Solving Woeginger's Hiking Problem
(Wonderful Partitions in Anonymous Hedonic Games)

Andrei Constantinescu, Pascal Lenzner, Rebecca Reiffenhäuser, Daniel Schmand, Giovanna Varricchio





ALGO'08

Gerhard Woeginger
(1964—2022)

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Comb. opt., scheduling, graphs

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Liked Puzzles!

A Puzzle

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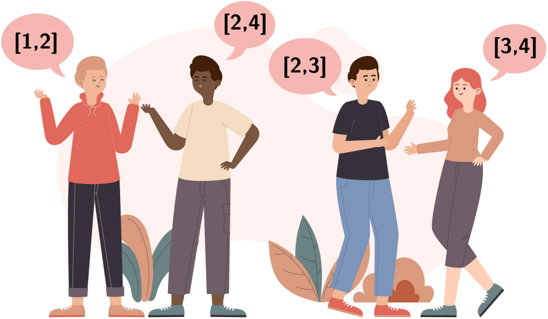
Desired group sizes:



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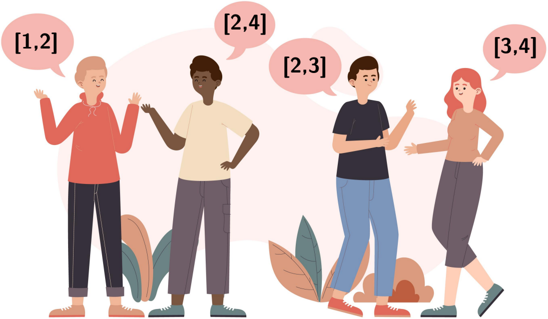

Partition hikers into subgroups such that everyone is satisfied with their group size.

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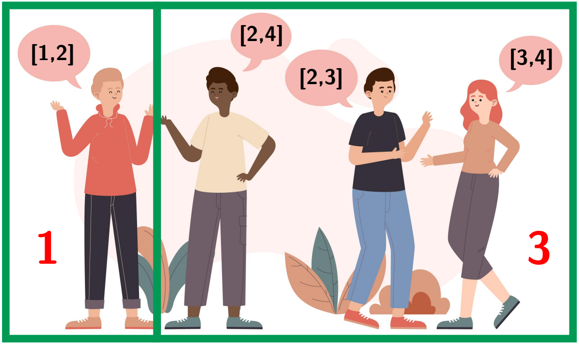

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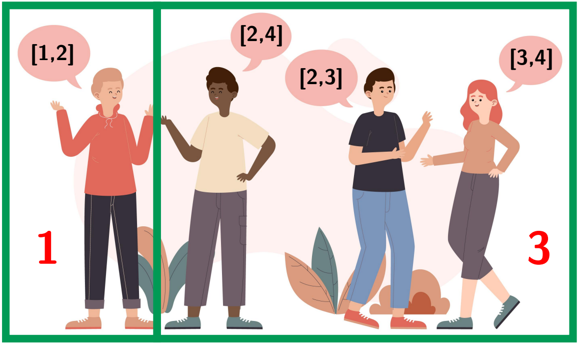

Partition hikers into subgroups such that everyone is satisfied with their group size. **Puzzle:** **Polynomial time?**

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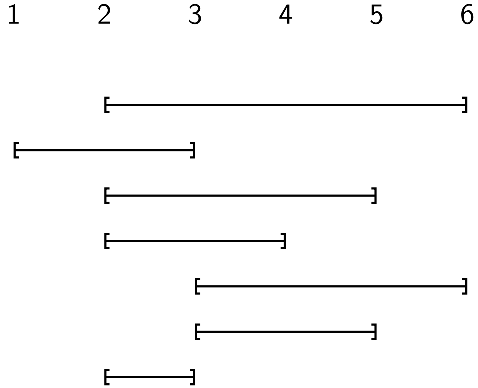

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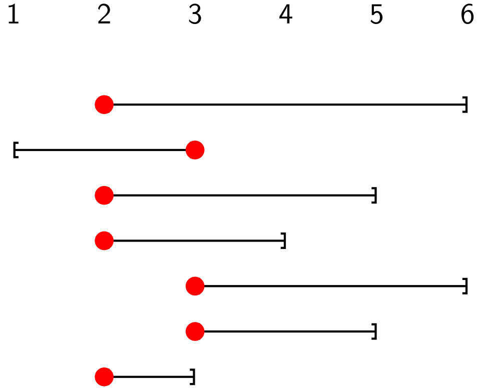

