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On the Windfall and price of friendship: Inoculation strategies on social networks

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ABSTRACT

This article investigates selfish behavior in games where players are embedded in a social context. A framework is presented which allows us to measure the *Windfall of Friendship*, i.e., how much players benefit (compared to purely selfish environments) if they care about the welfare of their friends in the social network.

As a case study, a virus inoculation game is examined. We analyze the Nash equilibria and show that the Windfall of Friendship can never be negative. However, we find that if the valuation of a friend is independent of the total number of friends, the social welfare may not increase monotonically with the extent to which players care for each other; intriguingly, in the corresponding scenario where the relative importance of a friend declines, the Windfall is monotonic again.

This article also studies convergence of best-response sequences. It turns out that in social networks, convergence times are typically higher and hence constitute a price of friendship. While such phenomena may be known on an anecdotal level, our framework allows us to quantify these effects analytically. Our formal insights on the worst case equilibria are complemented by simulations on larger social graphs, shedding light on robustness and fairness aspects, as well as on the structure of other equilibria.

1. Introduction

This article makes a first step to combine two active threads of research: social networks and game theory. We introduce a framework taking into consideration that people may care about the well-being of their friends. In particular, we define the *Windfall of Friendship* (WoF) which captures to what extent the social welfare improves in social networks compared to purely selfish systems.

In order to demonstrate our framework, as a case study, we provide a game-theoretic analysis of a *virus inoculation game*. Concretely, we assume that a virus spreads along the social network, and that the players have the choice between inoculating by buying anti-virus software and risking infection. We assume that a player p_i takes into account the cost (or utility) of a friend p_j, i.e., a *neighbor* $p_j \in \Gamma(p_i)$ in the social graph, by adding player p_j's cost to its own cost using a weighing factor $F_{i,j} \in [0,1]$. While many of our results hold for general $F_{i,j}$ values, two special scenarios are considered in more detail: (1) An *uniform friendship model* where players care about their friends to the same extent, i.e., for all i,j (where $p_j \in \Gamma(p_i)$), $F_{i,j} \equiv F$, independently of the number of friends; and a *relative friendship model* where the friendship value depends on the number of friends (i.e., neighbors) $|\Gamma(p_i)|$ of a given player $p_i : F_{i,j} = F/|\Gamma(p_i)|$ for some constant F and $p_j \in \Gamma(p_i)$. In other words, the relative importance of a friend decreases with the total number of friends, and a player with many friends cares less about the welfare of a specific friend compared to a player who only has one or two friends.

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Our analysis confirms the expectation that, in both the uniform and the relative friendship model, the players always benefit from caring about the other participants in the social network rather than being selfish. Intriguingly, however, we find that in the uniform friendship model, the Windfall of Friendship may not increase monotonically with stronger relationships. Despite the phenomenon being an "ever-green" in political debates, to the best of our knowledge, this is the first article to quantify this effect formally.

This article also presents upper and lower bounds on the Windfall of Friendship in simple, archetypical graphs. For example, the Windfall of Friendship under the uniform friendship model in a *complete graph* (describing a situation where all players are friendly) is at most $4/3$; this is tight in the sense that there are problem instances where the situation can indeed improve by this much. Moreover, we show that in *star graphs* (the other extreme, where friendship is concentrated on a single player), friendship can help to eliminate undesirable equilibria. Generally, we discover that even in simple graphs the Windfall of Friendship can attain a large spectrum of values, from constant ratios up to $\Theta(n)$, n being the network size, which is asymptotically maximal for general graphs.

For the relative friendship model where the importance of an individual friend declines with a larger number of friends, the Windfall of Friendship is still positive, we show that the non-monotonicity result is no longer applicable. Moreover, it is proved that in both models, computing the best and the worst friendship Nash equilibrium is $\mathcal{NP}$-hard.

The article also initiates the discussion of implications on convergence. We give a potential function argument to show convergence of best-response sequences in various models and for simple, cyclic graphs. Moreover, we report on our simulations which indicate that the convergence times are typically higher in social contexts, and hence constitute a certain price of friendship.

Finally, we complement our formal analysis with a simulation studying more general social graphs and equilibria, and we initiate the discussion of robustness and fairness.

1.1. Organization

The remainder of this article is organized as follows. Section 2 reviews related work and Section 3 formally introduces our model and framework. The Windfall of Friendship on general graphs is studied in Section 4. Section 5 takes a closer look at the Windfall of Friendship on special graphs and for the uniform friendship model. Section 6 studies the similarities and differences for the relative friendship model. Aspects of best-response convergence are examined in Section 7. In our simulations (Section 8) we study more realistic social network topologies and investigate additional aspects which were not studied analytically. Finally, we conclude the article in Section 9.

2. Related work

Social networks are a fascinating research area. Social networks are studied intensively not only by social scientists, psychologists, ethnologists, and economists, but also by mathematicians and computer scientists. The advent of social networks on the Internet, e.g., *Facebook*, *LinkedIn*, *MySpace*, *Orkut*, or *Xing*, to name but a few, heralded a new kind of social interactions, and the mere scale of online networks and the vast amount of data constitute an unprecedented treasure for scientific studies. The topological structure of these networks and the dynamics of the user behavior has interesting mathematical and algorithmic dimensions. Especially the famous *small world experiment* [29] conducted by Stanley Milgram 1967 has raised the interest of the algorithm community [21] and inspired research on topics such as decentralized search algorithms [22], routing on social networks [13,26] and the identification of communities [11,33]. The dynamics of epidemic propagation of information or diseases has been studied from an algorithmic perspective as well [23]. Knowledge on effects of this cascading behavior is useful to understand phenomena as diverse as word-of-mouth effects, the diffusion of innovation, the emergence of bubbles in a financial market, or the rise of a political candidate. It can also help to identify sets of influential players in networks where marketing is particularly efficient (*viral marketing*). For a good overview on economic aspects of social networks, we refer the reader to [6], which, i.a., compares random graph theory with game theoretic models for the formation of social networks.

Another area which currently experiences a renaissance and attracts much interest from mathematicians and computer scientists, is *game theory*. In particular, mathematicians and computer scientists are interested in the important algorithmic problems posed by game theory, e.g., on the existence of pure equilibria [34]. Moreover, game theory is also used to model distributed systems such as the Internet: the structure and organic growth of the Internet depends on various actors and stake-holders; accordingly, many specific aspects have been studied from a game-theoretic point of view, e.g., *routing* [35,36], *multicast transmissions* [10], or *network creation* [9,31].

This article seeks to combine the two research areas and to apply game theory to the domain of social networks. While game theory typically relies on the assumption of purely selfish behavior, we argue that in the context of social networks where different players may have different personal relationships to each other, new models are needed: in social networks, players may not be completely selfish and autonomous but have friends about whose well-being they care to some extent.

We demonstrate our mathematical framework with a virus inoculation game on social graphs. There is a large body of literature on the propagation of viruses [4,14,19,20,38]. Miscellaneous misuse of social networks has been reported, e.g., *email viruses*[1] have used address lists to propagate to the users' friends. Similar vulnerabilities have been exploited to spread worms on the *mobile phone network* [12] and on the Internet telephony tool *Skype*.[2]

[1] For example, the Outlook worm *Worm.ExploreZip*.

[2] See http://news.softpedia.com/news/Skype-Attacked-By-Fast-Spreading-Virus-52039.shtml.

Moreover, there already exists interesting work on game theoretic and epidemic models of propagation in social networks. For instance, Montanari and Saberi [30] attend to a game theoretic model for the diffusion of an innovation in a network and characterize the rate of convergence as a function of graph structure. The authors highlight crucial differences between game theoretic and epidemic models and find that the spread of viruses, new technologies, and new political or social beliefs do not have the same viral behavior.

The articles closest to ours are [2,32]. Our model is inspired by Aspnes et al. [2]. The authors apply a classic game-theoretic analysis and show that selfish systems can be very inefficient, as the Price of Anarchy is $\Theta(n)$, where n is the total number of players. They show that computing the social optimum is $\mathcal{NP}$-hard and give a reduction to the combinatorial problem *sum-of-squares partition*. They also present a $O(\log^2 n)$ approximation. Moscibroda et al. [32] have extended this model by introducing malicious players in the selfish network. This extension facilitates the estimation of the robustness of a distributed system to malicious attacks. They also find that in a non-oblivious model, intriguingly, the presence of malicious players may actually *improve* the social welfare. In a follow-up work [24] which generalizes the social context of [32] to arbitrary bilateral relationships, it has been shown that there is no such phenomenon in a simple network creation game. The *Windfall of Malice* has also been studied in the context of congestion games [3] by Babaioff et al. In contrast to these papers, our focus here is on social graphs where players are concerned about their friends' benefits.

There is other literature on game theory where players are influenced by their neighbors. In *graphical economics* [16,18], an undirected graph is given where an edge between two players denotes that free trade is allowed between the two parties, where the absence of such an edge denotes an embargo or an other restricted form of direct trade. The payoff of a player is a function of the actions of the players in its neighborhood only. In contrast to our work, a different equilibrium concept is used and no social aspects are taken into consideration.

Note that the nature of game theory on social networks also differs from *cooperative games* (e.g., [5]) where each coalition $C \subseteq 2^V$ of players V has a certain characteristic cost or payoff function $f : 2^V \to \mathbb{R}$ describing the collective payoff the players can gain by forming the coalition. In contrast to cooperative games, the "coalitions" are fixed, and a player participates in the "coalitions" of all its neighbors.

A preliminary version of this article appeared at ACM EC 2008 [28], and there have been several interesting results related to our work since then. For example, [8] studies auctions with spite and altruism among bidders, and presents explicit characterizations of Nash equilibria for first-price auctions with random valuations and arbitrary spite/altruism matrices, and for first and second price auctions with arbitrary valuations and so-called regular social networks (players have same out-degree). By rounding a linear program with region-growing techniques, Chen et al. [7] present a better, $O(\log z)$-approximation for the best vaccination strategy in the original model of [2], where z is the support size of the outbreak distribution. Moreover, the effect of autonomy is investigated: a benevolent authority may suggest which players should be vaccinated, and the authors analyze the "Price of Opting Out" under partially altruistic behavior; they show that with positive altruism, Nash equilibria may not exist, but that the price of opting out is bounded.

We extend the conference version of this article [28] in several respects. We generalize our model and results for *individual friendship factors* and *relative friendship*, initiate the discussion of *convergence* aspects, and complement the formal insights with an extensive simulation study on Kleinberg and Facebook graphs, also considering fairness and robustness properties. More specifically, we generalize our original model where each node values all its neighbors with the same friendship factor to a model where the weight of a neighbor's cost can vary. As a special case, we study a model where the relative importance of a neighbor declines with the total number of friends. We find that while friendship is still always beneficial, the non-monotonicity result no longer applies: unlike in the uniform friendship model, the Windfall of Friendship can only increase with stronger social ties. Regarding convergence times, it turns out that compared to purely selfish environments, it can take longer until an equilibrium is reached: this constitutes another price of friendship. We present a potential function argument to prove convergence in some simple cyclic networks, and complement our study with simulations on Kleinberg graphs. Our simulation study also looks at the welfare *distribution* among players (fairness), and considers a faulty scenario where supposedly inoculated players still propagate the virus, increasing the attack component. Finally, there are several minor improvements, e.g., we generalized the bound in Theorem 4.4 from $n > 7$ to $n > 3$.

