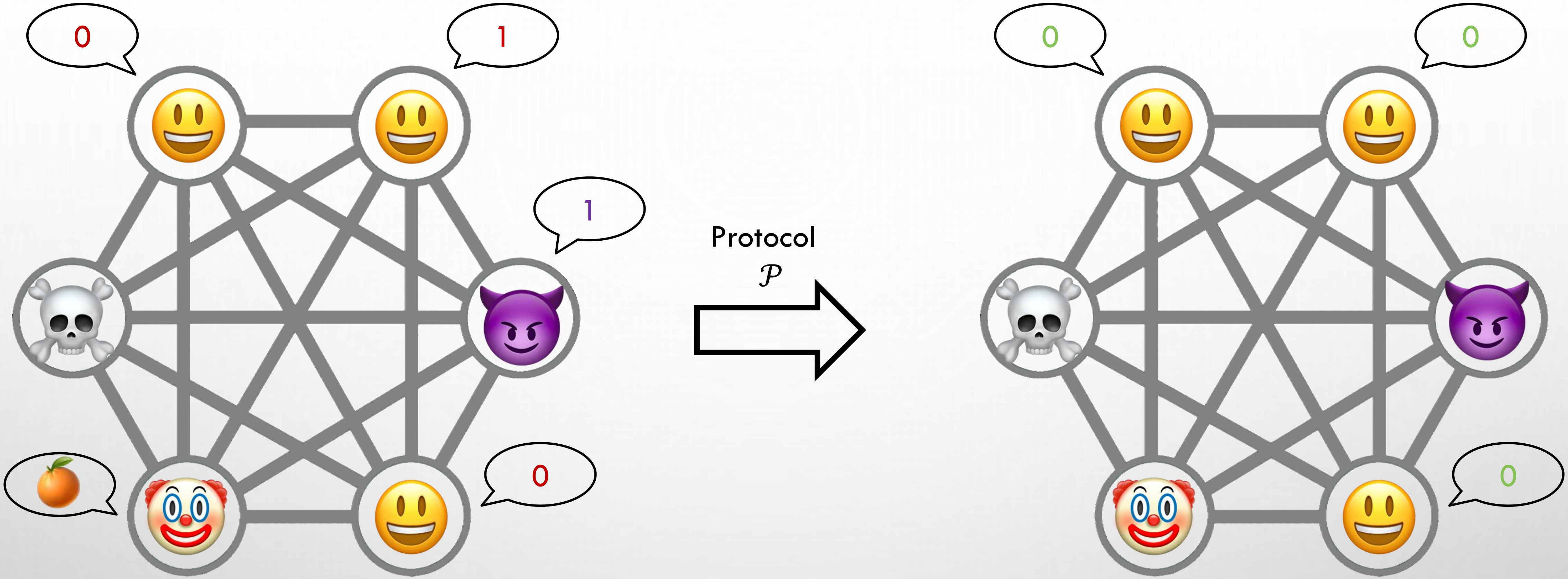
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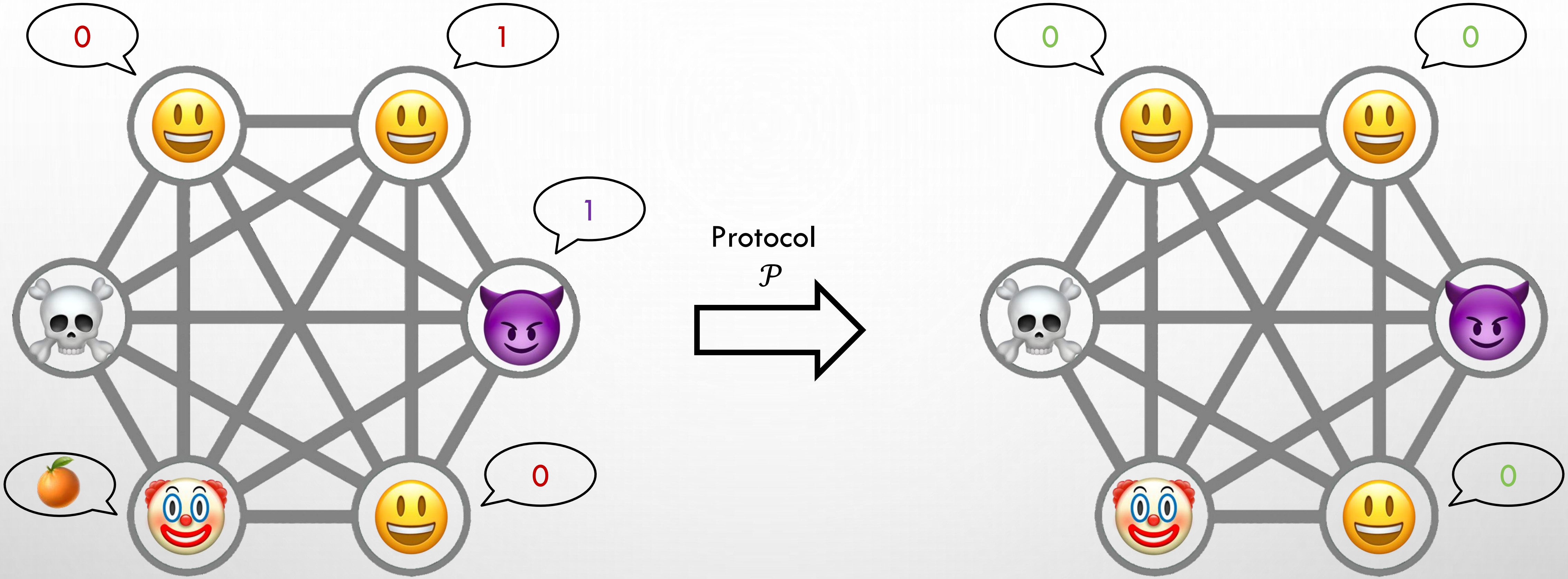


