On Local Fixing

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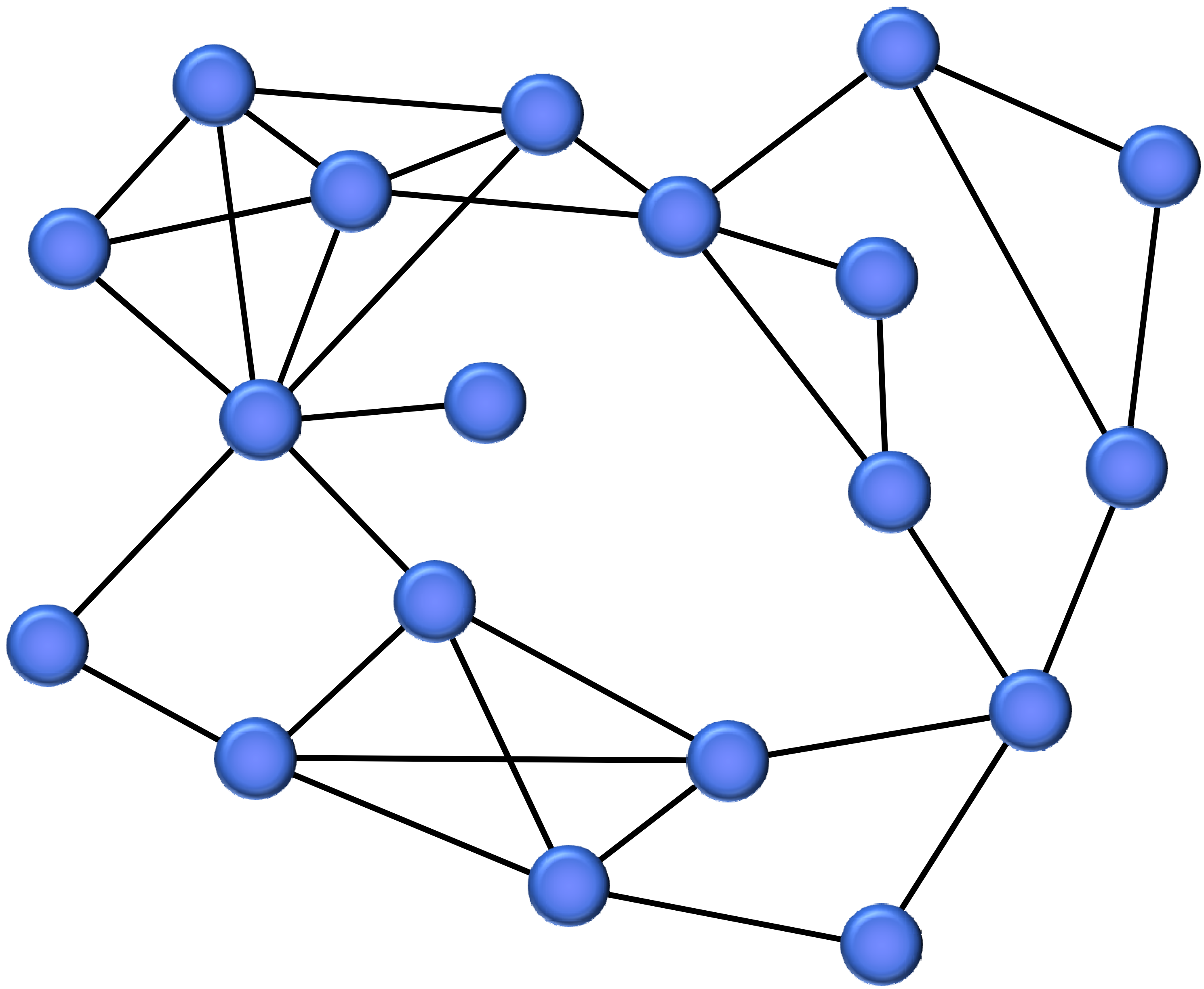
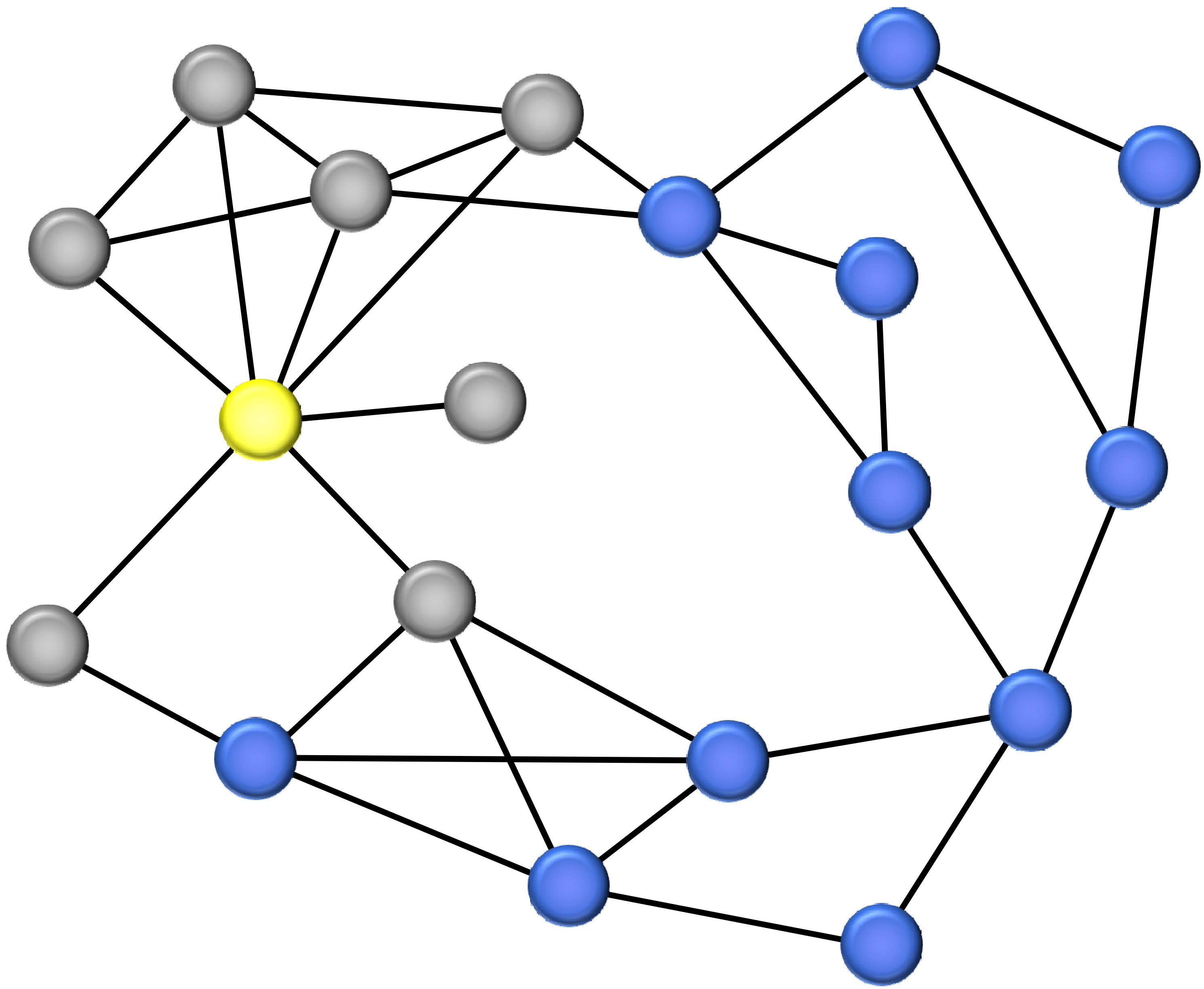
Motivation

