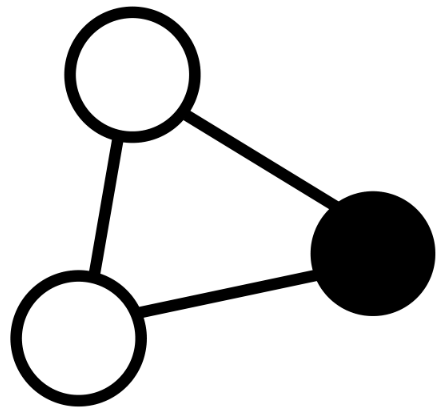
Approximating Fault-Tolerant Domination in General Graphs

Klaus-Tycho Förster

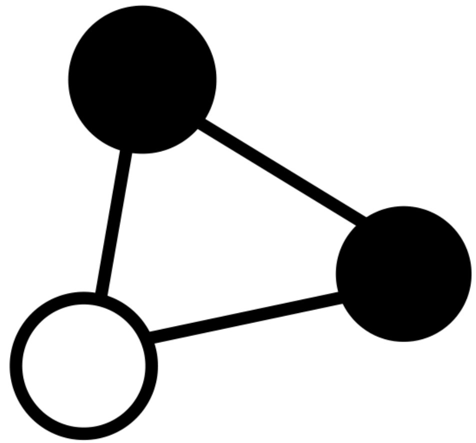
Minimum Dominating Set



- Can be approximated with ratio
  - $\mathbf{ln}(\boldsymbol{n}) - \mathbf{ln}(\mathbf{ln}(\boldsymbol{n})) + \mathbf{0.78}$ *[Slavík, 1996]*
  - $H_{\Delta+1} - 0.5 < \mathbf{ln}(\boldsymbol{\Delta} + \mathbf{1}) + \mathbf{0.5}$ *[Chlebík and Chlebíková, 2008]*

- NP-hard lower bound of
  - $\mathbf{0.2267}\,\mathbf{ln}(\boldsymbol{n})$ *[Alon, Moshkovitz and Safra, 2006]*

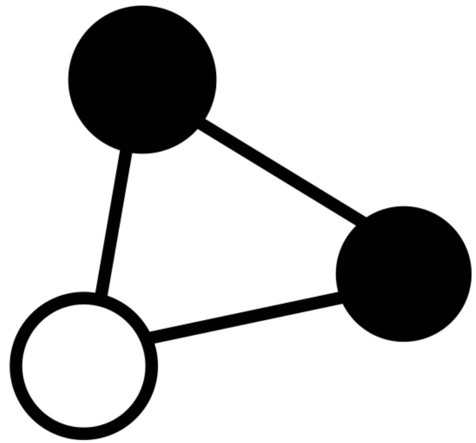
Minimum k-tuple Dominating Set



minimum **2-tuple** dominating set

- k-tuple dominating set:
  - Every node should have $\boldsymbol{k}$ dominating nodes in its **neighborhood** *[Harary and Haynes, 2000 and Haynes, Hedetniemi and Slater, 1998]*
- Can be approximated with ratio
  - $\mathbf{ln(\Delta+1)+1}$ *[Klasing and Laforest, 2004]*

Minimum k-Dominating Set



minimum **2-**dominating set

- k-dominating set:
  - Every node should be **in** the dominating set **or** have $\boldsymbol{k}$ dominating nodes in its **neighborhood** *[Fink and Jacobson, 1985]*

- Best known approx.-ratio

  $(\boldsymbol{e}^2 + \boldsymbol{e})\mathbf{ln}(\boldsymbol{\Delta})$ *[Kuhn, Moscibroda and Wattenhofer, 2006]*