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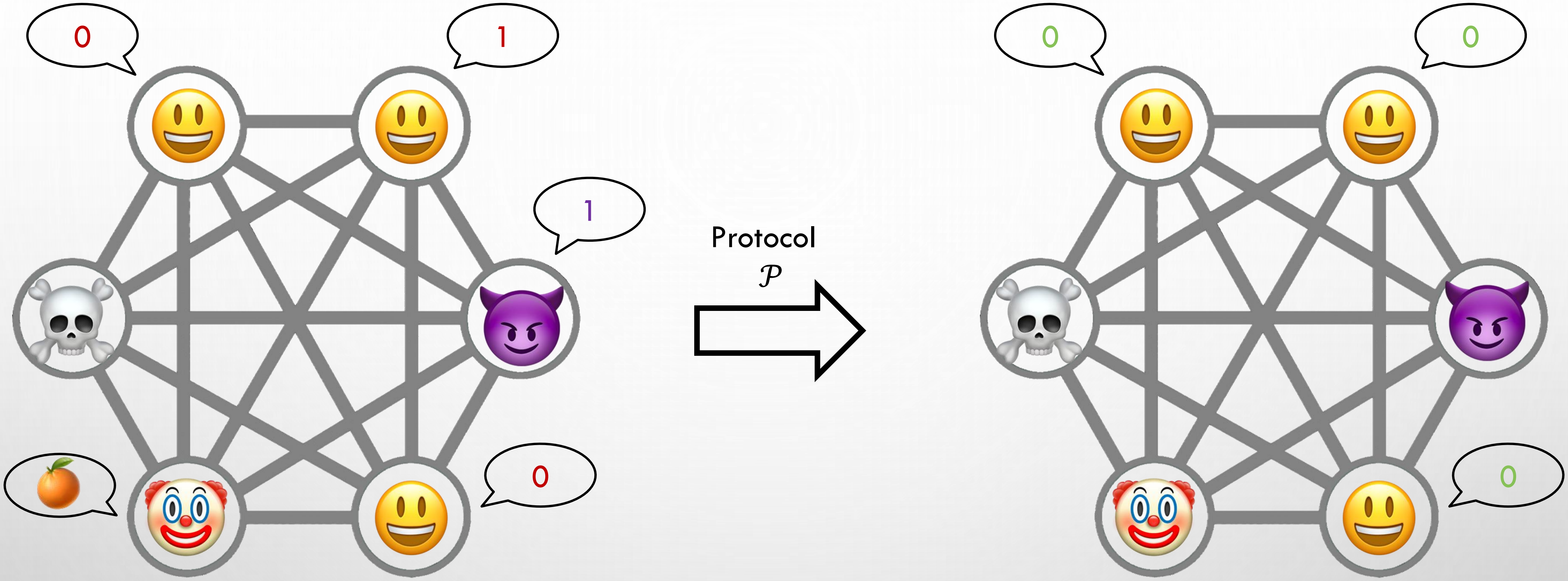


