Randomness vs. Time in Anonymous Networks

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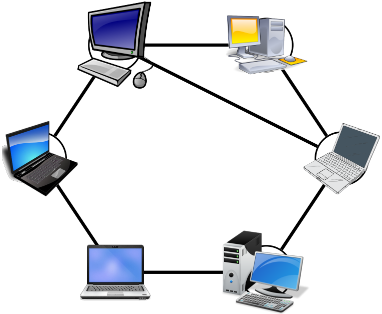
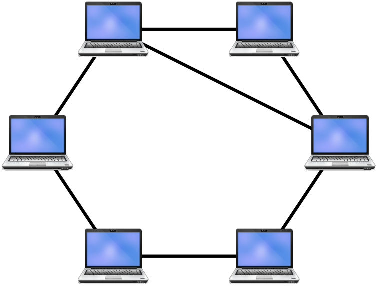
Anonymous Networks



Anonymous Networks



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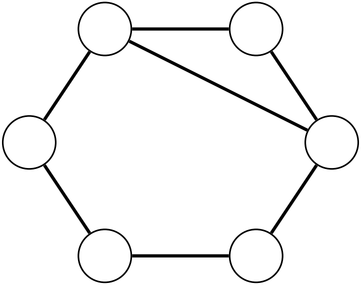

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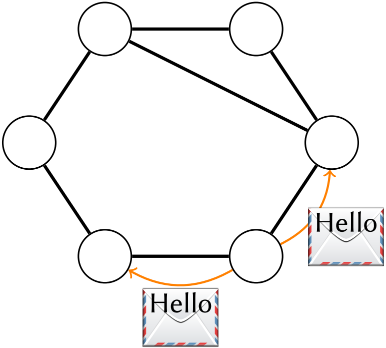
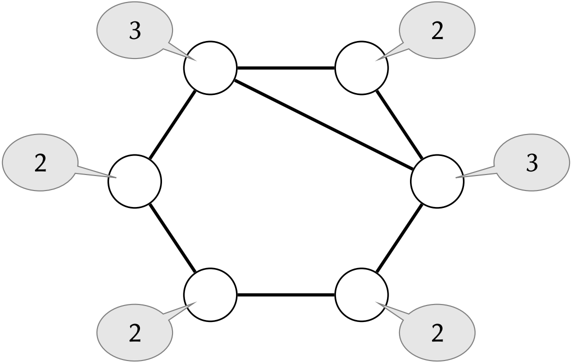
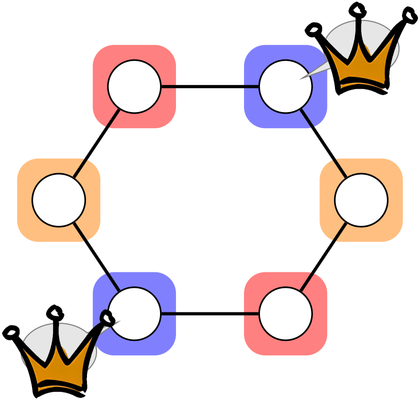
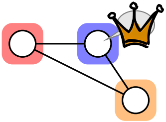