Overview of the remaining Talk

- k-tuple domination **vs** k-domination

- NP-hard **lower bound** for k-domination

- Improved **approximation ratio** for k-domination

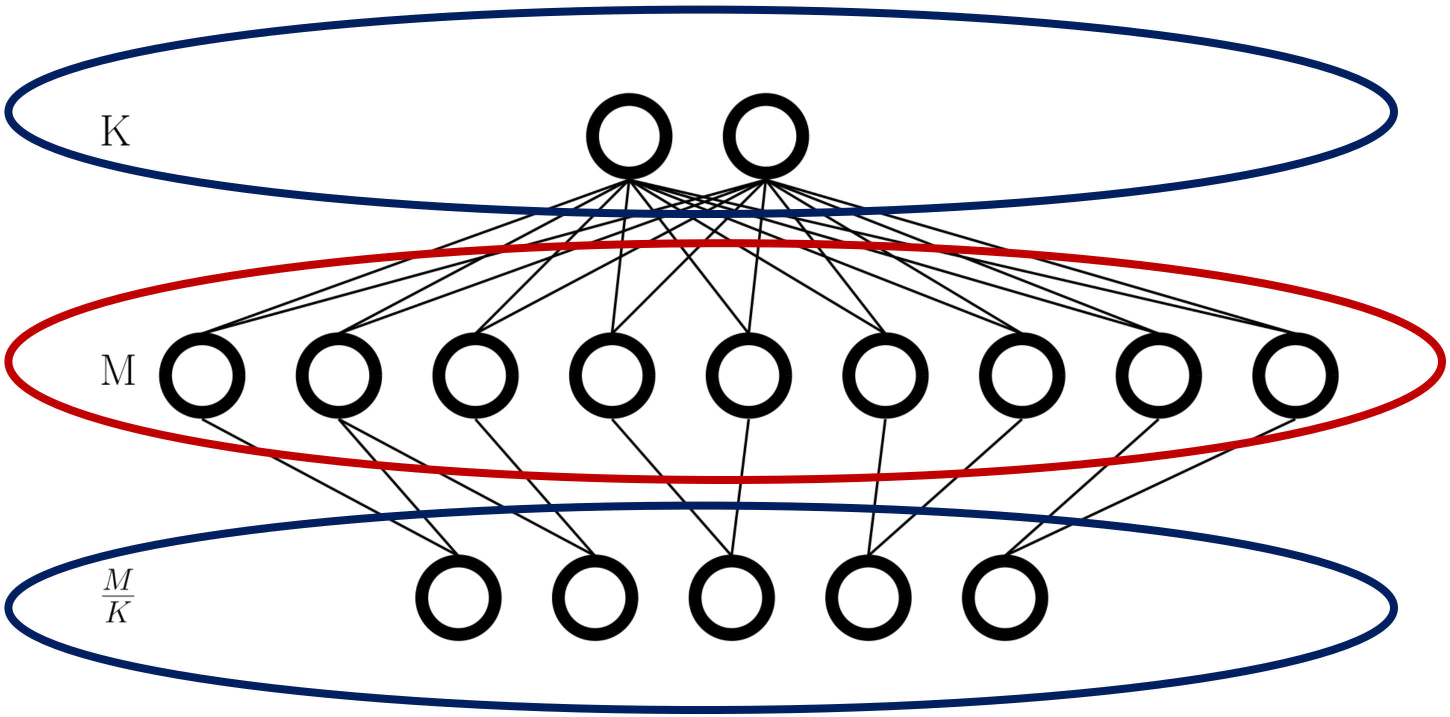
k-tuple Domination versus k-Domination

- k-tuple dominating set only exists if min. degree $\geq \boldsymbol{k-1}$

- **Every** k-tuple dominating set is a k-dominating set

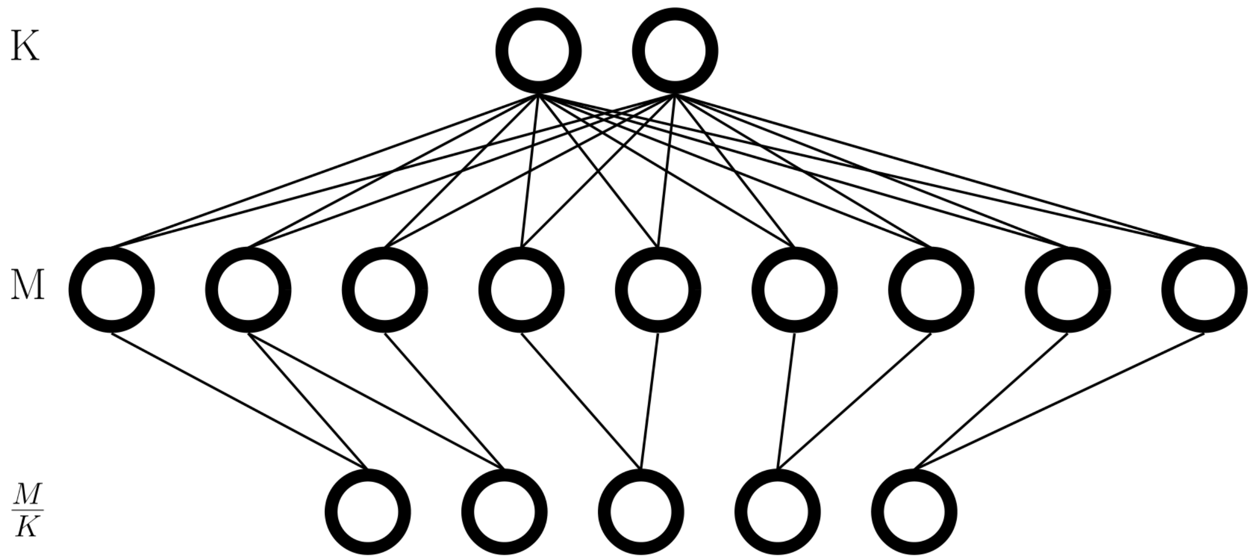
- But **how "*bad*"** can a k-tuple dom. set be in **comparison?**

k-tuple Domination versus k-Domination: With $\boldsymbol{k} = \mathbf{2}$



- $|K| = k = 2,\ |M| = 9,\ \left\lceil \frac{M}{K} \right\rceil = \left\lceil \frac{9}{2} \right\rceil = 5$