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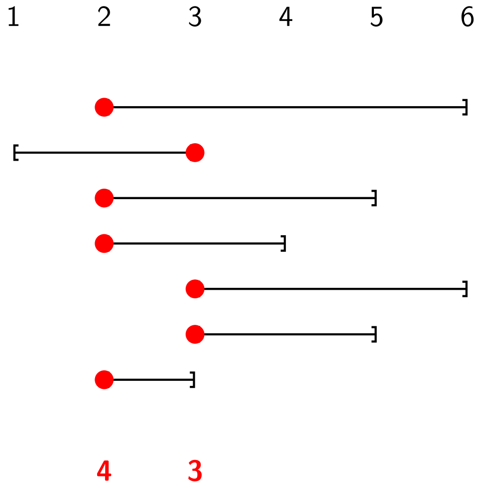

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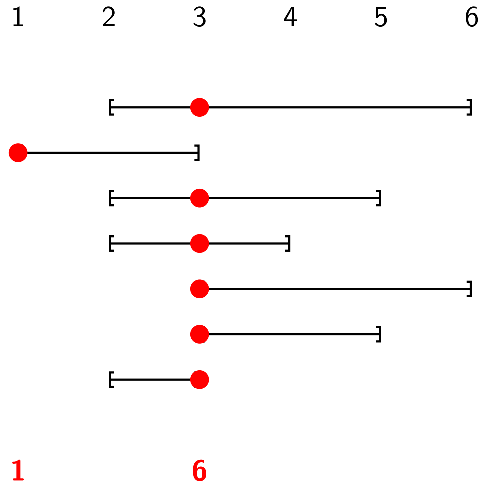

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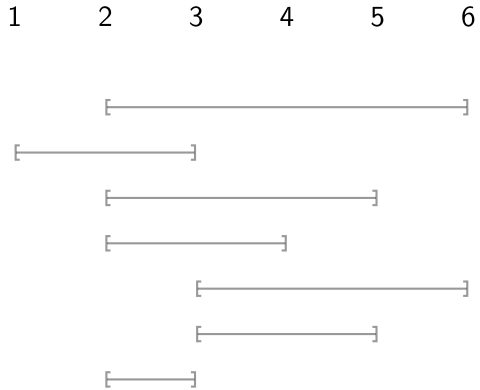

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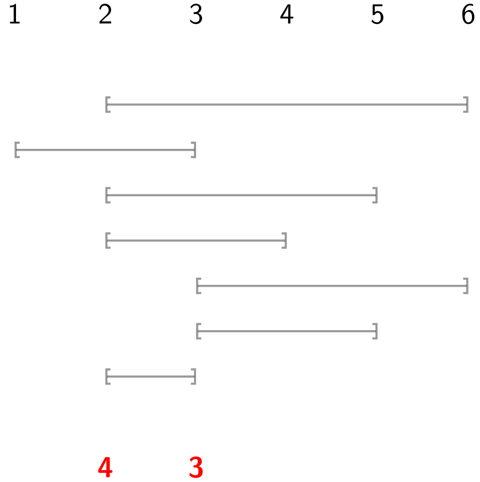

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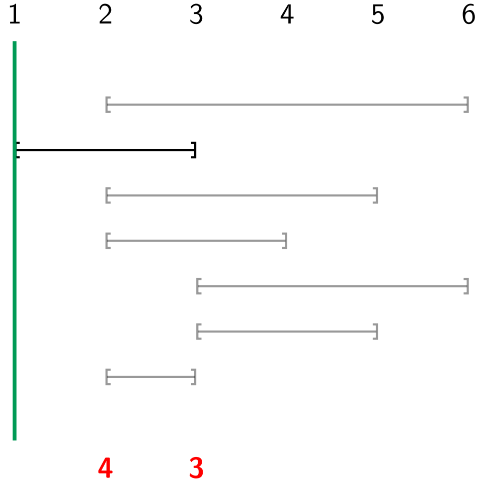