Anonymous Networks

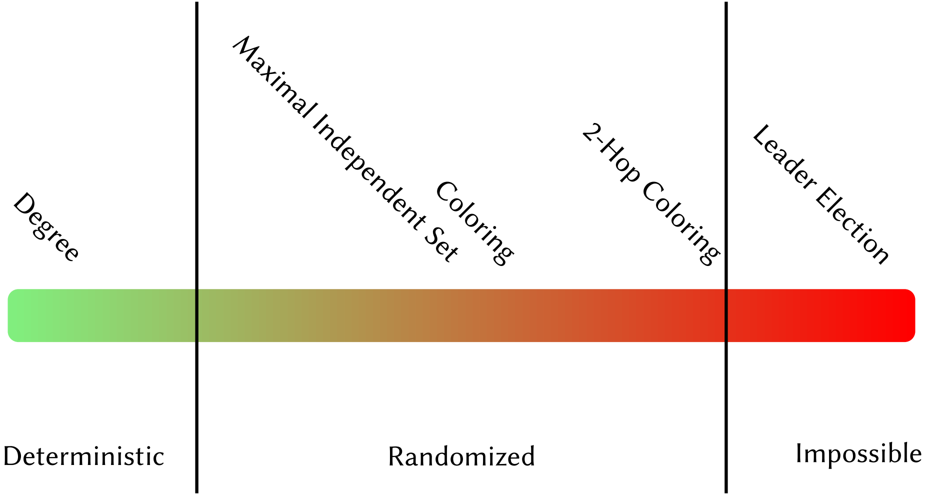


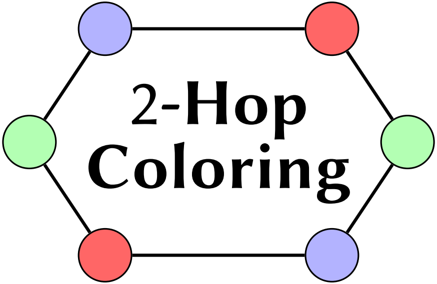
Leader Election



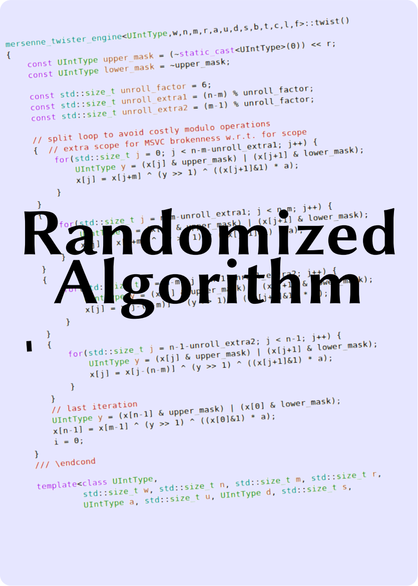


Computability

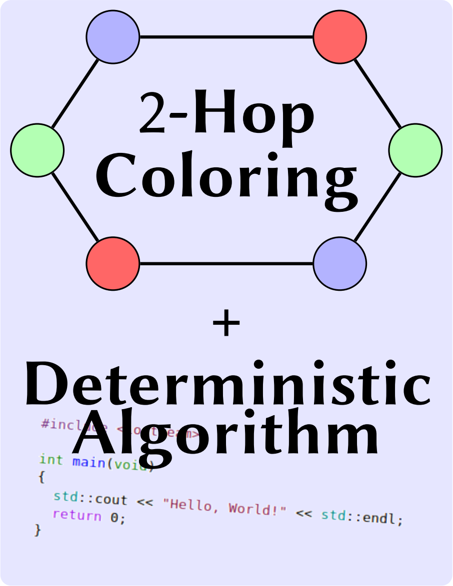



2-Hop
Coloring

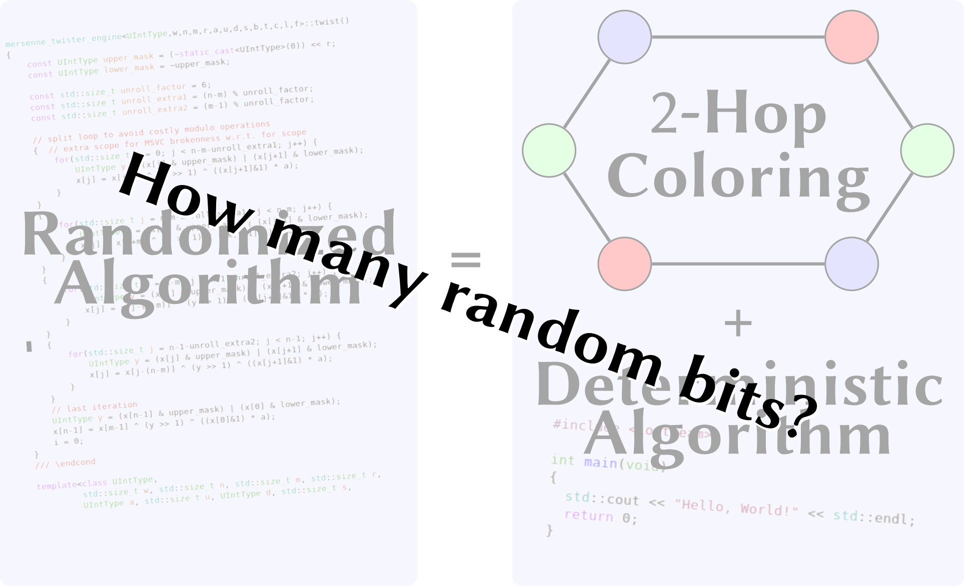
Theorem [PODC 2014]