3. Model

This section introduces our model and the game-theoretic framework for social networks. As a case study to gain insights into the Windfall of Friendship, we study a virus inoculation game on a social graph. We revisit the model of this game, and then show how it can be extended to incorporate social aspects.

3.1. Virus inoculation game

The virus inoculation game we consider in this article was introduced in [2]. We are given an undirected network graph $G = (V, E)$ of $n = |V|$ players (or synonymously: *nodes*) $p_i \in V$, for $i = 1, \ldots, n$, who are connected by a set of edges (or *links*) E. Every player has to decide whether it wants to *inoculate* (e.g., purchase and install anti-virus software) which costs C, or whether it prefers saving money and facing the risk of being infected. We assume that being infected yields a damage cost of L (e.g., a computer is out of work for L days). In other words, an instance I of a game consists of a graph $G = (V, E)$, the inoculation cost C and a damage cost L. We introduce a variable a_i

for every player p_i denoting p_i's chosen *strategy*. Namely, $a_i = 1$ describes that player p_i is protected whereas for a player p_j willing to take the risk, $a_j = 0$. In the following, we will assume that $a_j \in \{0, 1\}$, that is, we do not allow players to *mix* (i.e., use probabilistic distributions over) their strategies. These choices are summarized by the *strategy profile*, the vector $\vec{a} = (a_1, \ldots, a_n)$. After the players have made their decisions, a virus spreads in the network. The propagation model is as follows. First, one player p of the network is chosen uniformly at random as a starting point. If this player is inoculated, there is no damage and the process terminates. Otherwise, the virus infects p and all unprotected neighbors of p. The virus now propagates recursively to their unprotected neighbors. Hence, the more insecure players are connected, the more likely they are to be infected. The *vulnerable region* (set of unprotected players that form a connected component, also referred to as an *attack component*) in which an insecure player p_i lies is referred to as p_i's *attack component*.

We only consider a limited region of the parameter space to avoid trivial cases. If the cost C is too large, no player will inoculate, resulting in a totally insecure network and therefore all players eventually will be infected. On the other hand, if $C \ll L$, the best strategy for all players is to inoculate. Thus, we will assume that $C \leqslant L$ and $C > L/n$ in the following.

A player incurs the following expected costs.

Definition 3.1 (*Actual Individual Cost*). The actual individual cost of a player p_i is defined as

$$c_a(i, \vec{a}) = a_i \cdot C + (1 - a_i)L \cdot k_i/n$$

where k_i denotes the size of p_i's attack component. If p_i is inoculated, k_i stands for the size of the attack component that would result if p_i became insecure. In the following, let $c_a^0(i, \vec{a})$ refer to the actual cost of an insecure and $c_a^1(i, \vec{a})$ to the actual cost of a secure player p_i.

The total *social cost* of a game is defined as the sum of the cost of all participants: $C_a(\vec{a}) = \sum_{p_i \in V} c_a(i, \vec{a})$.

As in the classic prisoners' dilemma game, the cost/utility of a player's decision depends on the decision of other players and hence it might look favorable for a player to change its mind. Classic game theory assumes that all players act selfishly, i.e., each player seeks to minimize its individual cost. In order to study the impact of such selfish behavior, the solution concept of a *Nash equilibrium* (NE) is used. A Nash equilibrium is a strategy profile where no selfish player can unilaterally reduce its individual cost given the strategy choices of the other players. We can think of Nash equilibria as the stable strategy profiles of games with selfish players. We will only consider pure Nash equilibria in this article, i.e., players cannot use random distributions over their strategies but must decide whether they want to inoculate or not.

In a pure Nash equilibrium, it must hold for each player p_i that given a strategy profile $\vec{a} \forall p_i \in V,\ \forall a_i : c_a(i, \vec{a}) \leqslant c_a(i, (a_1, \ldots, 1 - a_i, \ldots, a_n))$, implying that player p_i cannot decrease its cost by choosing an alternative strategy $1 - a_i$. In order to quantify the performance loss due to selfishness, the (not necessarily unique) Nash equilibria are compared to the optimum situation where all players collaborate. To this end we consider the *Price of Anarchy* (PoA), i.e., the ratio of the social cost of the worst Nash equilibrium divided by the optimal social cost for a problem instance *I*. More formally, $PoA(I) = \max_{NE} C_{NE}(I)/C_{OPT}(I)$.

3.2. Social networks

In this article, we consider a setting where the virus spreads on a *social graph*, i.e., players are connected to their *friends*. Concretely, the virus inoculation game is adapted for the context of social networks as follows.

We define a *Friendship Factor Matrix* $\mathbf{F}$ which captures the extent to which players care about their *friends*, i.e., about the players *adjacent* to them in the social network. More formally, $F_{i,j}$ is the factor by which a player p_i takes the individual cost of its neighbor $p_j \in \Gamma(p_i)$ into account when deciding for a strategy (we assume that $F_{i,j} = 0$ if $i = j$ or p_j is not a neighbor). $F_{i,j}$ can assume any value between 0 and 1: $F_{i,j} = 0$ implies that the player i does not consider the cost of its neighbor j at all, whereas $F_{i,j} = 1$ implies that a player values the well-being of its neighbor j to the same extent as its own.

We will partition the neighbors $\Gamma(p_i)$ of player p_i into the set $\Gamma_{sec}(p_i) \subseteq \Gamma(p_i)$ of inoculated neighbors, and the set $\Gamma_{\overline{sec}}(p_i) = \Gamma(p_i) \setminus \Gamma_{sec}(p_i)$ of insecure neighbors.

We distinguish between a player's *actual cost* and a player's *perceived cost*. A player's actual individual cost is the expected cost arising for each player defined in Definition 3.1 used to compute a game's social cost. In our social network, the decisions of our players are steered by the players' *perceived cost*.

Definition 3.2 (*Perceived Individual Cost*). The perceived individual cost of a player p_i is defined as

$$c_p(i, \vec{a}) = c_a(i, \vec{a}) + \sum_{p_j \in \Gamma(p_i)} F_{i,j} \cdot c_a(j, \vec{a}).$$

In the following, we write $c_p^0(i, \vec{a})$ to denote the perceived cost of an insecure player p_i and $c_p^1(i, \vec{a})$ for the perceived cost of an inoculated player.

Fig. 1 illustrates our model.

This definition entails a new notion of equilibrium. We define a *friendship Nash equilibrium* (FNE) as a strategy profile $\vec{a}$ where no player can reduce its *perceived* cost by unilaterally changing its strategy given the strategies of the other players. Formally, $\forall p_i \in V,\ \forall a_i : c_p(i, \vec{a}) \leqslant c_p(i, (a_1, \ldots, 1 - a_i, \ldots, a_n))$. Given this equilibrium concept, we define the *Windfall of Friendship* Υ.

Definition 3.3 (*Windfall of Friendship (WoF)*). The Windfall of Friendship $\Upsilon(\mathbf{F}, I)$ is the ratio of the social cost of the worst Nash equilibrium for *I* and the social cost of the worst friendship Nash equilibrium for *I* under the friendship factor matrix $\mathbf{F}$:

$$\Upsilon(\mathbf{F}, I) = \frac{\max_{NE} C_{NE}(I)}{\max_{FNE} C_{FNE}(\mathbf{F}, I)}$$

where C_{NE} and C_{FNE} are the actual social cost of the Nash equilibrium *NE* and the friendship Nash equilibrium *FNE* respectively.

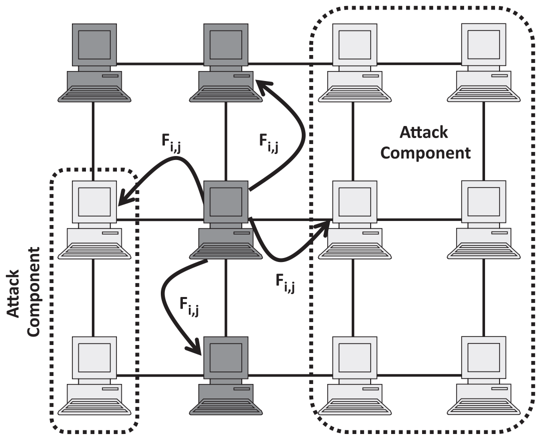


Fig. 1. Players (here: computers in an OSN) are embedded in a social context. Players care about their topological neighbors with a factor $F_{i,j}$. Players which are inoculated are depicted in darker color. The vulnerable parts are called *attack components*.

$\Upsilon(\mathbf{F}, I) > 1$ implies the existence of a real Windfall in the system, whereas $\Upsilon(\mathbf{F}, I) < 1$ denotes that the social cost can become *greater* in social graphs than in purely selfish environments.

We consider two special models (i.e., matrices $\mathbf{F}$) in more detail: (1) An *uniform friendship model* where all friendship factors are equal: for all i, j, if $p_j \in \Gamma(p_i)$ then $F_{i,j} = F$ for some given parameter F. We will sometime refer to a corresponding equilibrium by *UFNE*. (2) And a *relative friendship model* where friendship factors decline with the number of friends: for all i, j, if $p_j \in \Gamma(p_i)$ then $F_{i,j} = F/\mid \Gamma(p_i) \mid$ for some constant parameter F. We will refer to such an equilibrium by *RFNE*.

3.3. Remarks

We conclude the section with a short discussion. First, note that our model assumes that the virus propagation graph and the social network graph are *the same*: the virus propagates along the links describing friendship relationships. This especially makes sense in social networks describing human interactions: diseases may be propagated during personal meetings. But also computer viruses may spread along social links, e.g., a virus can propagate to the contacts in an instance messenger or Online Social Network. However, in other computer networks, friendship relationships may not be reflected, e.g., in the underlying physical network where the virus propagates. While theoretically our model applies to such a setting as well, it requires a methodology to describe both the physical network and the social network. The impact of friendship is likely to be smaller: friends may be remote in the underlying network and hence the perceived costs independent. We do not consider this generalization further in this article.

Second, as mentioned, throughout this article, we will assume that $F_{i,j} \leqslant 1$: a player never cares more about a friend than about himself. While some of our results even hold for $F_{i,j} > 1$, we do not consider this case in more detail.

Finally, when referring to the friendship matrix $\mathbf{F}$, we will always assume that $F_{i,j} = 0$ except for $p_j \in \Gamma(p_i)$. By slightly abusing notation, we will write $\mathbf{F} \neq 0$ or $\mathbf{F} > 0$ to denote a setting where there exists at least one neighbor with a positive friendship factor.