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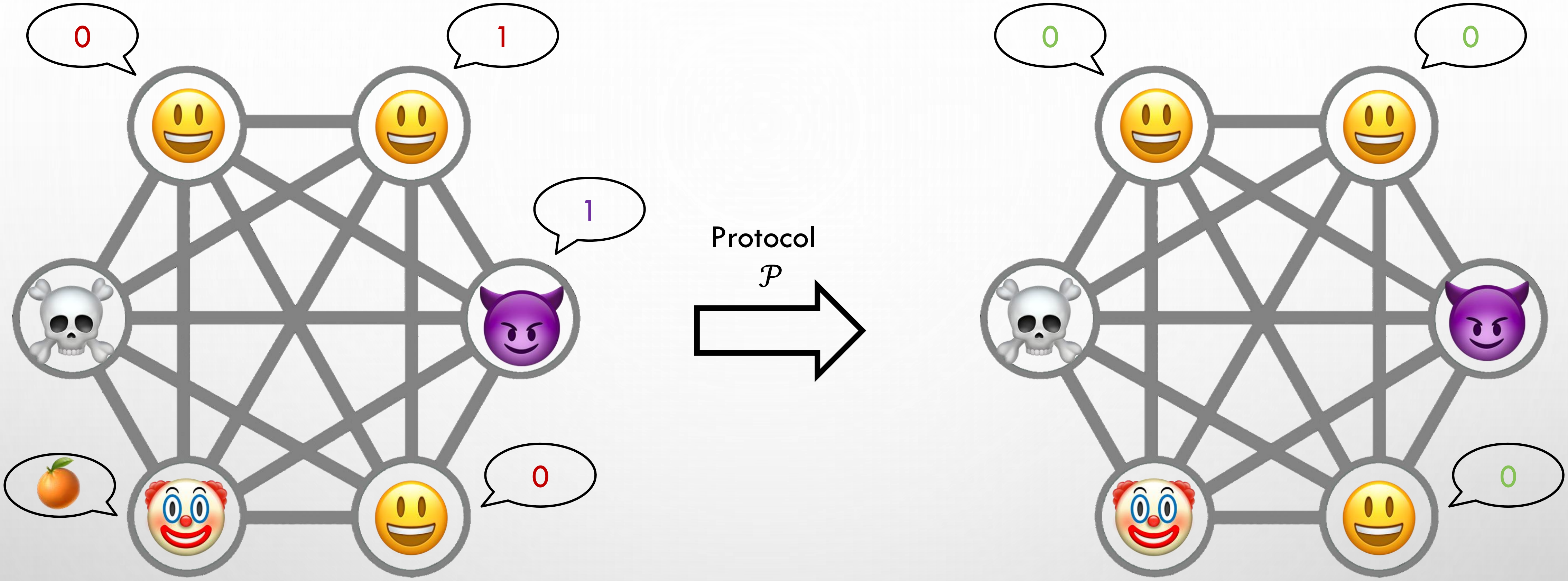


- **<u>Termination</u>**: All honest parties eventually decide a value
- **<u>Agreement</u>**: All honest parties decide the same value
- **<u>Validity</u>**: The value decided makes sense

# Validity Conditions

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**Weak Validity:**

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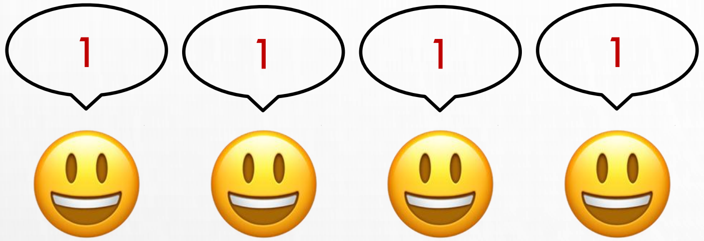