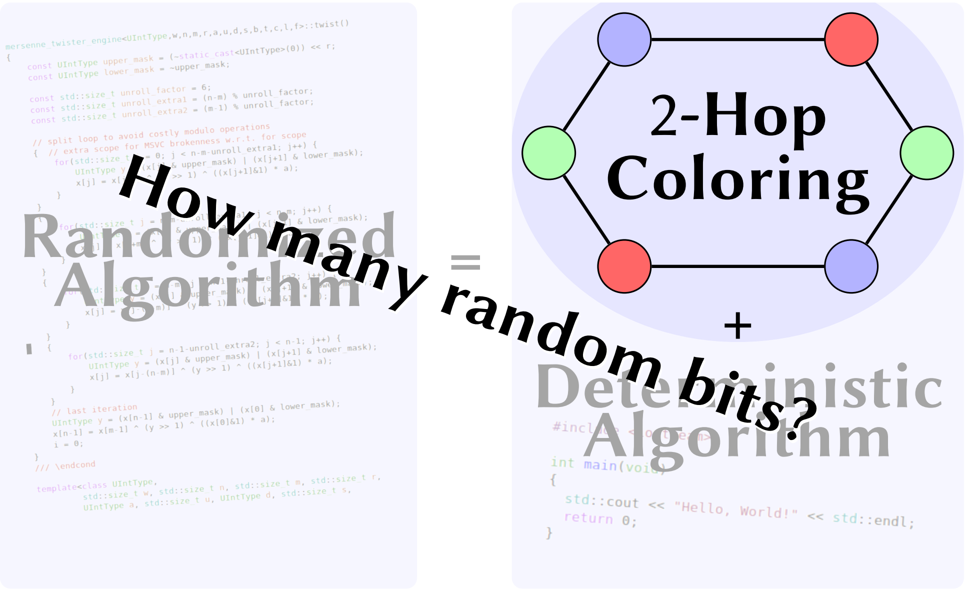
=



Theorem [PODC 2014]

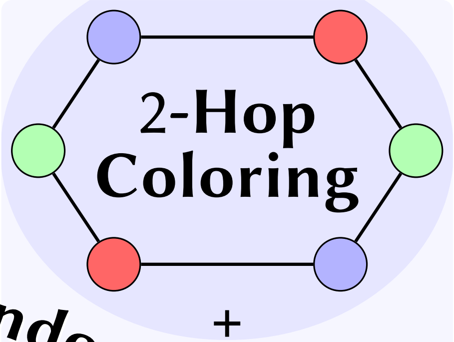


Theorem [PODC 2014]



Theorem [PODC 2014]

Randomized Algorithm =

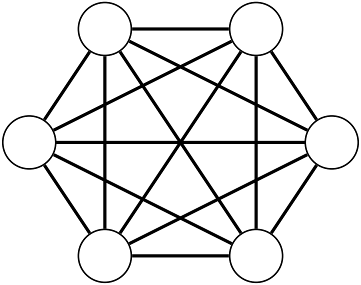


+ Deterministic Algorithm

How many random bits?

How fast?

How Many Random Bits?