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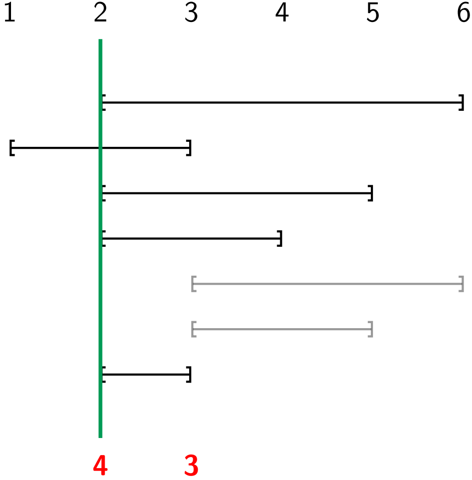

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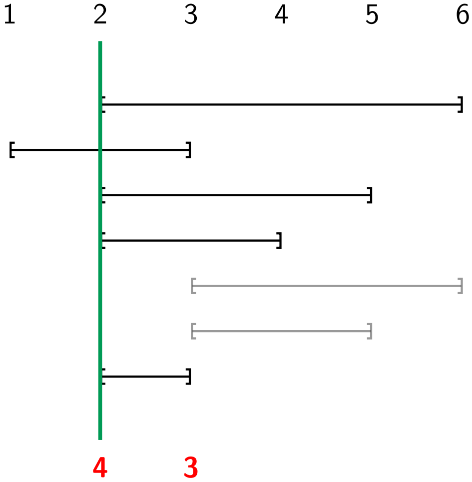

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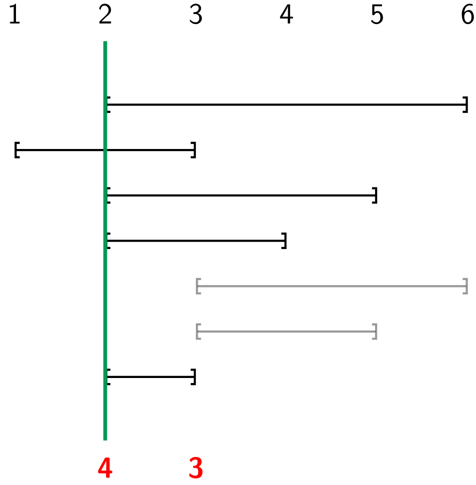

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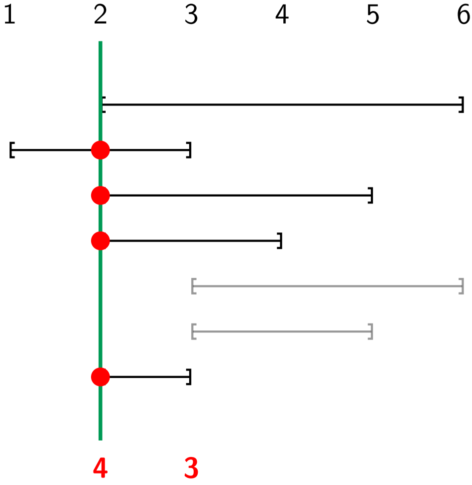

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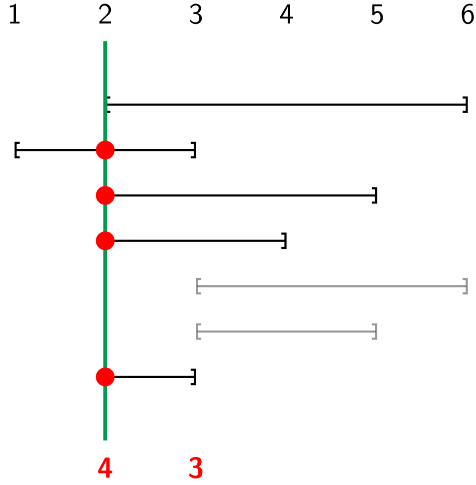

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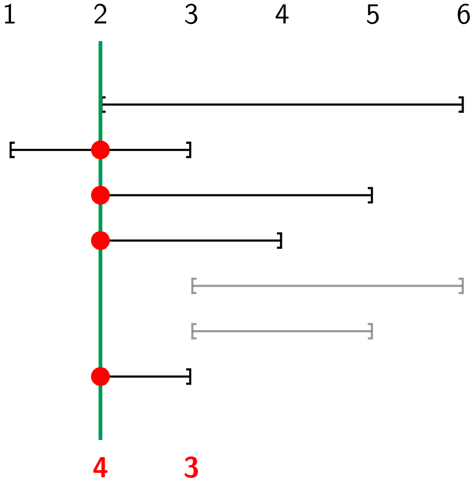