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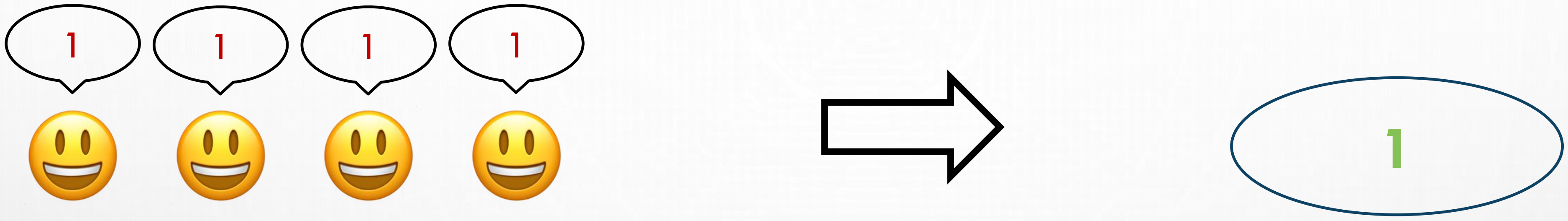


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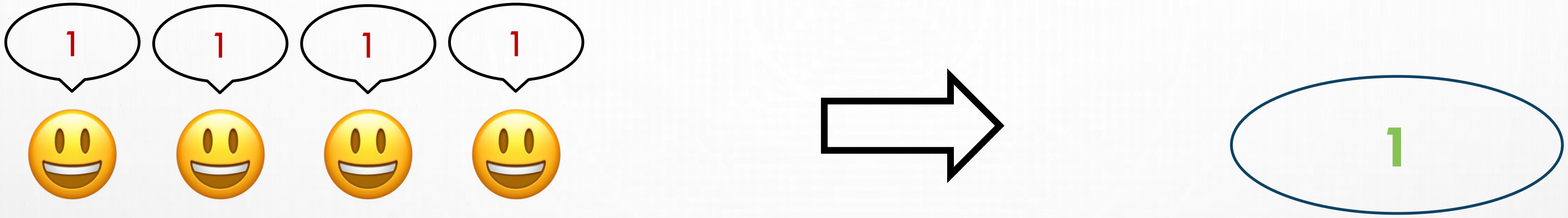


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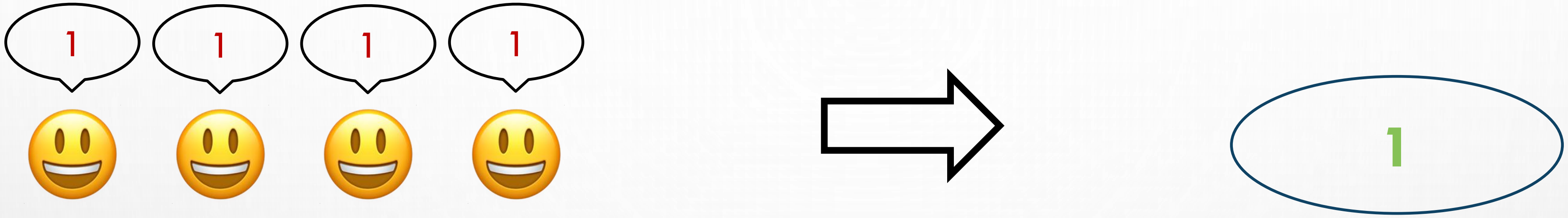


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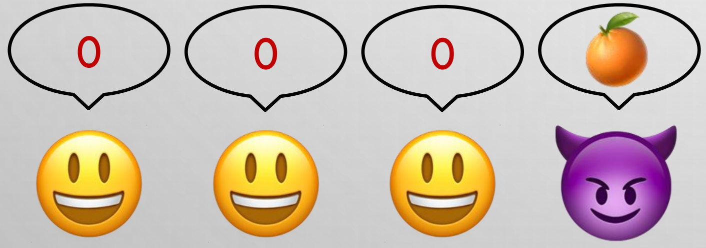


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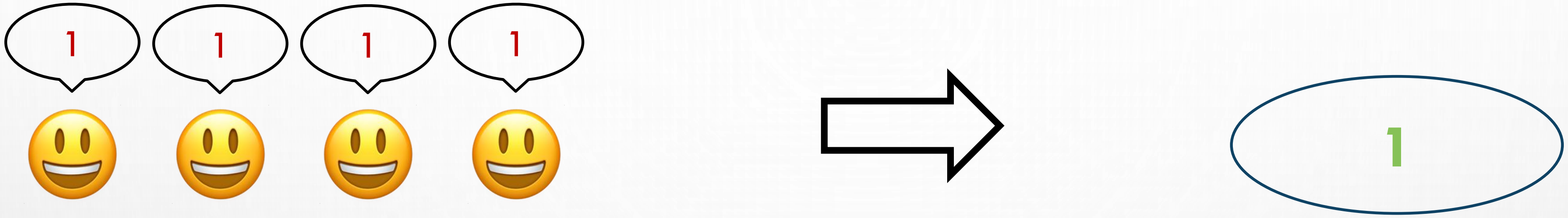