4. General analysis

We first present general characterizations of friendship Nash equilibria. In the following, we will assume general matrices $\mathbf{F}$, i.e., the factors $F_{i,j}$ can be arbitrary.

It has been shown [2] that in classic Nash equilibria ($\mathbf{F} = 0$), an attack component can never consist of more than Cn/L insecure players. A similar characteristic also holds for friendship Nash equilibria. As every player cares about its neighbors, the maximal attack component size in which an insecure player p_i still does not inoculate depends on the number of p_i's insecure neighbors and the size of their attack components. Therefore, it differs from player to player. We have the following helper lemma.

Lemma 4.1. *The player p_i will inoculate if and only if the size of its attack component is*

$$k_i > \frac{Cn/L + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot k_j}{1 + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}},$$

where the k_js are the attack component sizes of p_i's insecure neighbors assuming p_i is secure.

Proof. Player p_i will inoculate if and only if this choice lowers the perceived cost. The perceived individual cost of an inoculated player is

$$c_p^1(i, \vec{a}) = C + \sum_{p_j \in \Gamma_{sec}(p_i)} F_{i,j} \cdot C + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot L\frac{k_j}{n}$$

and for an insecure player we have

$$c_p^0(i, \vec{a}) = Lk_i/n + \sum_{p_j \in \Gamma_{sec}(p_i)} F_{i,j} \cdot C + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot Lk_i/n.$$

For p_i to prefer to inoculate it must hold that

$$\begin{aligned} c_p^0(i, \vec{a}) > c_p^1(i, \vec{a}) &\iff Lk_i/n + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot Lk_i/n \\ &> C + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot L\frac{k_j}{n} \iff k_i \\ &> \frac{Cn/L + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot k_j}{1 + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}}. \qquad \square \end{aligned}$$

An interesting question is whether social networks where players care about their friends yield better equilibria than selfish environments. The following theorem answers this questions affirmatively: the worst FNE costs never more than the worst NE.

Theorem 4.2. *For all instances of the virus inoculation game and friendship factor matrix F with $F_{i,j} \in [0, 1]$, it holds that*

$$1 \leqslant \Upsilon(F, I) \leqslant PoA(I)$$

Proof. The proof idea for $\Upsilon(\mathbf{F}, I) \geqslant 1$ is the following: for an instance I we consider an arbitrary FNE with $\mathbf{F} \neq 0$. Given this equilibrium, we show the existence of a NE with larger social cost (according to [2], our best response strategy always converges). Let $\vec{a}$ be any (e.g., the worst) FNE in the social model. If $\vec{a}$ is also a NE in the same instance with $\mathbf{F} = 0$ then we are done. Otherwise there is at least one player p_i that prefers to change its strategy. Assume p_i is insecure but favors inoculation. Therefore p_i's attack component has on the one hand to be of size at least $k_i' > Cn/L$ [2] and on the other hand of size at most $k_i'' = (Cn/L + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot k_j)/(1 + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}) \leqslant (Cn/L + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot (k_i'' - 1))/(1 + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}) \iff k_i'' \leqslant Cn/L - \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}$ (cf Lemma 4.1). This is impossible and yields a contradiction to the assumption that in the selfish network, an additional player wants to inoculate.

It remains to study the case where p_i is secure in the FNE but prefers to be insecure in the NE. Observe that, since every player has the same preference on the attack component's size when $F_{i,j} = 0$, a newly insecure player cannot trigger other players to inoculate. Furthermore, only the players inside p_i's attack component are affected by this change. The total cost of this attack component increases by at least

$$x = \underbrace{k_i/n \cdot L - C}_{p_i} + \underbrace{\sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} \left(k_i/n \cdot L - \frac{k_j}{n} L\right)}_{p_i\text{'s insecure neighbors}}$$

$$= k_i/n \cdot L - C + \frac{L}{n}(|\Gamma_{\overline{sec}}(p_i)|k_i - \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} k_j)$$

Applying Lemma 4.1 guarantees that

$$\sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} k_j \leqslant \frac{k_i\left(1 + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}\right) - Cn/L}{\sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}}$$

This results in

$$x \geqslant \frac{L}{n}\left(\sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot k_i - \frac{k_i\left(1 + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}\right) - Cn/L}{\sum_{p_j \in \Gamma_{\overline{sec}}} F_{i,j}}\right)$$

$$= \frac{k_i L}{n}\left(1 - \frac{1}{\sum_{p_j \in \Gamma_{\overline{sec}}} F_{i,j}}\right) - C\left(1 - \frac{1}{\sum_{p_j \in \Gamma_{\overline{sec}}} F_{i,j}}\right) > 0$$

since a player only gives up its protection if $C > \frac{k_i L}{n}$. If more players are unhappy with their situation and become vulnerable, the cost for the NE increases further. In conclusion, there exists a NE for every FNE (with positive friendship values) for the same instance which is at least as expensive.

The upper bound for the WoF, i.e., $PoA(I) \geqslant \Upsilon(\mathbf{F}, I)$, follows directly from the definitions: while the PoA is the ratio of the NE's social cost divided by the social optimum, $\Upsilon(\mathbf{F}, I)$ is the ratio between the cost of the NE and the FNE. As the FNE's cost must be at least as large as the social optimum cost the claim follows. □

Remark 4.3. Note that Aspnes et al. [2] proved that the Price of Anarchy never exceeds the size of the network, i.e., $n \geqslant PoA(I)$. Consequently, the Windfall of Friendship cannot be larger than n due to Theorem 4.2.

The above result leads to the question of whether the Windfall of Friendship grows monotonically with stronger social ties, i.e., with larger friendship factors. Intriguingly, this is not the case, even when all nodes have the same friendship factor.

Theorem 4.4. *For more than three players, there exist game instances where $\Upsilon(F^{(s)}, I) > \Upsilon(F^{(l)}, I)$ for two friendship matrices $F^{(s)}$ and $F^{(l)}$, although $F^{(l)}$ dominates $F^{(s)}$ in the sense that $F^{(s)}_{i,j} \leqslant F^{(l)}_{i,j}$ for all i,j.*

Proof. We prove the claim on the star graph S_n which has one center player and $n-1$ leaf players. We consider two friendship matrices, $\mathbf{F}^{(s)}$ and $\mathbf{F}^{(l)}$, where $\mathbf{F}^{(s)}$ represents weaker social ties than $\mathbf{F}^{(l)}$. Concretely, we assume that for two neighbors p_i and p_j, $F^{(s)}_{i,j} = \alpha_s$ for some small $\alpha_s > 0$, and similarly, $F^{(l)}_{i,j} = \alpha_l$ for some larger $\alpha_l > \alpha_s$.

We show that under $\mathbf{F}^{(l)}$, there exists a FNE, FNE_1, where only the center player and one leaf player remain insecure. For the same setting but under weaker social ties, at least two leaf players will remain insecure, which will trigger the center player to inoculate, yielding a FNE, FNE_2, where only the center player is secure.

Consider FNE_1 first. Let c be the insecure center player, let l_1 be the insecure leaf player, and let l_2 be a secure leaf player. In order for FNE_1 to constitute a Nash equilibrium, the following conditions must hold:

$$\begin{aligned}
&\text{player } c: \ \frac{2L}{n} + \frac{2\alpha_l L}{n} < C + \frac{\alpha_l L}{n} \\
&\text{player } l_1: \ \frac{2L}{n} + \frac{2\alpha_l L}{n} < C + \frac{\alpha_l L}{n} \\
&\text{player } l_2: \ C + \frac{2\alpha_l L}{n} < \frac{3L}{n} + \frac{3\alpha_l L}{n}
\end{aligned}$$

For FNE_2, let c be the insecure center player, let l_1 be one of the two insecure leaf players, and let l_2 be a secure leaf player. In order for the leaf players to be happy with their situation but for the center player to prefer to inoculate, it must hold that:

$$\begin{aligned}
&\text{player } c: \ C + \frac{2\alpha_s L}{n} < \frac{3L}{n} + \frac{6\alpha_s L}{n} \\
&\text{player } l_1: \ \frac{3L}{n} + \frac{3\alpha_s L}{n} < C + \frac{2\alpha_s L}{n} \\
&\text{player } l_2: \ C + \frac{3\alpha_s L}{n} < \frac{4L}{n} + \frac{4\alpha_s L}{n}
\end{aligned}$$

Now choose $C := 5L/(2n) + \alpha_l L/n$ (note that due to our assumption that $n > 3, C < L$). This yields the following conditions: $\alpha_l > \alpha_s + 1/2, \alpha_l < \alpha_s + 3/2$, and $\alpha_l < 4\alpha_s + 1/2$. These conditions are easily fulfilled, e.g., with $\alpha_l = 3/4$ and $\alpha_s = 1/8$. Observe that the social cost of the first FNE (for α_l) is $Cost(S_n, \vec{a}_{FNE1}) = (n-2)C + 4L/n$, whereas for the second FNE (for $\mathbf{F}_s$) $Cost(S_n, \vec{a}_{FNE2}) = C + (n-1)L/n$. Thus, $Cost(S_n, \vec{a}_{FNE1}) - Cost(S_n, \vec{a}_{FNE2}) = (n-3)C - (n-5)L/n > 0$ as we have chosen $C > 5L/(2n)$ and as, due to our assumption, $n > 3$. This concludes the proof. □

Reasoning about best and worst Nash equilibria raises the question of how difficult it is to compute such equilibria. We can generalize the proof given in [2] and show that computing the most economical and the most expensive FNE is hard for any friendship factor.

Theorem 4.5. *Computing the best and the worst pure FNE is $\mathcal{NP}$-complete for any social matrix F with $F_{i,j} \in [0,1]$.*

Proof. We prove this theorem by a reduction from two $\mathcal{NP}$-hard problems, VERTEX COVER [17] and INDEPENDENT DOMINATING SET [15]. Concretely, for the decision version of the problem, we show that answering the question whether there exists a FNE costing less than k, or more than k respectively, is at least as hard as solving vertex cover or independent dominating set. Note that verifying whether a proposed solution is correct can be done in polynomial time, hence the problems are indeed in $\mathcal{NP}$.