$\log n$

How Many Random Bits?

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$\log n$

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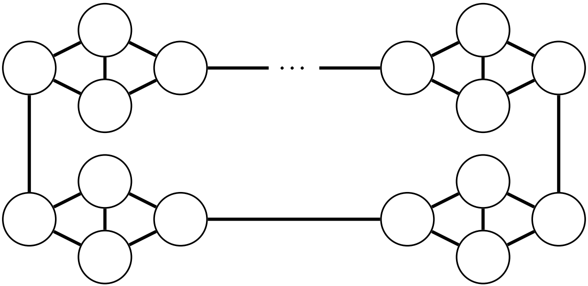
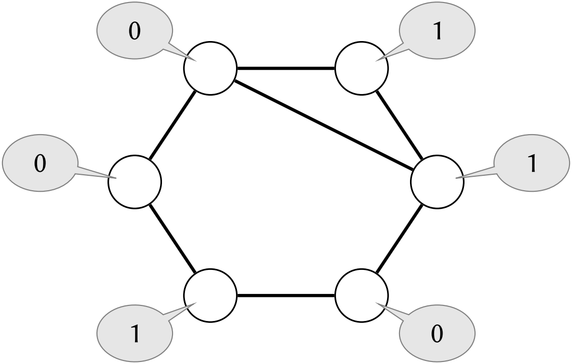


$\log n$

How Fast?



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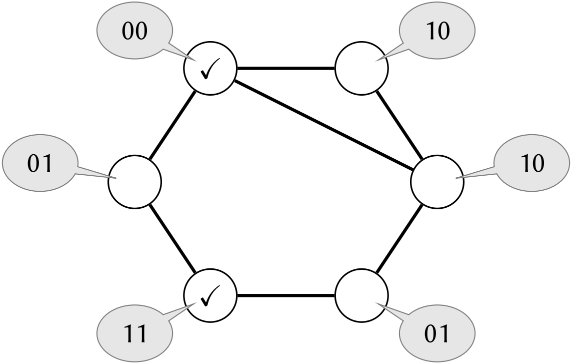
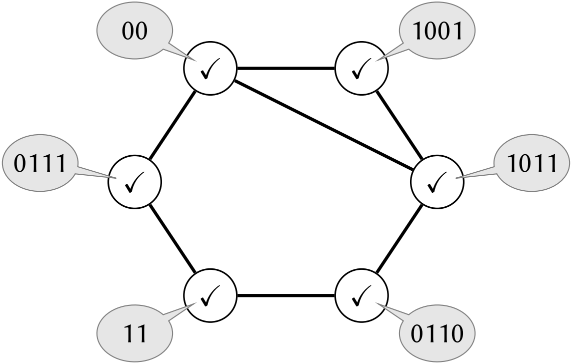
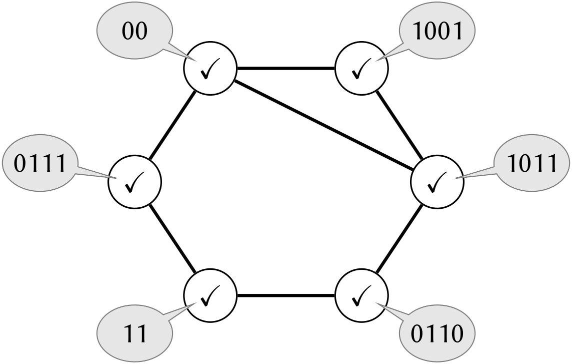


How Fast?

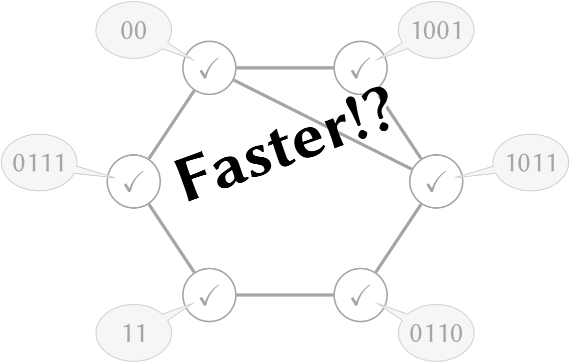


How Fast?