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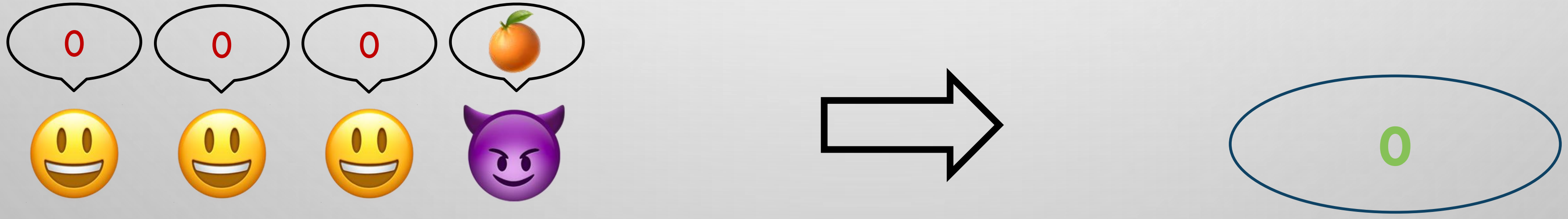


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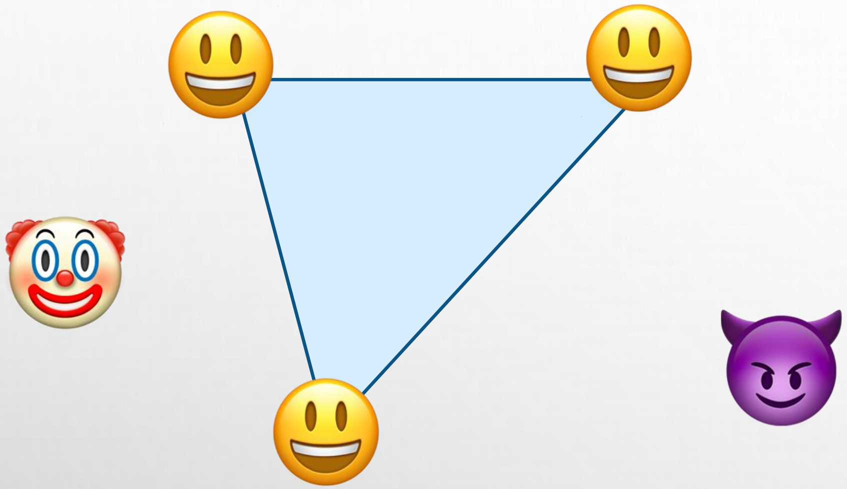

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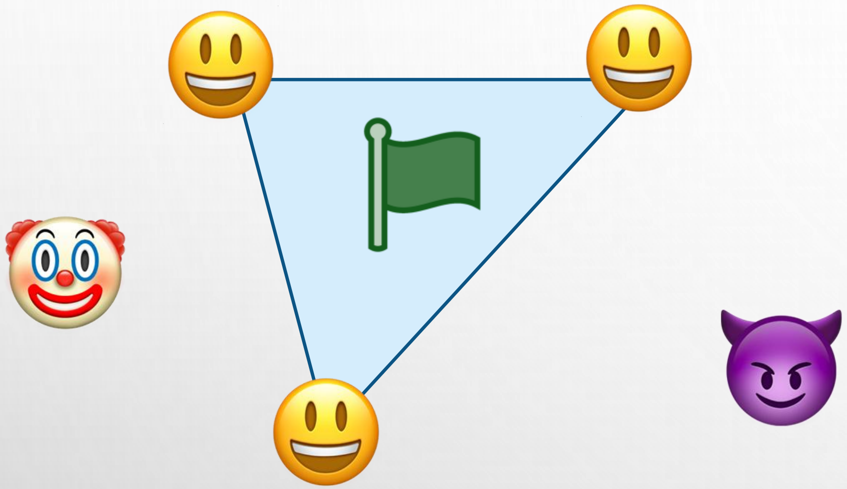