Fix some graph $G = (V, E)$ and set $C = 1$ and $L = n/1.5$. We show that the following two conditions are necessary and sufficient for a FNE: (a) all neighbors of an insecure player are secure, and (b) every inoculated player has at least one insecure neighbor. Due to our assumption that $C > L/n$, Condition (b) is satisfied in all FNE. To see that Condition (a) holds as well, assume the contrary, i.e., an attack component of size at least two. An insecure player p_i in this attack component bears the cost $k_i/nL + \sum_{p_j \in \Gamma_{sec}(p_i)} F_{i,j} \cdot C + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \cdot k_i/nL)$. Changing its strategy reduces its cost by at least $\Delta_i = k_i/nL + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} k_i/nL - C - \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j}(k_i - 1)/n \cdot L = k_i/nL + \sum_{p_j \in \Gamma_{\overline{sec}}(p_i)} F_{i,j} \frac{1}{n} L - C$. By our assumption that $k_i \geqslant 2$ it holds that $\Delta_i > 0$, resulting in p_i becoming secure. Hence, Condition (a) holds in any FNE as well. For the opposite direction assume that an insecure player wants to change its strategy even though (a) and (b) are true. This is impossible because in this case (b) would be violated because this player does not have any insecure neighbors. An inoculated player would destroy (a) by adopting another strategy. Thus (a) and (b) are sufficient for a FNE.

We now argue that G has a vertex cover of size k if and only if the virus game has a FNE with k or fewer secure players, or equivalently an equilibrium with social cost at most $Ck + (n-k)L/n$, as each insecure player must be in a component of size 1 and contributes exactly L/n expected cost. Given a minimal vertex cover $V' \subseteq V$, observe that installing the software on all players in V' satisfies Condition (a) because V' is a vertex cover and (b) because V' is minimal. Conversely, if V' is the set of secure players in a FNE, then V' is a vertex cover by Condition (a) which is minimal by Condition (b).

For the worst FNE, we consider an instance of the independent dominating set problem. Given an independent dominating set V', installing the software on all players except the players in V' satisfies Condition (a) because V' is independent and (b) because V' is a dominating set. Conversely, the insecure players in any FNE are independent by Condition (a) and dominating by Condition (b). This shows that G has an independent dominating set of size at most k if and only if it has a FNE with at least $n-k$ secure players. □

5. Windfall for special graphs

While the last section has presented general results on equilibria in social networks and the Windfall of Friendship, we now present upper and lower bounds on the Windfall of Friendship for concrete topologies, namely the *complete graph* K_n and the *star graph* S_n. In the following, we will focus on the uniform friendship model (i.e., *UFNE* equilibria) where $F_{i,j} = F$, for all neighboring players p_i and p_j.

5.1. Complete graphs

In order to initiate the study of the Windfall of Friendship, we consider a very simple topology, the complete graph K_n where all players are connected to each other. First consider the classic setting where players do not care about their neighbors ($F = 0$). We have the following result:

Lemma 5.1. *In the graph K_n, there are two Nash equilibria with social cost*

$$NE_1 : Cost(K_n, \vec{a}_{NE1}) = C(n - \lceil Cn/L \rceil + 1) + L/n(\lceil Cn/L \rceil - 1)^2$$

and

$$NE_2 : Cost(K_n, \vec{a}_{NE2}) = C(n - \lfloor Cn/L \rfloor) + L/n(\lfloor Cn/L \rfloor)^2$$

If $\lceil Cn/L \rceil - 1 = \lfloor Cn/L \rfloor$, there is only one Nash equilibrium.

Proof. Let $\vec{a}$ be a NE. Consider an inoculated player p_i and an insecure player p_j, and hence $c_a(i, \vec{a}) = C$ and $c_a(j, \vec{a}) = L\frac{k_j}{n}$, where k_j is the total number of insecure players in K_n. In order for p_i to remain inoculated, it must hold that $C \leqslant (k_j + 1)L/n$, so $k_j \geqslant \lceil Cn/L - 1 \rceil$; for p_j to remain insecure, it holds that $k_j L/n \leqslant C$, so $k_j \leqslant \lfloor Cn/L \rfloor$. As the total social cost in K_n is given by $Cost(K_n, \vec{a}) = (n - k_j)C + k_j^2 L/n$, the claim follows. □

Observe that the equilibrium size of the attack component is roughly twice the size of the attack component of the social optimum, as $Cost(K_n, \vec{a}) = (n - k_j)C + k_j^2 L/n$ is minimized for $k_j = Cn/2L$.

Lemma 5.2. *In the social optimum for K_n, the size of the attack component is either $\lfloor \frac{1}{2} Cn/L \rfloor$ or $\lceil \frac{1}{2} Cn/L \rceil$, yielding a total social cost of*

$$Cost(K_n, \vec{a}_{OPT}) = \left(n - \left\lfloor \frac{1}{2} Cn/L \right\rfloor\right) C + \left(\left\lfloor \frac{1}{2} Cn/L \right\rfloor\right)^2 \frac{L}{n}$$

or

$$Cost(K_n, \vec{a}_{OPT}) = \left(n - \left\lceil \frac{1}{2} Cn/L \right\rceil\right) C + \left(\left\lceil \frac{1}{2} Cn/L \right\rceil\right)^2 \frac{L}{n}$$

In order to compute the Windfall of Friendship, the friendship Nash equilibria in social networks have to be identified.

Lemma 5.3. *In K_n, there are two friendship Nash equilibria with social cost*

$$FNE_1 : Cost(K_n, \vec{a}_{FNE1}) = C\left(n - \left\lceil \frac{Cn/L - 1}{1+F} \right\rceil\right) + L/n\left(\left\lceil \frac{Cn/L - 1}{1+F} \right\rceil\right)^2$$

and

$$FNE_2 : Cost(K_n, \vec{a}_{FNE2}) = C\left(n - \left\lfloor \frac{Cn/L + F}{1+F} \right\rfloor\right) + L/n\left(\left\lfloor \frac{Cn/L + F}{1+F} \right\rfloor\right)^2$$

If $\lceil (Cn/L - 1)/(1+F) \rceil = \lfloor (Cn/L + F)/(1+F) \rfloor$, there is only one FNE.

Proof. According to Lemma 4.1, in a FNE, a player p_i remains secure if otherwise the component had size at least $k_i = k_j + 1 \geqslant (Cn/L + Fk_j^2)/(1 + Fk_j)$ where k_j is the number of insecure players. This implies that $k_j \geqslant \lceil (Cn/L - 1)/(1+F) \rceil$. Dually, for an insecure player p_j it holds that $k_j \leqslant (Cn/L + F(k_j - 1)^2)/(1 + F(k_j - 1))$ and therefore $k_j \leqslant \lfloor (Cn/L + F)/(1+F) \rfloor$. Given these bounds on the total number of insecure players in a FNE, the social cost can be obtained by substituting k_j in $Cost(K_n, \vec{a}) = (n - k_j)C + k_j^2 L/n$. As the difference between the upper and the lower bound for k_j is at most 1, there are at most two equilibria and the claim follows. □

Given the characteristics of the different equilibria, we have the following theorem.

Theorem 5.4. *In K_n, the Windfall of Friendship is at most $\Upsilon(F, I) = 4/3$ for an arbitrary network size. This is tight in the sense that there are indeed instances where the worst FNE is a factor $4/3$ better than the worst NE.*

Proof. *Upper Bound.* We first derive the upper bound on $\Upsilon(F, I)$.

$$\begin{aligned}\Upsilon(F,I) &= \frac{Cost(K_n, \vec{a}_{NE})}{Cost(K_n, \vec{a}_{\text{FNE}})} \\ &\leqslant \frac{Cost(K_n, \vec{a}_{NE})}{Cost(K_n, \vec{a}_{OPT})} \\ &\leqslant \frac{(n - \lceil Cn/L - 1 \rceil)C + (\lfloor Cn/L \rfloor)^2 \frac{L}{n}}{(n - \frac{1}{2}Cn/L)C + (\frac{1}{2}Cn/L)^2 \frac{L}{n}}\end{aligned}$$

as the optimal social cost (cf Lemma 5.2) is smaller or equal to the social cost of any FNE. Simplifying this expression yields

$$\Upsilon(F,I) \leqslant \frac{n(1 - C/L)C + C^2 n/L}{n(1 - \frac{1}{2}C/L)C + \frac{1}{4}C^2 n/L} = \frac{1}{1 - \frac{1}{4}C/L}$$

This term is maximized for $L = C$, implying that $\Upsilon(F, I) \leqslant 4/3$, for arbitrary n.

Lower Bound. We now show that the ratio between the equilibria cost reaches $4/3$.

There exists exactly one social optimum of cost $Ln/2 + (n/2)^2 L/n = 3n \cdot L/4$ for even n and $C = L$ by Lemma 5.2. For $F = 1$ this is also the only friendship Nash equilibrium due to Lemma 5.3. In the selfish game however the Nash equilibrium has fewer inoculated players and is of cost $n \cdot L$ (see Lemma 5.1). Since these are the only Nash equilibria they constitute the worst equilibria and the ratio is

$$\Upsilon(F,I) = \frac{Cost(K_n, \vec{a}_{NE})}{Cost(K_n, \vec{a}_{\text{FNE}})} = \frac{n \cdot L}{3/4n \cdot L} = 4/3 \qquad \square$$

To conclude our analysis of K_n, observe that friendship Nash equilibria always exist in complete graphs, and that in environments where one player at a time is given the chance to change its strategy in a best response manner quickly results in such an equilibrium as all players have the same preferences.

5.2. Star

While the analysis of K_n was simple, it turns out that already slightly more sophisticated graphs are challenging. In the following, we investigate the Windfall of Friendship in star graphs S_n. Note that in S_n, the social welfare is maximized if the center player inoculates and all other players do not. The total inoculation cost then is C and the attack components are all of size 1, yielding a total social cost of $Cost(S_n, \vec{a}_{OPT}) = C + (n-1)L/n$.

Lemma 5.5. *In the social optimum of the star graph S_n, only the center player is inoculated. The social cost is*

$Cost(S_n, \vec{a}_{OPT}) = C + (n-1)L/n.$

The situation where only the center player is inoculated also constitutes a NE. However, there are more Nash equilibria.

Lemma 5.6. *In the star graph S_n, there are at most three Nash equilibria with social cost*

$$NE_1 : Cost(S_n, \vec{a}_{NE1}) = C + (n-1)L/n$$

$$NE_2 : Cost(S_n, \vec{a}_{NE2}) = C(n - \lceil Cn/L \rceil + 1) + L/n(\lceil Cn/L \rceil - 1)^2$$

and

$$NE_3 : Cost(S_n, \vec{a}_{NE3}) = C(n - \lfloor Cn/L \rfloor) + L/n(\lfloor Cn/L \rfloor)^2$$

If $Cn/L \notin \mathbb{N}$, only two equilibria exist.

Proof. If the center player is the only secure player, changing its strategy costs L but saves only C. When a leaf player becomes secure, its cost changes from L/n to C. These changes are unprofitable, and the social cost of this NE is $Cost(S_n, \vec{a}_{NE1}) = C + (n-1)L/n$.