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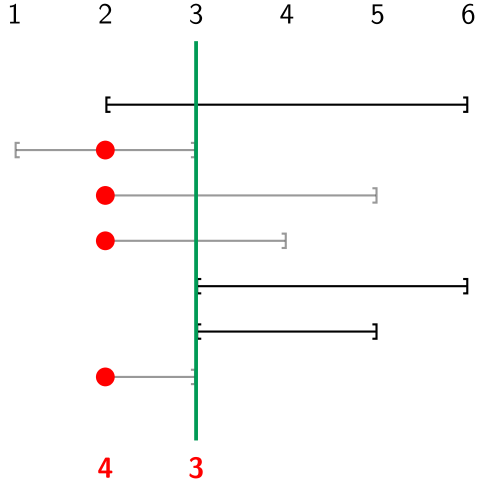

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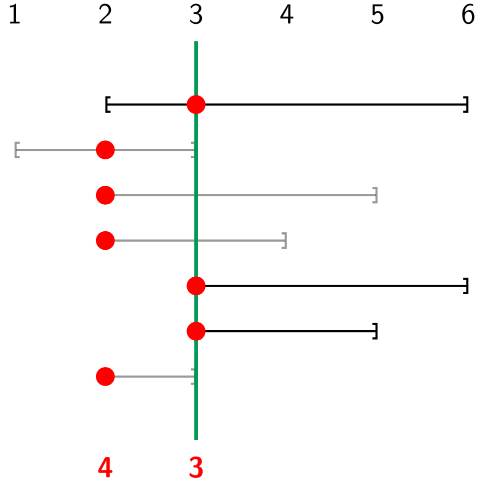

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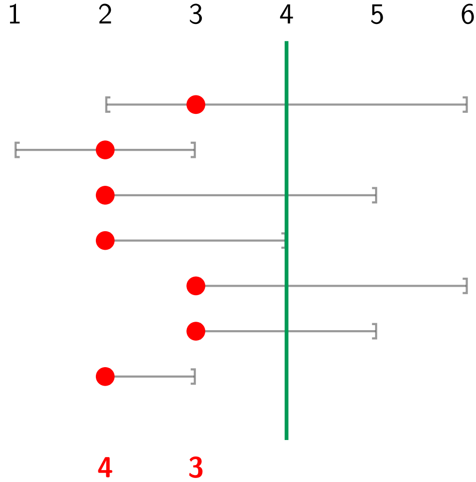

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Problem: exponentially many states!

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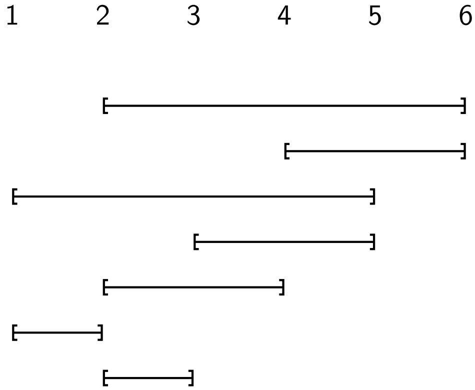

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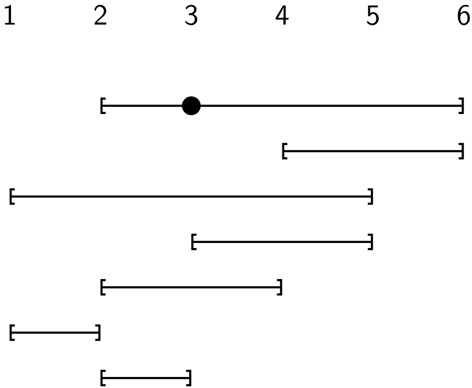

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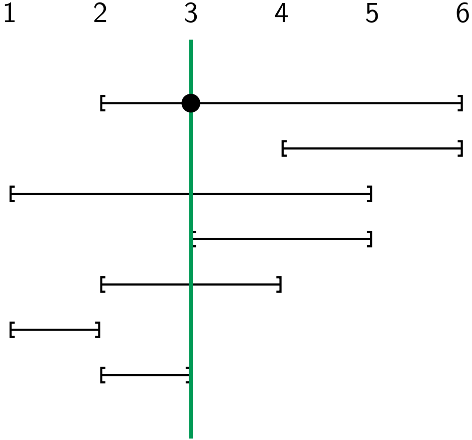