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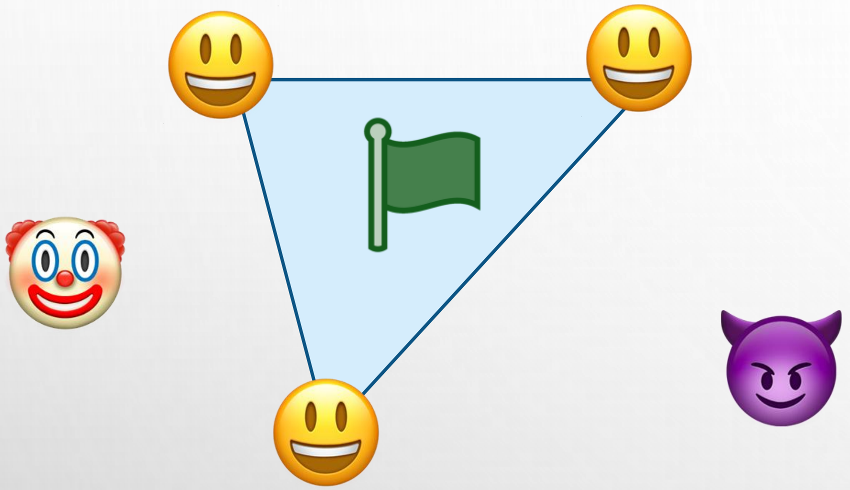

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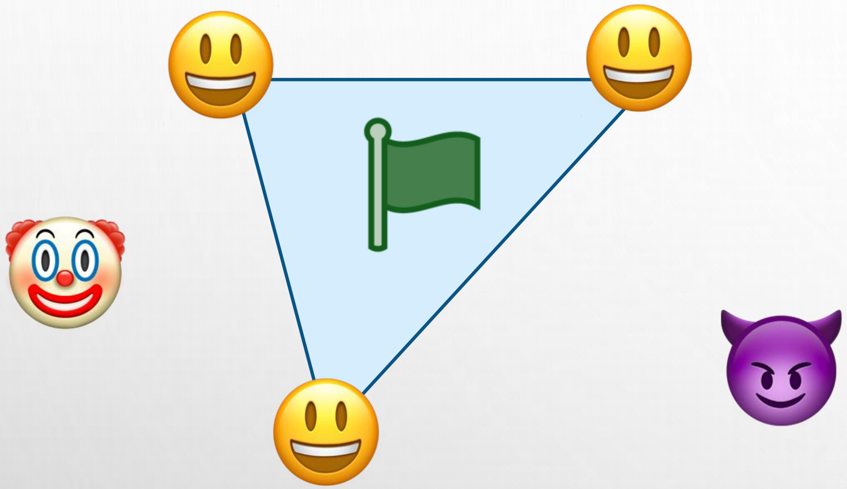

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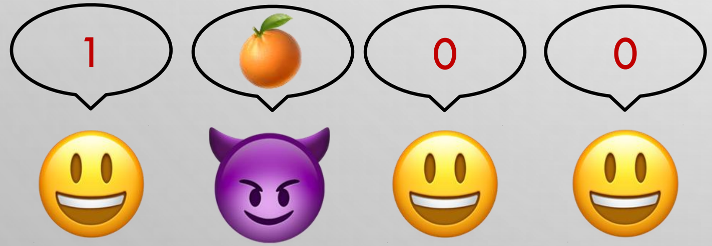


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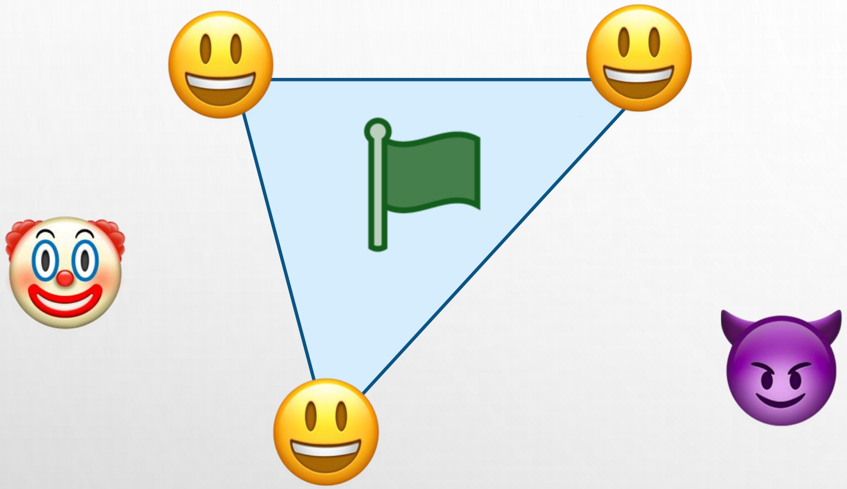