For the other Nash equilibria the center player is not inoculated. Let the number of insecure leaf players be n_0. In order for a secure player to remain secure, it must hold that $C \leqslant (n_0 + 2)L/n$, and hence $n_0 \geqslant \lceil Cn/L - 2 \rceil$. For an insecure player to remain insecure, it must hold that $(1 + n_0)L/n \leqslant C$, thus $n_0 \leqslant \lfloor Cn/L - 1 \rfloor$. Therefore, we can conclude that there are at most two Nash equilibria, one with $\lceil Cn/L - 1 \rceil$ and one with $\lfloor Cn/L \rfloor$ many insecure players. The total social cost follows by substituting n_0 in the total social cost function. Finally, observe that if

$Cn/L \in \mathbb{N}$ and $Cn/L > 3$, all three equilibria exist in parallel; if $Cn/L \notin \mathbb{N}$, NE_2 and NE_3 become one equilibrium. □

Let us consider the social network scenario again.

Lemma 5.7. *In S_n, there are at most three friendship Nash equilibria with social cost*

$$FNE_1 : Cost(S_n, \vec{a}_{FNE1}) = C + (n-1)L/n$$
$$FNE_2 : Cost(S_n, \vec{a}_{FNE2}) = C(n - \lceil Cn/L - F\rceil + 1) + L/n(\lceil Cn/L - F\rceil - 1)^2$$

and

$$FNE_3 : Cost(S_n, \vec{a}_{FNE3}) = C(n - \lfloor Cn/L - F\rfloor) + L/n(\lfloor Cn/L - F\rfloor)^2$$

If $Cn/L - F \notin \mathbb{N}$, at most 2 friendship Nash equilibria exist.

Proof. First, observe that having only an inoculated center player constitutes a FNE. In order for the center player to remain inoculated, it must hold that $C + F(n-1)L\frac{1}{n} \leqslant n \cdot L/n + F(n-1)L\frac{n}{n} = L + F(n-1)L$. All leaf players remain insecure as long as $L/n + FC \leqslant C + FC \iff L/n \leqslant C$. These conditions are always true, and we have $Cost(S_n, \vec{a}_{FNE1}) = C + (n-1)L/n$. If the center player is not inoculated, we have n_0 insecure and $n - n_0 - 1$ inoculated leaf players. In order for a secure leaf player to remain secure, it is necessary that $C + F\frac{n_0+1}{n}L \leqslant \frac{n_0+2}{n}L + F\frac{n_0+2}{n}L$, so $n_0 \geqslant \lceil Cn/L - F\rceil - 2$. For an insecure leaf player, it must hold that $\frac{n_0+1}{n}L + F\frac{n_0+1}{n}L \leqslant C + F\frac{n_0}{n}L$, so $n_0 \leqslant \lfloor Cn/L - F\rfloor - 1$. The claim follows by substitution. □

Note that there are instances where FNE_1 is the only friendship Nash equilibrium. We already made use of this phenomenon in Section 4 to show that $\Upsilon(F, I)$ is not monotonically increasing in F. The next lemma states under which circumstances this is the case.

Lemma 5.8. *In S_n, there is a unique FNE equivalent to the social optimum if and only if*

$$\lfloor Cn/L - F\rfloor - \left\lfloor \frac{1}{2F}\left(\sqrt{1 - 4F(1 - Cn/L)} - 1\right)\right\rfloor - 2 \geqslant 0$$

Proof. S_n has only one FNE if and only if every (insecure) leaf player is content with its chosen strategy but the insecure center player would rather inoculate. In order for an insecure leaf player to remain insecure we have $n_0 \leqslant \lfloor Cn/L - 1 - F\rfloor$ and the insecure center player wants to inoculate if and only if

$$C + F(n - n_0 - 1)C + Fn_0\frac{1}{n}L < (n_0 + 1)\frac{L}{n} + F(n - n_0 - 1)C + Fn_0\frac{n_0 + 1}{n}L$$

which is equivalent to $Fn_0^2 + n_0 + 1 - Cn/L > 0$. This implies that $n_0 \geqslant \lfloor \frac{1}{2F}(\sqrt{1 - 4F(1 - Cn/L)} - 1) + 1\rfloor$. Therefore there is only one FNE if and only if there exists an integer n_0 such that $\lfloor \frac{1}{2F}(\sqrt{1 - 4F(1 - Cn/L)} - 1) + 1\rfloor \leqslant n_0 \leqslant \lfloor Cn/L - 1 - F\rfloor$. □

Given the characterization of the various equilibria, the Windfall of Friendship can be computed.

Theorem 5.9. *If $\lfloor \frac{1}{2F}(\sqrt{1 - 4F(1 - Cn/L)} - 1)\rfloor + 2 - \lfloor Cn/L - F\rfloor \leqslant 0$, the Windfall of Friendship is*

$$\Upsilon(F, I) \geqslant \frac{(n-2)C + L/n}{C + (n-1)L/n}, \text{ else } \Upsilon(F, I) \leqslant \frac{n+1}{n-3}$$

Proof. According to Lemma 5.8, the friendship Nash equilibrium is unique and hence equivalent to the social optimum if

$$\lfloor Cn/L - F\rfloor - \left\lfloor \frac{1}{2F}\left(\sqrt{1 - 4F(1 - Cn/L)} - 1\right)\right\rfloor - 2 \geqslant 0$$

On the other hand, observe that there always exist suboptimal Nash equilibria where the center player is not inoculated. Hence, we have

$$\begin{aligned}\Upsilon(F, I) &= \frac{Cost(S_n, \vec{a}_{NE})}{Cost(S_n, \vec{a}_{FNE})} = \frac{Cost(S_n, \vec{a}_{E})}{Cost(S_n, \vec{a}_{OPT})} \\ &\geqslant \frac{(n - \lfloor Cn/L - 1\rfloor)C + (\lceil Cn/L\rceil - 1)^2 L/n}{C + (n-1)L/n} \\ &\geqslant \frac{C(n-2) + L/n}{C + (n-1)L/n}\end{aligned}$$

Otherwise, i.e., if there exist friendship Nash equilibria with an insecure center player, an upper bound for the WoF can be computed

$$\begin{aligned}\Upsilon(F, I) &= \frac{Cost(S_n, \vec{a}_{NE})}{Cost(S_n, \vec{a}_{FNE})} \\ &\leqslant \frac{(n - \lceil Cn/L - 1\rceil)C + (\lfloor Cn/L\rfloor)^2 L/n}{(n - \lfloor Cn/L - F\rfloor)C + (\lceil Cn/L - 1 - F\rceil)^2 L/n} \\ &\leqslant \frac{(n+1)C}{nC + FC - 2C(1+F) + (1+F)^2 L/n} \\ &< \frac{(n+1)C}{C(n + F - 2(1+F))} < \frac{n+1}{n-3} \quad \square\end{aligned}$$

Theorem 5.9 reveals that caring about the cost incurred by friends is particularly helpful to reach more desirable equilibria. In large star networks, the social welfare can be much higher than in Nash equilibria: in particular, the Windfall of Friendship can increase linearly in n, and hence indeed be asymptotically as large as the Price of Anarchy. However, if $\lfloor Cn/L - F\rfloor - \lfloor \frac{1}{2F}(\sqrt{1 - 4F(1 - Cn/L)} - 1)\rfloor - 2 \geqslant 0$ does not hold, social networks are not much better than purely selfish systems: the maximal gain is constant.

Finally observe that in stars friendship Nash equilibria always exist and can be computed efficiently (in linear time) by any best response strategy.

5.3. Discussion

This section has focused on a small set of very simple topologies only and we regard the derived results as a first step towards more complex graph classes; we will discuss Kleinberg and Facebook networks in more detail in the simulation section (Section 8).

Nevertheless, our findings have some implications for general topologies already. For example, we could show that even in simple graphs such as the star graph, the Windfall of Friendship can assume all possible values, from constant ratios up to ratios linear in n. This is asymptotically maximal for general graphs as well since the Price of Anarchy is bounded by n [2].

Finally, note that we focused on the *worst* equilibria, and the situation looks quite different for the best equilibria. In fact, it is easy to see that in the star network, the center node will always inoculate in the best equilibrium. We do not explore best equilibria further in this article; for some insights on the distribution of "average equilibria", we refer the reader to the simulation section.

6. The relative friendship model

Let us now focus on the relative friendship model RFNE. We will highlight some interesting commonalities and differences between the UFNE and the RFNE model.

First, recall that the *relative perceived individual cost* of a player p_i is defined as

$$c_r(i,\vec{a}) = c_a(i,\vec{a}) + F \cdot \frac{\sum_{p_j \in \Gamma(p_i)} c_a(j,\vec{a})}{|\Gamma(p_i)|}.$$

In the following, we write $c_r^0(i,\vec{a})$ to denote the relative perceived cost of an insecure player p_i and $c_r^1(i,\vec{a})$ for the relative perceived cost of an inoculated player.

While most results for general friendship equilibria still hold, there is a crucial difference. Namely, the phenomenon of a non-monotonic welfare increase with larger F does no longer hold in the star graph S_n. To see this, note that there are only at most two distinct RFNE in S_n (apart from the trivial situations where all players are either insecure or secure): the "good equilibrium" where the center player is secure and all the leave players insecure, and the "bad equilibrium" where the center is insecure and a fraction of the leaves secure. The following theorem shows that the example of Theorem 4.4 for FNE is no longer true for RFNE.

Theorem 6.1. *The Windfall of Friendship is monotonic in F for S_n under the relative cost model.*

Proof. Consider a friendship factor F. Clearly, the equilibrium where only the center player is secure always exists (w.l.o.g., we focus on "reasonable values" C and L). When is there an equilibrium where the center is insecure? Consider such an equilibrium where x leave players are insecure. In order for this to constitute an equilibrium, it must hold for the center player that:

$$\frac{(x+1)L}{n} + \frac{F}{n-1} \cdot \frac{(x+1)L}{n} + \frac{F \cdot C \cdot (n-x-1)}{n-1}$$
$$< C + \frac{F}{n-1} \cdot \frac{x \cdot L}{n} + \frac{F \cdot C \cdot (n-x-1)}{n-1}$$
$$\iff \frac{(x+1)L}{n} + \frac{F}{n-1} \cdot \frac{L}{n} < C$$

On the other hand, for an insecure leaf player we have:

$$\frac{(x+1)L}{n} + \frac{FL(x+1)}{n} < C + \frac{FLx}{n} \iff \frac{(x+1)L}{n} + \frac{FL}{n} < C$$

Unlike in the FNE scenario, the center player is less likely to inoculate, i.e., leaf players inoculate first. Thus, a larger F can only render the existence of such an equilibrium more unlikely. □

Finally, note that the hardness result of Theorem 4.5 is also applicable to relative *FNE*s.