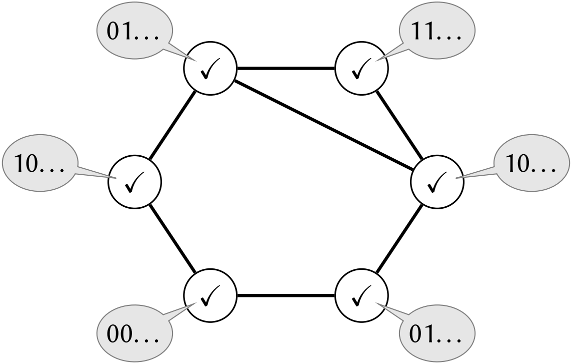


log log n rounds, log n bits
(w.h.p. & in expectation)

How Fast?

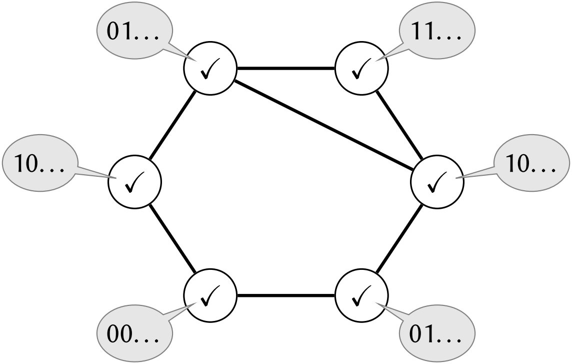


How Fast?



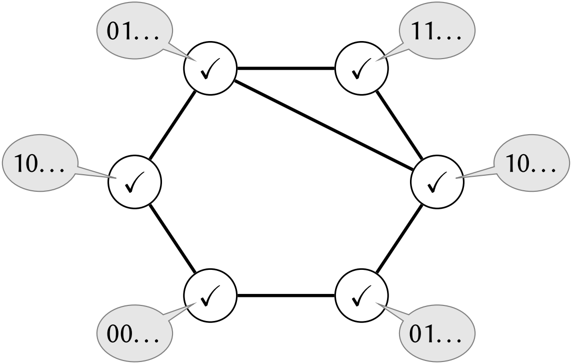
∞ random bits $\Rightarrow$ 1 round

How Fast?



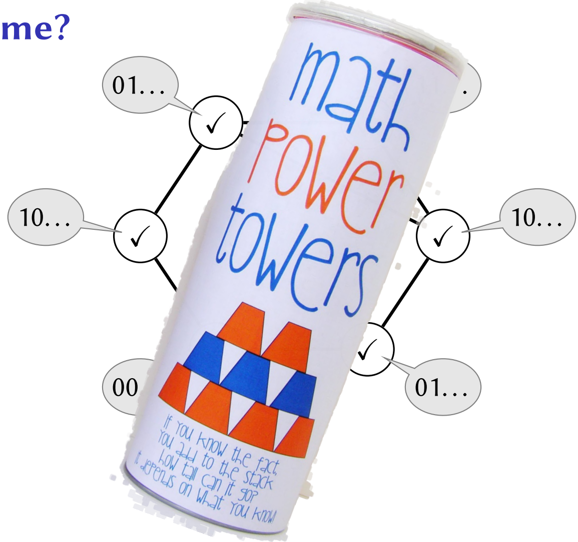
$< \log^* n$ rounds $\Rightarrow$  $\Rightarrow \infty$ random bits

$\log^*$ Runtime?