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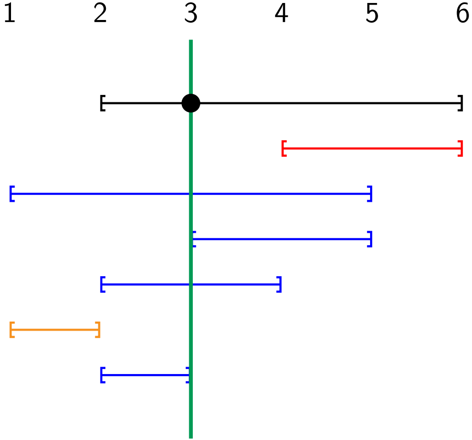

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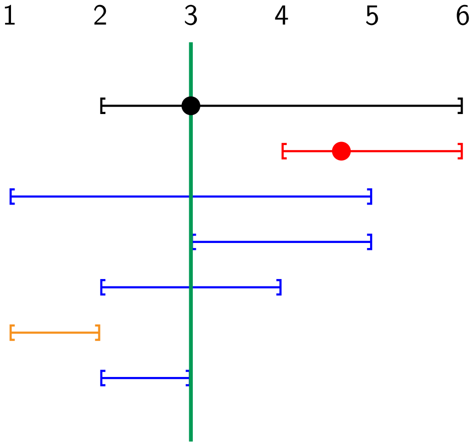

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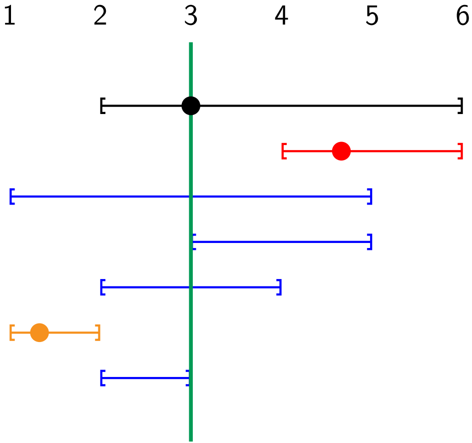

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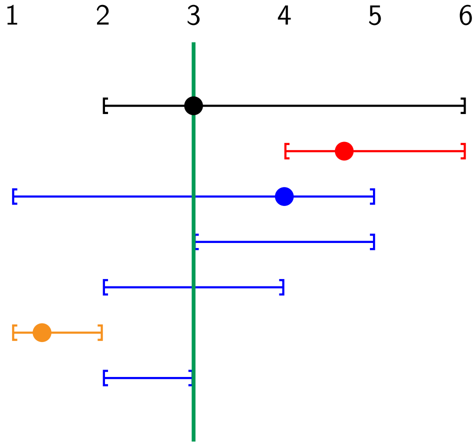

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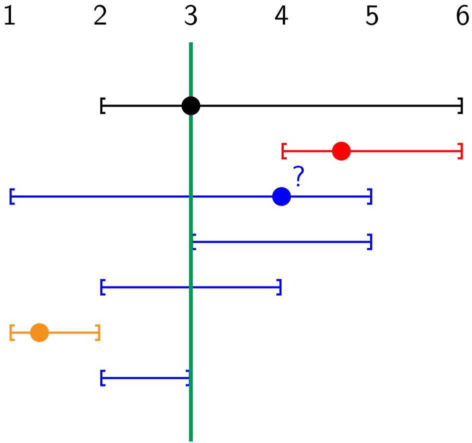

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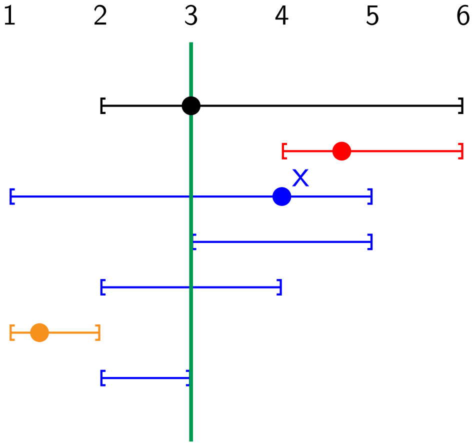