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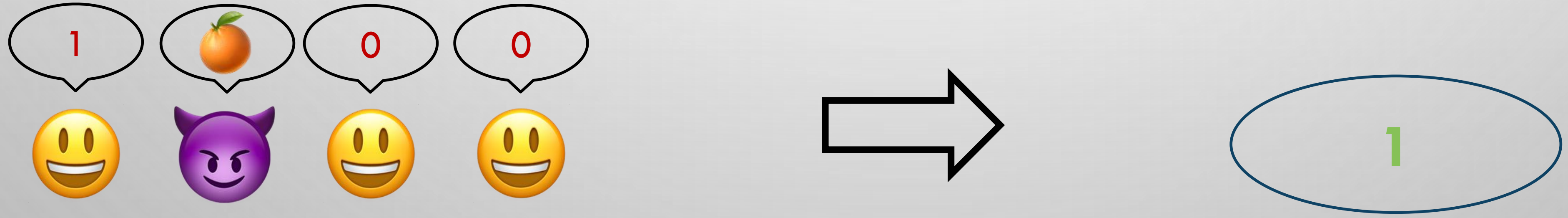


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A validity property is trivial if there is a value which can always be decided!

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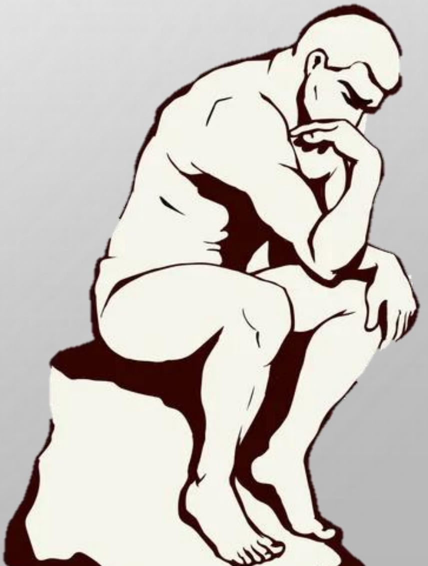
We need to assume $V_{IN}$ or $V_{OUT}$ is not uncountable !

## What are tight conditions for a given validity property to be solvable in the network-agnostic setting?



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Deterministic!

- **Network Agnostic:** This work

# Synchronous setting

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**Theorem**: Without PKI, if a validity property is solvable, then:

$$t_S < n/3$$

# Network agnostic setting

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**Theorem**: If a validity property is solvable, then:

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# Necessary conditions

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Necessary and sufficient for weak validity, strong validity !

**Is it enough?**

No:

- Reliable broadcast: Not solvable if $t_S > 0$.
- Convex validity: Solvable only if $n > \max(\omega \cdot t_S, \omega \cdot t_A + t_S)$

[Constantinescu, Ghinea, Wattenhofer, Westermann DISC'24]

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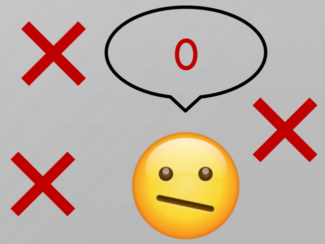
How far away is it from the input configuration?

- **<u>Synchrony</u>**: A byzantine party may act as honest with an arbitrary input value.


- **<u>Asynchrony</u>**: Messages from an honest party may be delayed until after the common view is established.



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Deciding a value in $\bigcap_{J \in \mathrm{SIMILAR}(\mathcal{J})} \mathrm{val}(J)$ satisfies the validity property!

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A validity property is solvable in the network agnostic setting if and only if:

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- There exists a turing-computable function $\sigma\colon \mathcal{I} \mapsto \mathrm{V_{OUT}}$ such that for all $I \in \mathcal{I}$:

$$\sigma(I) \in \bigcap_{J \in SIMILAR(I)} \mathrm{val}(J)$$

Protocol $\mathcal{P}$

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$\sigma$

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Run σ locally on the common view
Decide $\sigma(I)$

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# QUESTIONS?