Idea: Use the inverse of $\log^*$

$\log^*$ **Runtime?**



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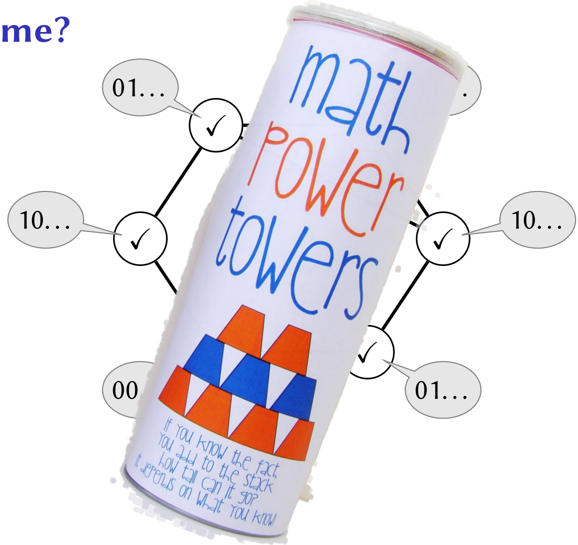
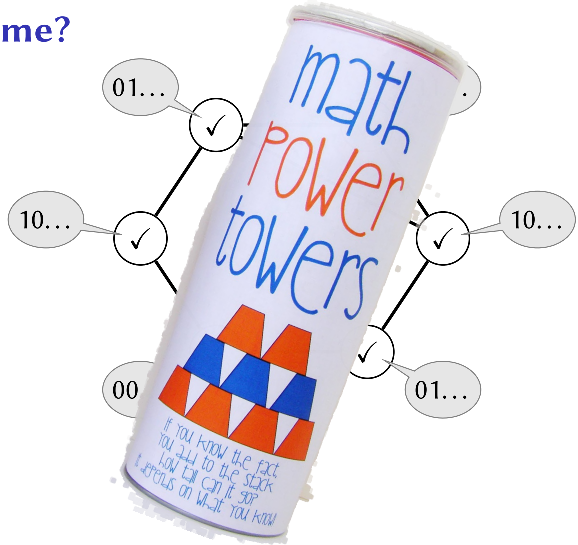


use $\log 2^{\cdot^{\cdot^{\cdot^{2}}}}$ bits upto round $i \Rightarrow \log^* n$ rounds, n bits

$\log^*$ Runtime?