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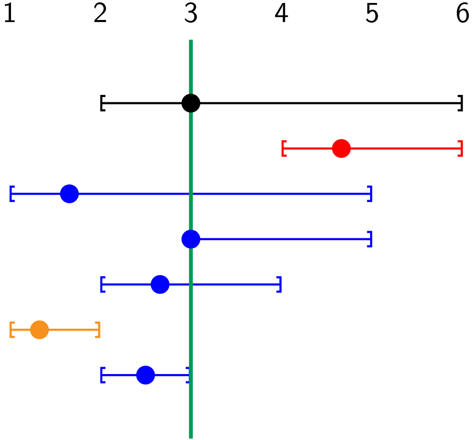

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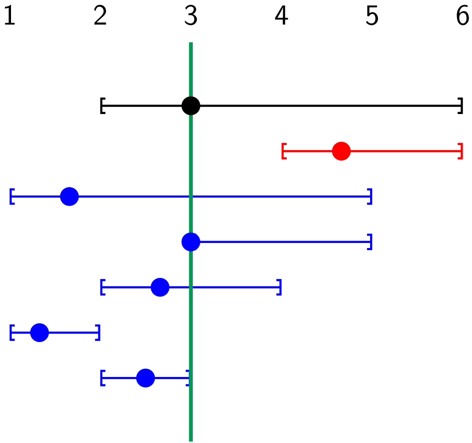

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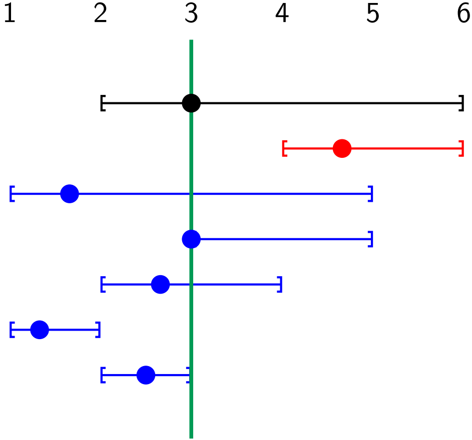

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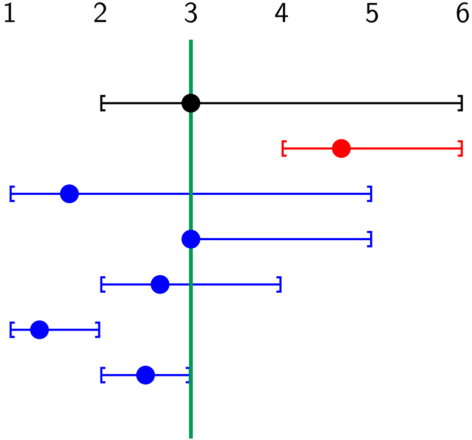

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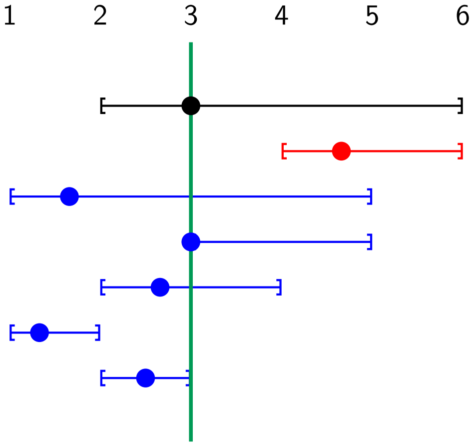

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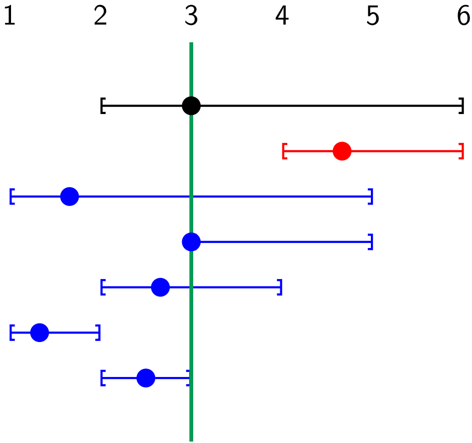