Theorem 6.2. *Computing the best and the worst pure RFNE is $\mathcal{NP}$-complete for any $F \in [0,1]$.*

Proof. (*Sketch*) Again, deciding the existence of a RFNE with cost less than k or more than k is at least as hard as solving the *vertex cover* or *independent dominating set* problem, respectively. Note that verifying whether a proposed solution is correct can be done in polynomial time, hence the problems are indeed in $\mathcal{NP}$. The proof is similar to Theorem 4.5, and we only point out the difference for Condition (a): an insecure player p_i in the attack component bears the cost $k_i/n \cdot L + F \mid \Gamma_{sec}(p_i) \mid C + \mid \Gamma_{\overline{sec}}(p_i) \mid \cdot (k_i L/n)/\mid \Gamma(p_i) \mid$, and changing its strategy reduces the cost by at least $\Delta_i = k_i L/n + F \mid \Gamma_{\overline{sec}}(p_i) \mid k_i L/(\mid \Gamma(p_i) \mid n) - C - F \mid \Gamma_{\overline{sec}}(p_i) \mid (k_i - 1)L/(\mid \Gamma(p_i) \mid n) = k_i L/n - C + FL \mid \Gamma_{\overline{sec}}(p_i) \mid /(\mid \Gamma(p_i) \mid n)$. By our assumption that $k_i \geqslant 2$, and hence $\mid \Gamma_{\overline{sec}}(p_i) \mid \geqslant 1$, it holds that $\Delta_i > 0$, resulting in p_i becoming secure. □

7. Convergence

According to Lemma 4.2, the social context can only improve the overall welfare of the players, both in the uniform and the relative friendship model. However, there are implications beyond the players' welfare in the equilibria: in social networks, the dynamics of how the equilibria are reached is different.

In [2], Aspnes et al. have shown that best-response behavior quickly leads to some pure Nash equilibrium, from any initial situation. Their potential function argument however relies on a "symmetry" of the players in the sense insecure players in the same attack component have the same cost. This no longer holds in the social context where different players take into account their neighborhood: a player with four insecure neighbors may be more likely to inoculate than a player with just one, secure neighbor. Thus, the distinction between "big" and "small" components used in [2] cannot be applied, as different players require a different threshold.

Nevertheless, convergence can be shown in certain scenarios. For example, the hardness proofs of Lemmas 4.5 and 6.2 imply that equilibria always exist in the corresponding areas of the parameter space, and it is easy to see that the equilibria are also reached by best-response sequences. Similarly, in the star and complete networks, best-response sequences converge in linear time. Linear convergence time also happens in more complex, cyclic graphs. For example, consider the cycle graph C_n where each player is connected to one left and one right neighbor in a circular fashion. To prove best response convergence from arbitrary initial states, we distinguish between an initial phase where certain structural invariants are established, and a second phase where a potential function argument can be applied with respect to the view of only one type of players. We will use the term *round* to refer to a sequence of best-response decisions where each player could update its decision exactly once (round-robin).

Theorem 7.1. *From any initial state in the cycle graph C_n, any round-robin best-response scheme results in an equilibrium after $O(1)$ rounds, both in case of uniform and relative friendship equilibria and for any F.*

Proof. After two rounds where each player is given the chance to make a best response twice (at most $2n$ changes), it holds that an insecure player p_1 which is adjacent to a secure player p_2 cannot become secure: since p_1 preferred to be insecure at some time t, the only reason to become secure again is the event that a player p_3 becomes insecure in p_1's attack component at time $t' > t$; however, since p_1 has a secure neighbor p_2 and hence p_3 can only have more insecure neighbors than p_1, p_3 cannot prefer a larger attack component than p_1, which yields a contradiction to the assumption that p_1 becomes secure while its neighbor p_2 is still secure. Moreover, by the same arguments, there cannot be three consecutive secure players.

Therefore, in the best response rounds after the two initial ones, there are the following cases. Case (A): a secure player having two insecure neighbors becomes insecure; Case (B): a secure player with one secure neighbor becomes insecure; and Case (C): an insecure player with two insecure neighbors becomes secure.

In order to prove convergence, the following potential function Φ is used:

$$\Phi(\vec{a}) = \sum_{A \in \mathcal{S}_{big}(\vec{a})} |A| - \sum_{A \in \mathcal{S}_{small}(\vec{a})} |A|$$

where the attack components A in $\mathcal{S}_{big}$ contain more than $t = nC/(FL) - L/F + 1$ players and the attack components A in $\mathcal{S}_{small}$ contain at most t players in case of uniform friendship equilibria; for relative friendship equilibria we use $t = 2Cn/(FL) - 2L/F + 1$. In other words, the threshold t to distinguish between small and big components is chosen with respect to players having *two insecure neighbors*; in case of uniform *FNE*s:

$$C + F \cdot \frac{L \cdot (t-1)}{n} = \frac{t \cdot L}{n} + 2F \cdot \frac{L \cdot t}{n} \iff \frac{Cn}{FL} - \frac{L}{F} + 1 = t$$

and in case of relative *FNE*s:

$$C + F/2 \cdot \frac{L \cdot (t-1)}{n} = \frac{t \cdot L}{n} + F \cdot \frac{L \cdot t}{n} \iff \frac{2Cn}{FL} - \frac{2L}{F} + 1 = t$$

Note that it holds that $-n \leqslant \Phi(\vec{a}) \leqslant n, \forall \vec{a}$. We now show that Case (A) and (C) reduce $\Phi(\vec{a})$ by at least one unit in each best response. Moreover, Case (B) can increase the potential by at most one. However, since we have shown that Case (B) incurs less than n times, the claim follows by an amortization argument. *Case (A):* In this case, a new insecure player p_1 is added to an attack component in $\mathcal{S}_{small}$. *Case (B):* A new insecure player p_1 is added to an attack component in $\mathcal{S}_{small}$ or to an attack component in $\mathcal{S}_{big}$ (since p_1 is "on the edge" of the attack component, it prefers a larger attack component). *Case (C):* An insecure player is removed from an attack component in $\mathcal{S}_{big}$. $\square$

The proof of Theorem 7.1 can be adapted to show linear convergence in general networks where players have degree at most two. In order to gain deeper insights into the convergence behavior of other graph classes, we conducted several experiments described (among many other findings), in the following section.

8. Simulations

We complement our formal worst-case analysis with a simulation study, and investigate the equilibria and convergence times on larger social networks *on average*. Furthermore we initiate discussions on robustness and fairness aspects.

8.1. Methodology

To study the virus inoculation game in more complex social networks, we generated graphs according to the Kleinberg model [21]. The reason for choosing Kleinberg graphs is that they exhibit the small-world property and are frequently used to model social networks. Concretely, Kleinberg graphs with n^2 nodes are based on a $n \times n$ toroidal lattice; each node u has four local connections, one to each of its neighbors. In addition, one long range connections to some node v where v is chosen randomly according to probability proportional to $d(u,v)^{-\alpha}$, where $d(u,v)$ is the lattice distance between u and its long-range contact v, and where α is the so-called clustering exponent. Previous studies have shown that a clustering exponent of $\alpha = 2.0$ is most appropriate [21]. We repeat all our experiments 100 times on different Kleinberg graphs.

In addition, we also run some experiments on a 252-node connected subgraph of Facebook (obtained from [37]). The Facebook graph is less symmetric than the Kleinberg graph, in the sense that degrees, densities, connectivity, etc. exhibit a higher variance over the graph (see also Fig. 2). In the following, if not stated otherwise, plots are shown for Kleinberg graphs only: their regular structure simplifies the interpretation of the simulation results and hence provides interesting insights into the convergence behavior and the nature of the equilibria. In particular, as we will see, despite the symmetric topology, the variance of the size of attack components is high and the cost can be distributed unfairly among players.

8.2. Equilibria and convergence

A first takeaway from our simulations is that in the thousands of experiments we conducted, we did not encounter a single instance which did not converge; however, a formal proof for (or against) the existence of equilibria remains an open research question. Moreover, our

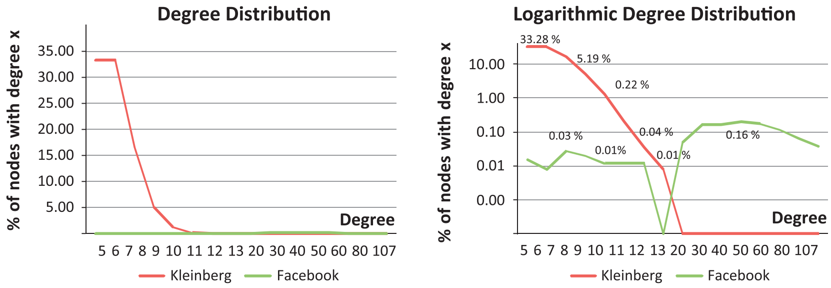


Fig. 2. Degree distribution of Kleinberg graphs with 250 nodes and of Facebook subgraph with 252 nodes.

experiments indicate that the initial configuration (i.e., the set of secure and insecure players) as well as the relationship of L to C typically has a negligible effect on the convergence time, and hence, unless stated otherwise, the following experiments assume an initially completely insecure network and $C = 1$ and $L = 4$.

8.3. Average Windfall of Friendship

All our experiments showed a positive Windfall of Friendship that increases monotonically in F, both for the relative and the uniform friendship model. Fig. 3 shows a typical result for Kleinberg graphs. Maybe surprisingly, it turns out that the Windfall of Friendship is often not due to a higher fraction of secure players, but rather the fact that the secure players are located at strategically more beneficial locations (see also Fig. 4). We can conclude that there is a Windfall of Friendship not only for the worst but also for "average equilibria".

The boxplots in Fig. 5 give a more detailed picture of the cost for $F \in \{0, 1\}$. The overall cost of pure NE is typically higher than the cost of RFNE which is in turn higher than the cost of UFNE.

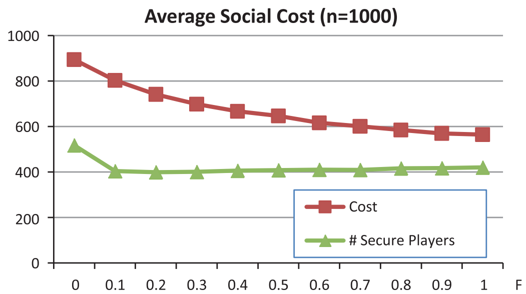


Fig. 3. Average social cost and average number of secure players as a function of F, in the FNE resulting from round-robin best response sequences starting from an initially completely insecure Kleinberg graph. Note that a FNE with $F = 0$ is a *NE*.

8.4. Convergence time

Besides social cost, we are mainly interested in convergence times. In the following, we will consider the convergence time in *best response rounds* until an equilibrium is reached; in one round, each player makes one best response. We find that while the convergence time typically increases already for a small $F > 0$, the magnitude of F plays a minor role. Fig. 6 shows the typical convergence times in best response rounds as a function of F on Kleinberg graphs. Notice that the convergence time more than doubles when changing from the selfish to the social model but is roughly constant for all values of F.