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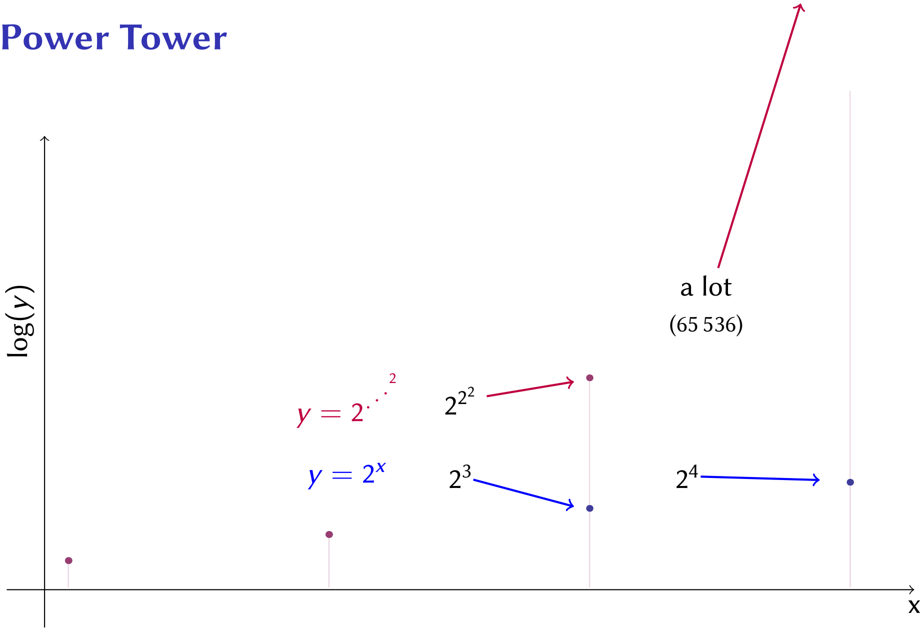
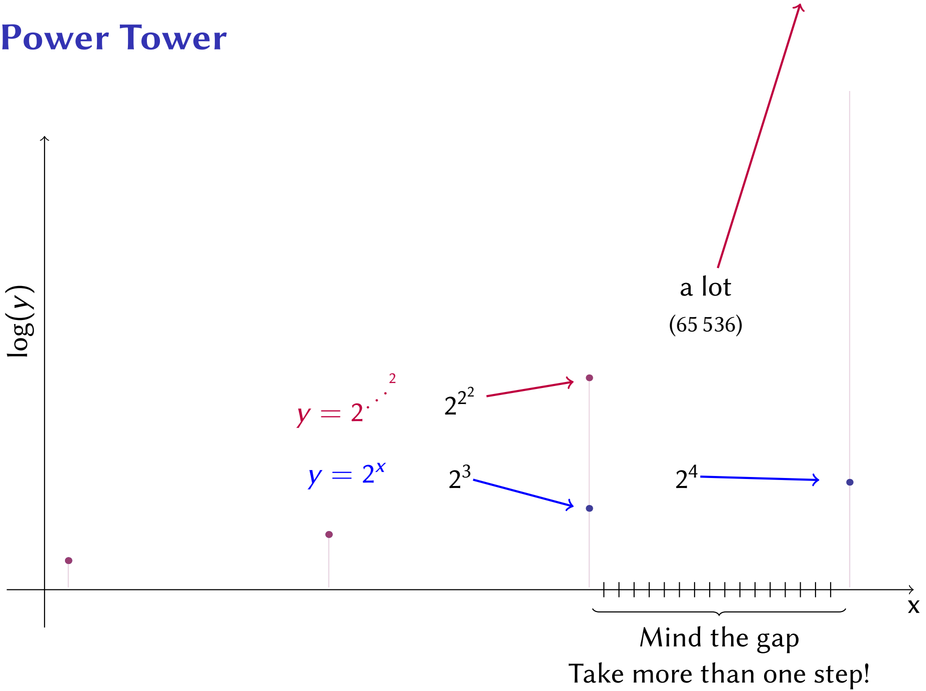
Lower bound?

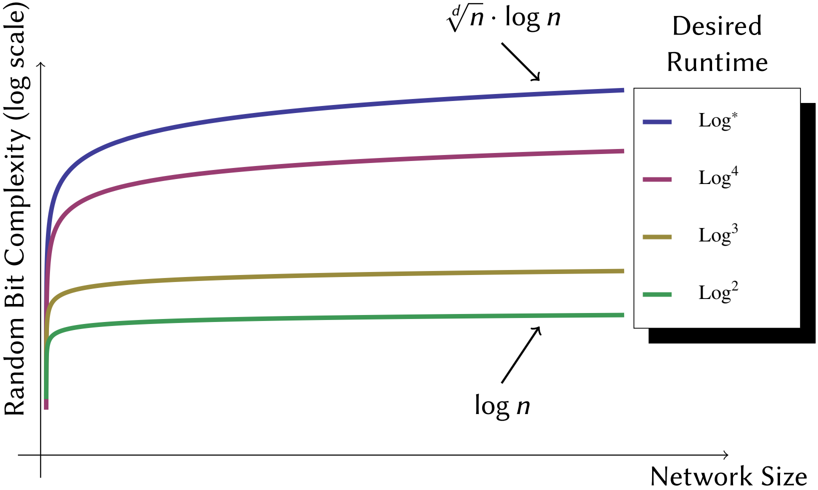
Power Tower



Power Tower


a lot
(65 536)
$y = 2^{\cdot^{\cdot^{\cdot^{2}}}}$
2^{2^2}
$y = 2^x$
2^3
2^4
$\log(y)$
x
Mind the gap
Take more than one step!