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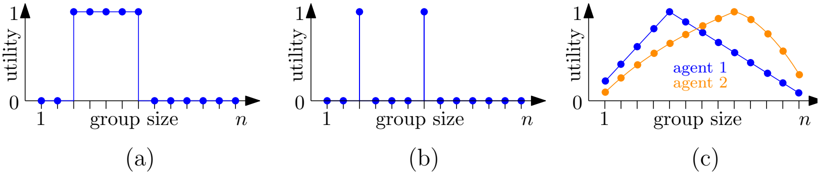

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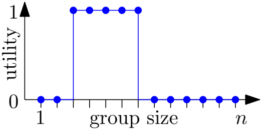

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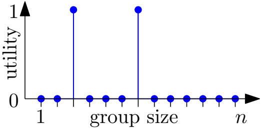

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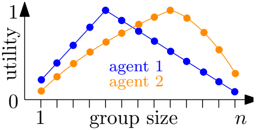

(c)

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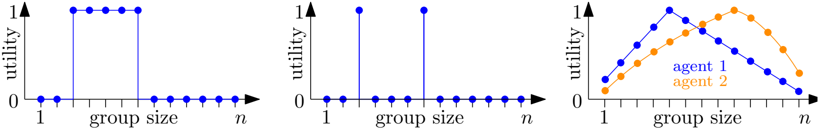

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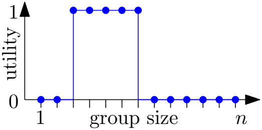

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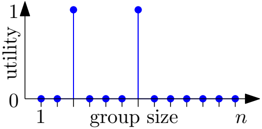

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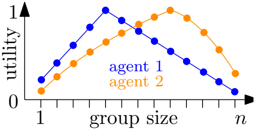

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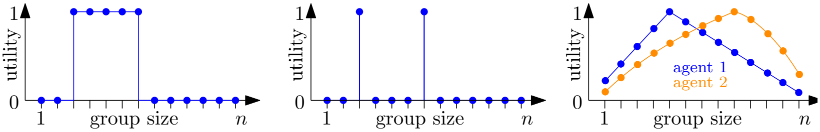

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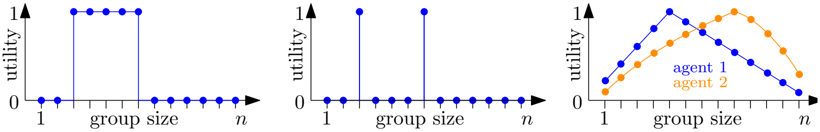

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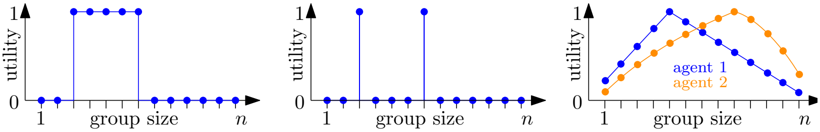

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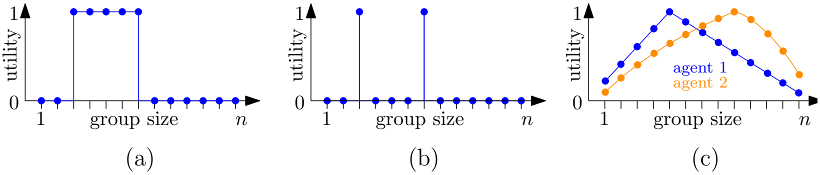

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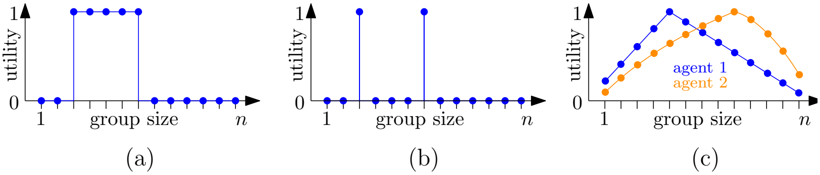

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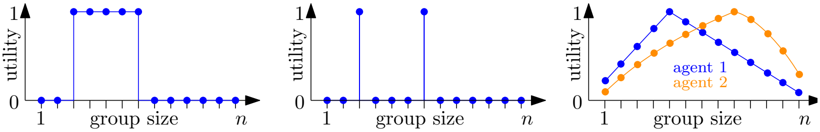

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Gerhard Woeginger (1964—2022) RIP