The boxplots in Fig. 7 provide a more thorough characterization of the convergence times, for different network sizes and using one hundred repetitions each (i.e., random order of best-response sequence). The plots show that

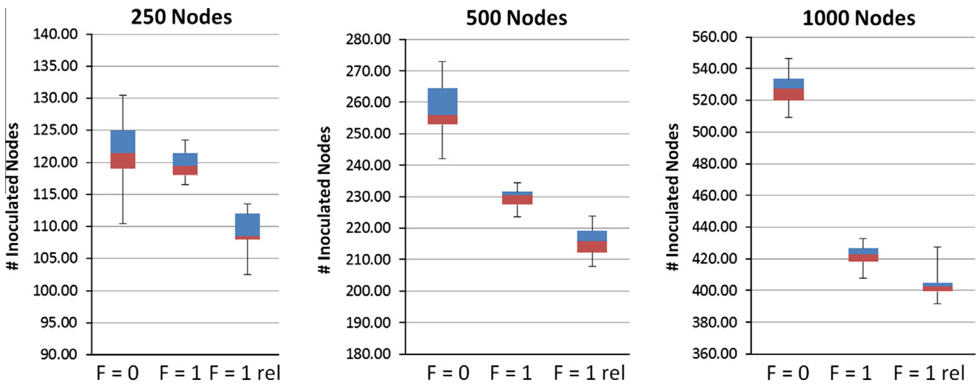


Fig. 4. Number of secure players in different models using $L = 16$: friendship does not imply more secure players, but better locations (blue represents the results between the median and the 75-percentile, red the results between the median and the 25-percentile). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

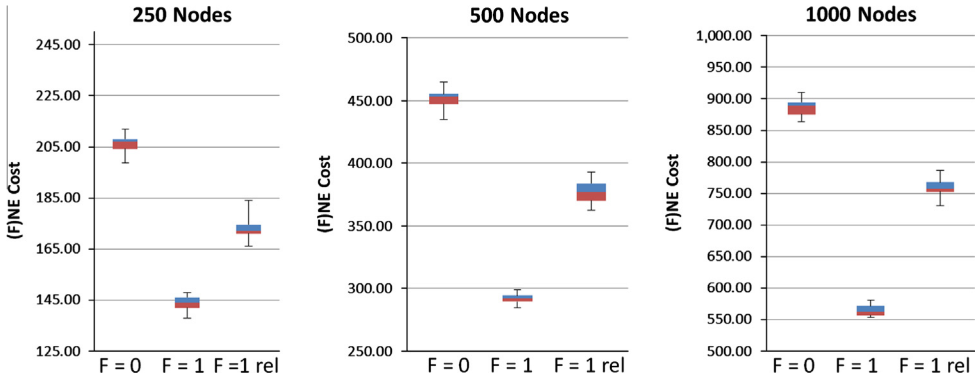


Fig. 5. Boxplots of social cost in different scenarios. The considered equilibria resulted from round-robin best response sequences starting from an initially completely insecure graph (blue represents the results between the median and the 75-percentile, red the results between the median and the 25-percentile). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

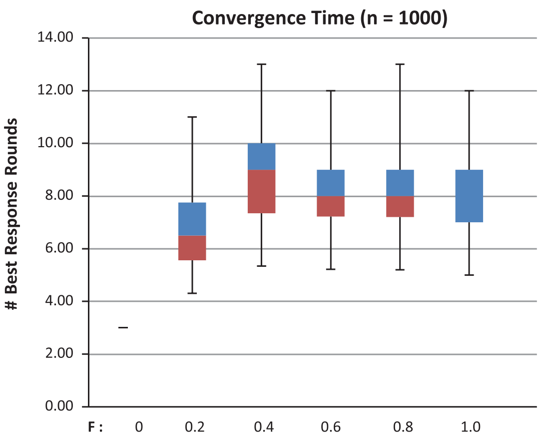


Fig. 6. Boxplot of number of best response rounds until convergence to FNE with $L = 4$, starting from an initially completely insecure graph (blue represents the results between the median and the 75-percentile, red the results between the median and the 25-percentile). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

runtimes in social settings exhibit quite a high variance, and that the convergence times to uniform FNEs are usually higher than the convergence times to relative FNEs; this may be due to larger dependency of the players' interactions.

Moreover we found that while the average cost per node is more or less independent of the starting scenario, the largest convergence time until an equilibrium is reached occurs when no nodes are inoculated in the beginning. For example, to find relative equilibria takes 1.9 rounds longer on average when no nodes are inoculated than when nodes start being inoculated with probability 0.5; this corresponds to an increase of more than 30% (see Fig. 8 for more details).

8.5. Robustness

As already observed in Fig. 4, the number of secure players does not increase for larger F; rather, a smaller number of secure players are located a more strategic locations in the graph. This naturally raises the questions: What happens if a secure player fails to stop the virus propagation? Are friendship equilibria less robust? We conducted experiments studying the consequences of supposedly secure nodes forwarding the virus. More precisely we analyzed the attack component size and the cost before and after the making 50 secure nodes insecure (out of 1000 nodes in the network). On average the difference of the attack component size is rather small, i.e., the best response

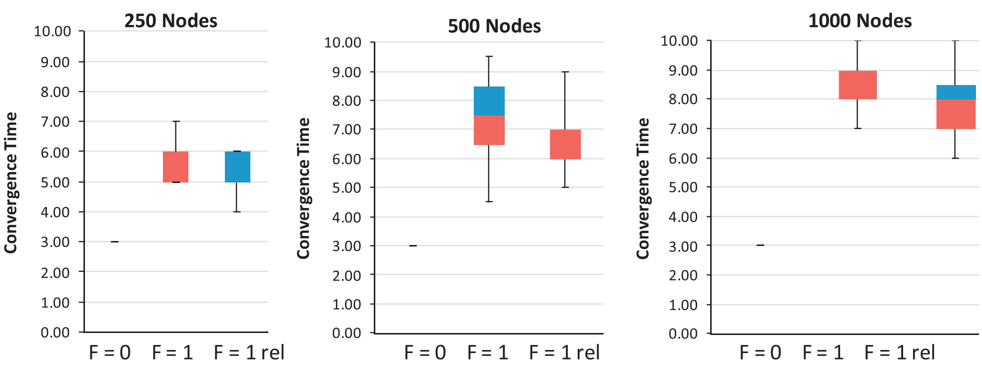


Fig. 7. Boxplot of convergence times for $L = 16$ with round-robin best response sequences from initially completely insecure graph.

dynamic leads to a robust equilibrium (Fig. 9). In contrast, the cost difference is large and grows in F; this indicates that taking neighbor costs into account to a larger extent helps to reach equilibria where players in strategically good locations are inoculated.

8.6. Fairness

Related to the robustness discussion above is the *fairness* issue: how fair is the distribution of the perceived cost among the players? Fig. 10 shows a histogram for the number of players occurring a certain actual cost in the Kleinberg graph. (Recall from Fig. 2 that the Kleinberg network is otherwise quite regular.) Note that while in the considered setting, the actual costs cannot exceed a value of one unit, independently of F, the average perceived cost increases with F. In other words, a larger F will increase the fraction of players with a high perceived cost: the distribution becomes more unfair. Thus, fairness may constitute another price in the social setting.

Fig. 10 shows a histogram of the actual costs incurred by different players as a function of F. (Note the logarithmic y-axis in the second plot of Fig. 10.) We observe that a larger F implies a higher perceived cost for some of the nodes. The *Gini Coefficient*, measuring the inequality among the players costs (0 expresses perfect equality), varies between 7% and 29% for these scenarios, with the perceived cost being distributed in a fairer way than the actual cost.

Cost / # nodes	F=0	F=0.5	F=1.0	rel F=0.5	rel F= 1.0
random inoculation					
125	0.92	0.66	0.58	0.81	0.78
250	0.82	0.61	0.58	0.78	0.72
500	0.91	0.69	0.60	0.82	0.78
1000	0.89	0.68	0.60	0.80	0.80
Social cost / # nodes	**0.89**	**0.66**	**0.59**	**0.80**	**0.77**
no inoculation					
125	0.91	0.67	0.58	0.82	0.78
250	0.83	0.64	0.55	0.78	0.73
500	0.92	0.69	0.58	0.83	0.79
1000	0.86	0.69	0.60	0.82	0.79
Social cost / # nodes	**0.88**	**0.67**	**0.58**	**0.81**	**0.77**

Rounds	F=0	F=0.5	F=1.0	rel F=0.5	rel F= 1.0
random inoculation					
125	3.00	4.80	5.20	5.00	4.00
250	3.00	5.40	5.20	4.60	4.60
500	3.00	5.40	6.60	6.20	5.40
1000	3.00	8.60	7.40	7.40	7.20
Avg # rounds	**3.00**	**6.05**	**6.10**	**5.80**	**5.30**
no inoculation					
125	3.00	5.60	5.60	6.00	5.40
250	3.00	5.40	5.60	6.00	7.40
500	3.00	6.60	7.80	9.20	7.20
1000	3.00	8.00	8.20	9.60	8.20
Avg # rounds	**3.00**	**6.40**	**6.80**	**7.70**	**7.05**

Fig. 8. Cost and number best response rounds until convergence for starting scenarios without inoculation compared to starting scenarios where the probability to be inoculated is 0.5. The cost per node of the found equilibria depends only on F but the convergence complexity is higher if no nodes are inoculated.

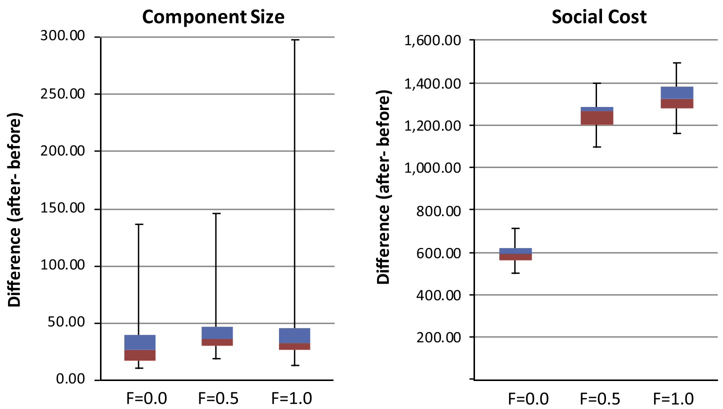
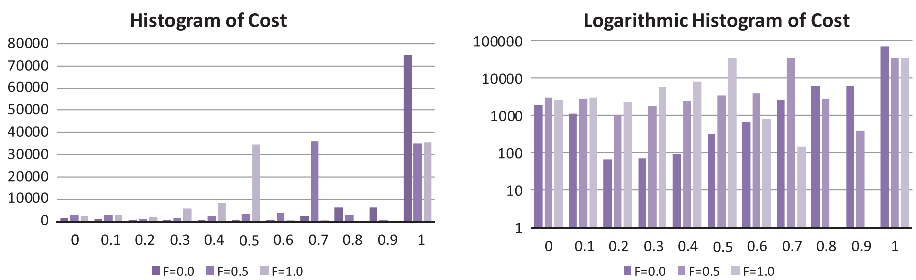


Fig. 9. Differences of average attack component size (*left*) and cost (*right*) when making 50 supposedly secure nodes insecure. While the attack component sizes are not affected by much (although the difference can reach 260 nodes), the cost rises significantly for larger F.