Time vs. Random Bit Complexity



Time vs. Random Bit Complexity

Theorem

Anonymous networks can be 2-hop colored in $O(d \cdot f(n))$ rounds.[1]

The achieved random bit complexity is

$$O\left(\sqrt[d]{\frac{\lceil \log f^{-1}(f(n)+1) \rceil}{\lceil \log f^{-1}(f(n)) \rceil}} \cdot \log n\right) .$$

[1] With high probability and in expectation. For reasonable desired runtime f, i.e., between $\log^*$ and $\log\log$.

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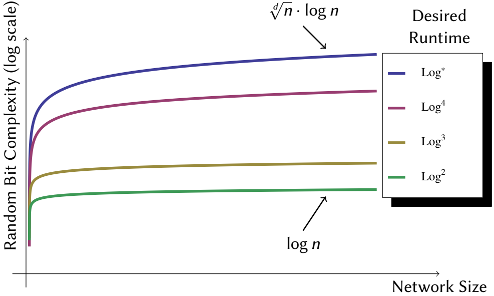
Theorem

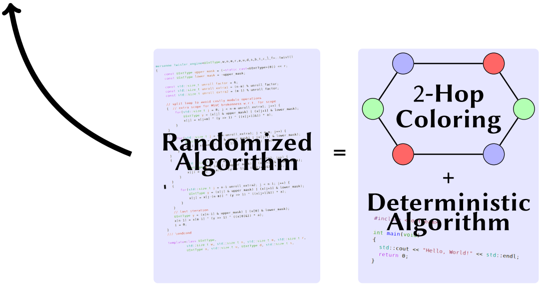
This is asymptotically optimal.

[1]With high probability and in expectation. For reasonable desired runtime f, i.e., between $\log^*$ and $\log\log$.

Recap

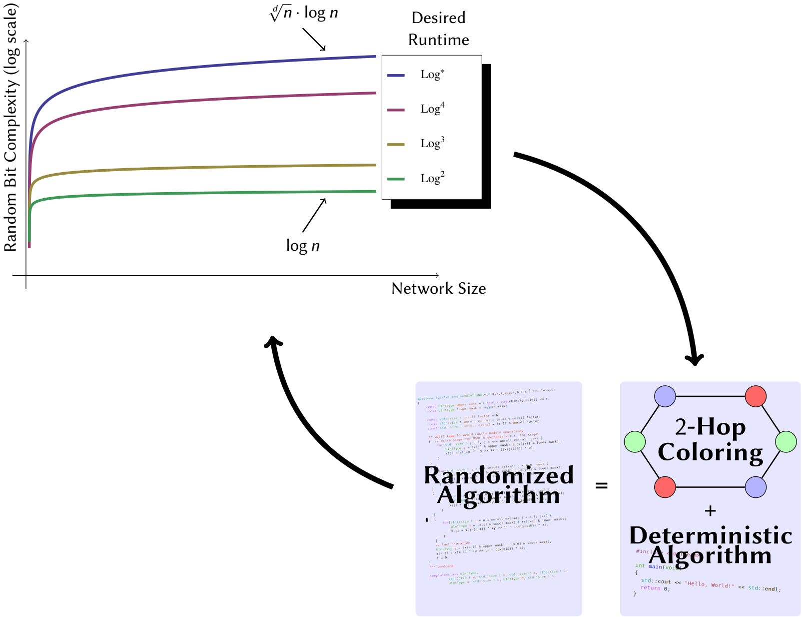


Random Bit Complexity (log scale)
$\sqrt[d]{n} \cdot \log n$
Desired Runtime
Log*
Log⁴
Log³
Log²
$\log n$
Network Size



Randomized Algorithm
=
2-Hop Coloring
+
Deterministic Algorithm

Recap



$\sqrt[d]{n} \cdot \log n$
Desired Runtime
Log*
Log4
Log3
Log2
log n
Random Bit Complexity (log scale)
Network Size
Randomized Algorithm
=
2-Hop Coloring
+
Deterministic Algorithm

Thank You