- At least $|M| = 9$ nodes for a k-tuple dom. set
- But $|K| + \left\lceil \frac{M}{K} \right\rceil = 7$ nodes suffice for a k-dom. set

k-tuple Domination versus k-Domination

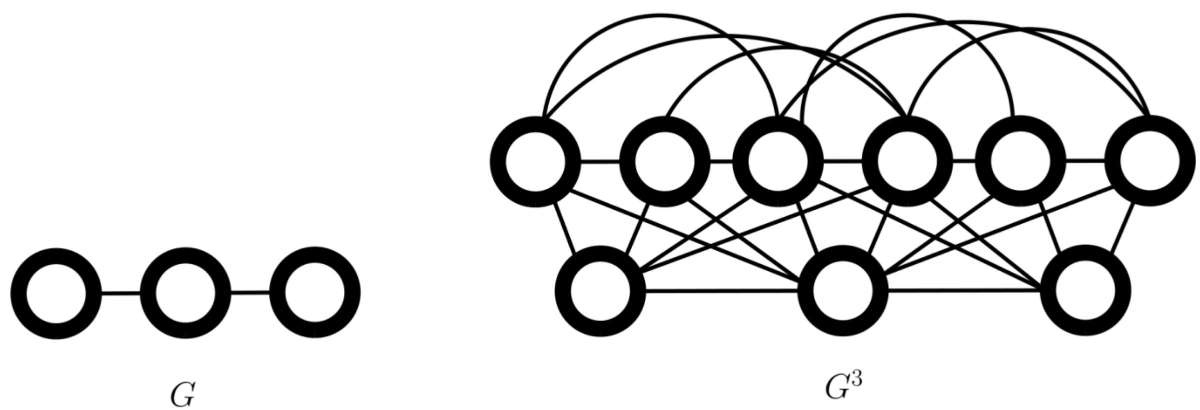


- $M \rightarrow \infty$: Off by a factor of **nearly $\boldsymbol{k}$**!
- For $1 < \alpha < k$ and $n \geq k - 1 + \frac{(k-1)^2}{\alpha-1}$: Off by a factor $\geq \boldsymbol{\frac{k}{\alpha}}$ **(tight)**

NP–hard lower bound for k–domination

- NP-hard lower bound for 1-domination
  - $\mathbf{0.2267\ ln(n)}$ *[Alon, Moshkovitz and Safra, 2006]*

- If we could approx. k-dom. set with ratio of $s(n)$
  - Then build a k-multiplication graph:



Example for $k = 3$

- NP-hard lower bound for k-domination
  - $\mathbf{0.2267/k\ \ ln(n/k)}$

Improved approximation ratio for k−domination

- Utilizes a **greedy**-algorithm

- Use **“degree” of $\boldsymbol{k}$−domination** per node
  - $\boldsymbol{k}$, if in the k-dominating set
  - else **#neighbors** in the k-dominating set, but at most k

- Pick a node that **improves total sum** of degree the **most**

When does the Greedy Algorithm **finish?**

- Let a fixed **optimal solution** have $\boldsymbol{r > 1}$ nodes
- Greedy does at least $\boldsymbol{1/r}$ of remaining work **per step**
- If it does more, also good ☺
- Total amount of work is $\boldsymbol{n \cdot k}$
- This gives an approximation ratio of roughly $\boldsymbol{\ln(n \cdot k) + 1}$

When to stop when chopping off…

- When is chopping off $\mathbf{1}/\boldsymbol{r}$ of the remaining work **ineffective?**
- When remaining work **is less than r**
- Then **at most r more steps** are needed
- Stop chopping after $ln\left(\frac{\boldsymbol{nk}}{\boldsymbol{r}}\right) / \, ln\left(\frac{\boldsymbol{r}}{\boldsymbol{r-1}}\right)$ steps
- Gives an approx. ratio of $\mathbf{1} + \ln\left(\frac{\boldsymbol{nk}}{\boldsymbol{r}}\right) / \boldsymbol{r} \cdot \ln\left(\frac{\boldsymbol{r}}{\boldsymbol{r-1}}\right)$

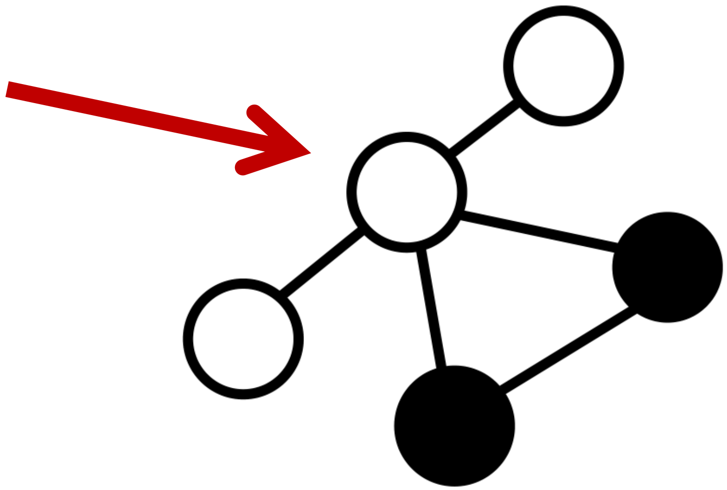
Calculating the approximation ratio

- $1 + ln\left(\frac{nk}{r}\right)/r \cdot ln\left(\frac{r}{r-1}\right)$ does not look too nice…
- 1) $\frac{1}{\ln\left(\frac{r}{r-1}\right)} \leq r\left(1 - \frac{1}{2r}\right) < r$
- 2) $\frac{nk}{\Delta+k} \leq r \Leftrightarrow \frac{nk}{r} \leq \Delta + k$
- Yields: Approx. ratio of less than $\mathbf{ln(\Delta + \mathit{k}) + 1}$
- $\mathbf{ln(\Delta) + 1.7} \quad < \ln(n) + 1.7$

Extending the Domination Range

- Instead of dominating the **1-**neighborhood…
- … dominate the $\boldsymbol{h}$**-**neighborhood
- Often called h**-step domination** *cf. [Hage and Harary, 1996]*

Extending the Domination Range



- The black nodes form a **2-step** dominating set
- But **not** a 2-step **2-**dominating set !

Extending the Domination Range

- Instead of having k dominating nodes in the h-neighborhood …
  - (unless you are in the dominating set)
- … have k **node-disjoint paths** of length at most h
- Results in approximation ratio of:
- $\mathbf{ln}(\boldsymbol{\Delta_h} + \boldsymbol{k}) + \mathbf{1} < \boldsymbol{ln}(\boldsymbol{n}) + \mathbf{1.7}$

Thank you

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