Gini Coefficient	
AC F=0.0	0.07
AC F=0.5	0.21
AC F=1.0	0.29
PC F=0.5	0.11
PC F=1.0	0.14

Fig. 10. The fairness price.

At the same time a higher F results in a higher Gini Coefficient in both actual and perceived cost, i.e., fairness descreases with growing F.

8.7. Impact of social network topology

Finally, we also ran experiments with alternative social network graphs, namely Facebook networks and Kleinberg graphs with smaller and larger clustering exponents α. While many results are qualitatively similar to the ones presented above for the Kleinberg graph with $\alpha = 2$, there are also difference. The main insights from the simulations on alternative topologies can be summarized as follows: (1) The Windfall of Friendship is always positive. (2) The social costs do not depend much on α, i.e., our results are relatively robust to the social network topology. (3) In contrast, the convergence time does depend on the clustering exponent. In particular, our experiments show that a larger α yields lower convergence times. For example, $\alpha = 3.0$ converges up to 50% faster than $\alpha = 1.0$. (3) As expected, the higher the clustering exponent, the smaller the number of secure nodes in the equilibrium.

9. Conclusion

This article presented a framework to study and quantify the effects of game-theoretic behavior in social networks. For example, this framework allows us to formally describe phenomena which are known on an anecdotal level only. For instance, we find that the Windfall of Friendship is always positive, and that players embedded in a social context may be subject to longer convergence times. Moreover, interestingly, we find that the Windfall of Friendship does not always increase monotonically with stronger social ties.

We believe that our work opens interesting directions for future research. We have focused on a virus inoculation game, and additional insights must be gained by studying alternative and more general games such as potential games, or games that do and do not exhibit a Braeßparadox. Also alternative equilibria (e.g., the *best* equilibria and the Price of Stability) as well as the implications on the games' dynamics need to be investigated in more detail (e.g., [39]). It may also be interesting to study scenarios where players care not only about their friends but also, to a smaller extent, about friends of friends. Finally, in our model, we so far assumed that the social graph and the virus propagation graph coincide; in many Internet networks, the two graphs may differ, and the Windfall of Friendship is likely to be smaller.

What about practical implications? One intuitive takeaway of our work is that in case of large benefits of social behavior, it may make sense to design distributed systems where neighboring players have good relationships. However, if the resulting convergence times are large and the price of the dynamics higher than the possible gains, such connections should be discouraged; moreover, a more social environment may lead to less robust configurations if secure players fail. Our game-theoretic tools can be used to compute these benefits and convergence times, and may hence be helpful during the design phase of such a system.

Acknowledgments

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Appendix A. Differences to EC 2008 Paper

A preliminary version of this work appeared at the ACM EC 2008 conference [28]. The article at hand extends the conference paper in the following respects.

1. *Generalized friendship:* We generalize our results to settings where $F_{i,j}$ can differ from neighbor to neighbor. In addition to the case study of uniform friendship, we introduce a model where the relative importance of a neighbor decreases with the total number of friends (new Section 6). The relative friendship model offers some interesting insights: While friendship is still always beneficial, we show that the non-monotonicity result no longer applies: unlike in the uniform friendship model, the Windfall of Friendship can only increase with stronger social ties. We also show that the best and worst relative friendship equilibrium is still $\mathcal{NP}$-hard.
2. *Convergence*: We initiate the study of convergence issues (Section 7) in social networks. It turns out that it takes longer until an equilibrium is reached compared to purely selfish environments and hence constitutes a price of friendship. We present a potential function argument to prove convergence of best-response sequences in some simple cyclic networks, and perform a simulation study on Kleinberg and Facebook graphs.
3. *Simulations (new Section 8)*: We report on insights gained on alternative equilibria reached by best-response sequences by simulations. We investigate different social networks, and initiate the study of fairness and robustness aspects.
4. We improve the bound in Theorem 4.4: we show that non-monotonicity already holds for $n > 3$ (and not only for $n > 7$ as proved in the EC paper).
5. We updated and extended the related work section.
6. Minor improvements in the presentation.

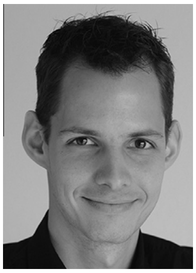
References

[1] J. Aspnes, K. Chang, A. Yampolskiy, Inoculation strategies for victims of viruses and the sum-of-squares partition problem, in: Proc. 16th ACM-SIAM Annual Symposium on Discrete Algorithms (SODA), 2005.

[2] M. Babaioff, R. Kleinberg, C.H. Papadimitriou, Congestion games with malicious players, in: Proc. 8th ACM Conference on Electronic Commerce (EC), 2007. Also Appeared in Games and Economic Behavior (GEB), vol. 67(1), 2007.

[3] N.T. Bailey, The Mathematical Theory of Infectious Diseases and its Applications, Hafner Press, 1975.

[4] J.M. Bilbao, Cooperative Games on Combinatorial Structures, Springer, 2000.
[5] R. Blundell, W. Newey, T. Persson (Eds.), Advances in Economics and Econometrics (Chapter 1: The Economics of Social Networks) (2006).
[6] P.-A. Chen, M. David, D. Kempe, Better vaccination strategies for better people, in: Proc. 11th ACM Conference on Electronic Commerce (EC), 2010.
[7] P.-A. Chen, D. Kempe, Bayesian auctions with friends and foes, in: Proc. 2nd International Symposium on Algorithmic Game Theory (SAGT), 2009.
[8] A. Fabrikant, A. Luthra, E. Maneva, C.H. Papadimitriou, S. Shenker, On a network creation game, in: Proc. 22nd Annual Symposium on Principles of Distributed Computing (PODC), 2003.
[9] J. Feigenbaum, C.H. Papadimitriou, S. Shenker, Sharing the cost of multicast transmissions, J. Comput. Syst. Sci. 63 (1) (2001) 21–41.
[10] G.W. Flake, S. Lawrence, C.L. Giles, F.M. Coetzee, Self-organization and identification of web communities, Computer 35 (3) (2002) 66–71.
[11] C. Fleizach, M. Liljenstam, P. Johansson, G.M. Voelker, A. Mehes, Can you infect me now? Malware propagation in mobile phone networks, in: Proc. 2007 ACM Workshop on Recurring Malcode (WORM), 2007.
[12] P. Fraigniaud, C. Gavoille, C. Paul, Eclecticism shrinks even small worlds, in: Proc. 23rd Annual ACM Symposium on Principles of Distributed Computing (PODC), 2004.
[13] J.C. Frauenthal, Mathematical Modeling in Epidemiology, Springer-Verlag, 1980.
[14] M.R. Garey, D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, W.H. Freeman, 1979.
[15] S.M. Kakade, M. Kearns, L.E. Ortiz, Graphical economics, in: Proc. 17th Annual Conference on Learning Theory (COLT), 2004.
[16] R.M. Karp, Reducibility among combinatorial problems, Complexity Comput. Comput. (1972) 85–103.
[17] M. Kearns, M. Littman, S. Singh, Graphical models for game theory, in: Proc. Conference on Uncertainty in Artificial Intelligence (UAI), 2001.
[18] J.O. Kephart, S.R. White, Measuring and modeling computer virus prevalence, in: Proc. IEEE Symposium on Security and Privacy, 1993.
[19] J.O. Kephart, S.R. White, D.M. Chess, Computers and epidemiology, IEEE Spectrum 30 (5) (1993) 20–26.
[20] J. Kleinberg, The small-world phenomenon: an algorithmic perspective, in: Proc. 32nd ACM Symposium on Theory of Computing (STOC), 2000.
[21] J. Kleinberg, Complex networks and decentralized search algorithms, in: Proc. International Congress of Mathematicians (ICM), 2006.
[22] J. Kleinberg, Algorithmic game theory, in: N. Nisan, T. Roughgarden, E. Tardos, V.V. Vazirani (Eds.), Chapter 24: Cascading Behavior in Networks: Algorithmic and Economic Issues, Cambridge University Press, 2007.
[23] P. Kuznetsov, S. Schmid, Towards network games with social preferences, in: 17th International Colloquium on Structural Information and Communication Complexity (SIROCCO), 2010.
[24] D. Liben-Nowell, J. Novak, R. Kumar, P. Raghavan, A. Tomkins, Geographic routing in social networks, Proc. National Academy of Sciences, vol. 33, National Acad Sciences, 2005, pp. 11623–11628.
[25] D. Meier, Y.A. Oswald, S. Schmid, R. Wattenhofer, On the windfall of friendship: inoculation strategies on social networks, in: Proc. 9th ACM Conference on Electronic Commerce (EC), 2008.
[26] S. Milgram, The small world problem, Psychol. Today (1967) 60–67.
[27] A. Montanari, A. Saberi, Convergence to equilibrium in local interaction games, SIGecom Exch. (2009) 60–67.
[28] T. Moscibroda, S. Schmid, R. Wattenhofer, On the topologies formed by selfish peers, in: Proc. 25th Annual ACM Symposium on Principles of Distributed Computing (PODC), 2006.
[29] T. Moscibroda, S. Schmid, R. Wattenhofer, When selfish meets evil: byzantine players in a virus inoculation game, in: Proc. 25th Annual ACM Symposium on Principles of Distributed Computing (PODC), 2006. Also Appeared in Journal Internet Mathematics (IM), vol. 6(2), 2009.
[30] M.E.J. Newman, M. Girvan, Finding and evaluating community structure in networks, Phys. Rev. E 69 (2004).
[31] Ch. Papadimitriou, Algorithms, games, and the internet, in: Proc. 33rd Annual ACM Symposium on Theory of Computing (STOC), 2001.
[32] T. Roughgarden, Selfish Routing and the Price of Anarchy, MIT Press, 2005.
[33] T. Roughgarden, Éva Tardos, How bad is selfish routing?, Journals of the ACM 49 (2) (2002) 236–259
[34] B. Viswanath, A. Mislove, M. Cha, K. Gummadi, On the evolution of user interaction in facebook, in: Proc. 2nd ACM Workshop on Online Social Networks (WOSN), 2009.
[35] C. Wang, J.C. Knight, M.C. Elder, On computer viral infection and the effect of immunization, in: Proc. 16th Annual Computer Security Applications Conference (ACSAC), 2000.
[36] J. Wright, K. Leyton-Brown, Beyond equilibrium: predicting human behavior in normal-form games, in: Proc. 24th AAAI Conference on Artificial Intelligence (AAAI), 2010.



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