Motivation

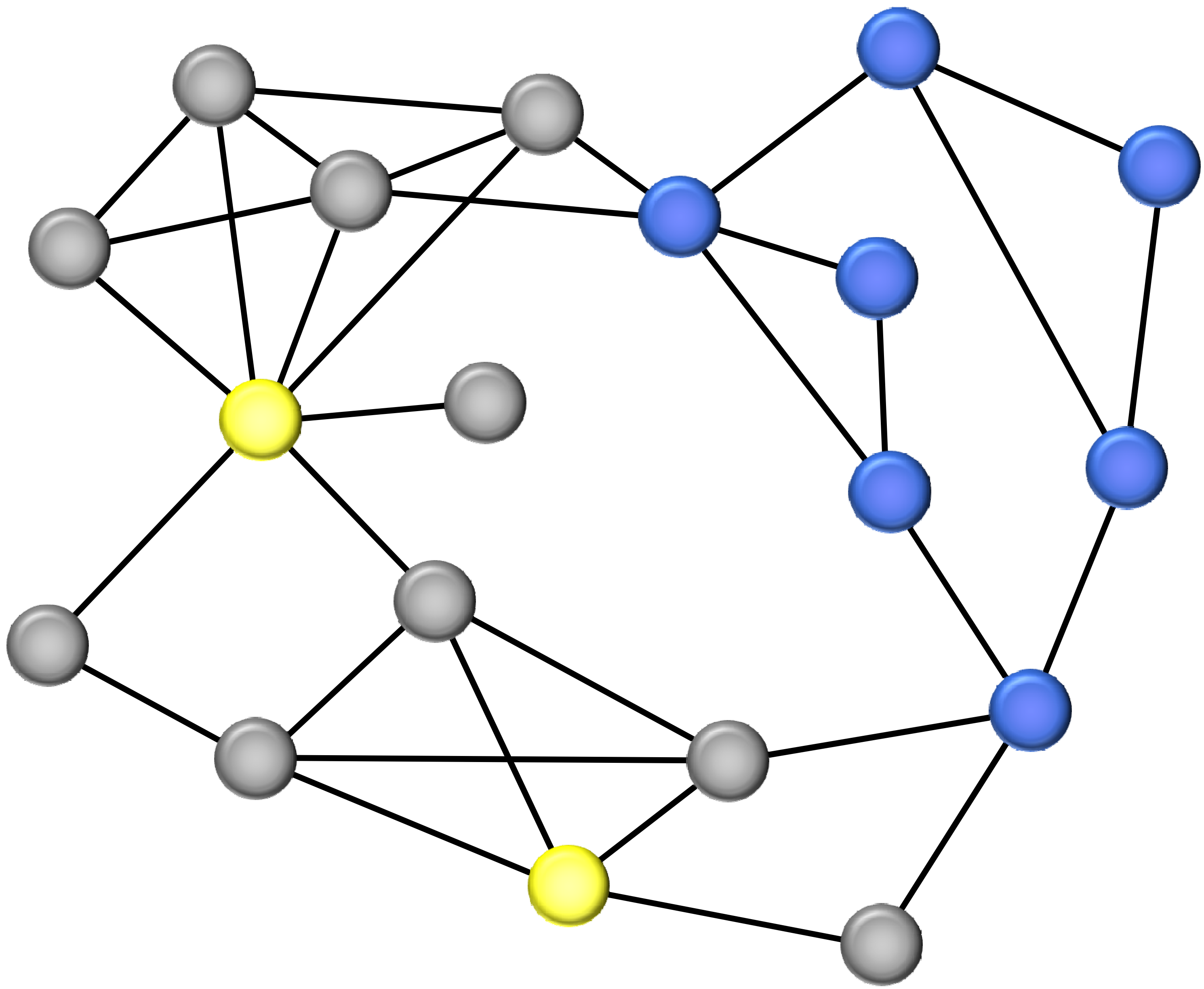
Motivation by Example



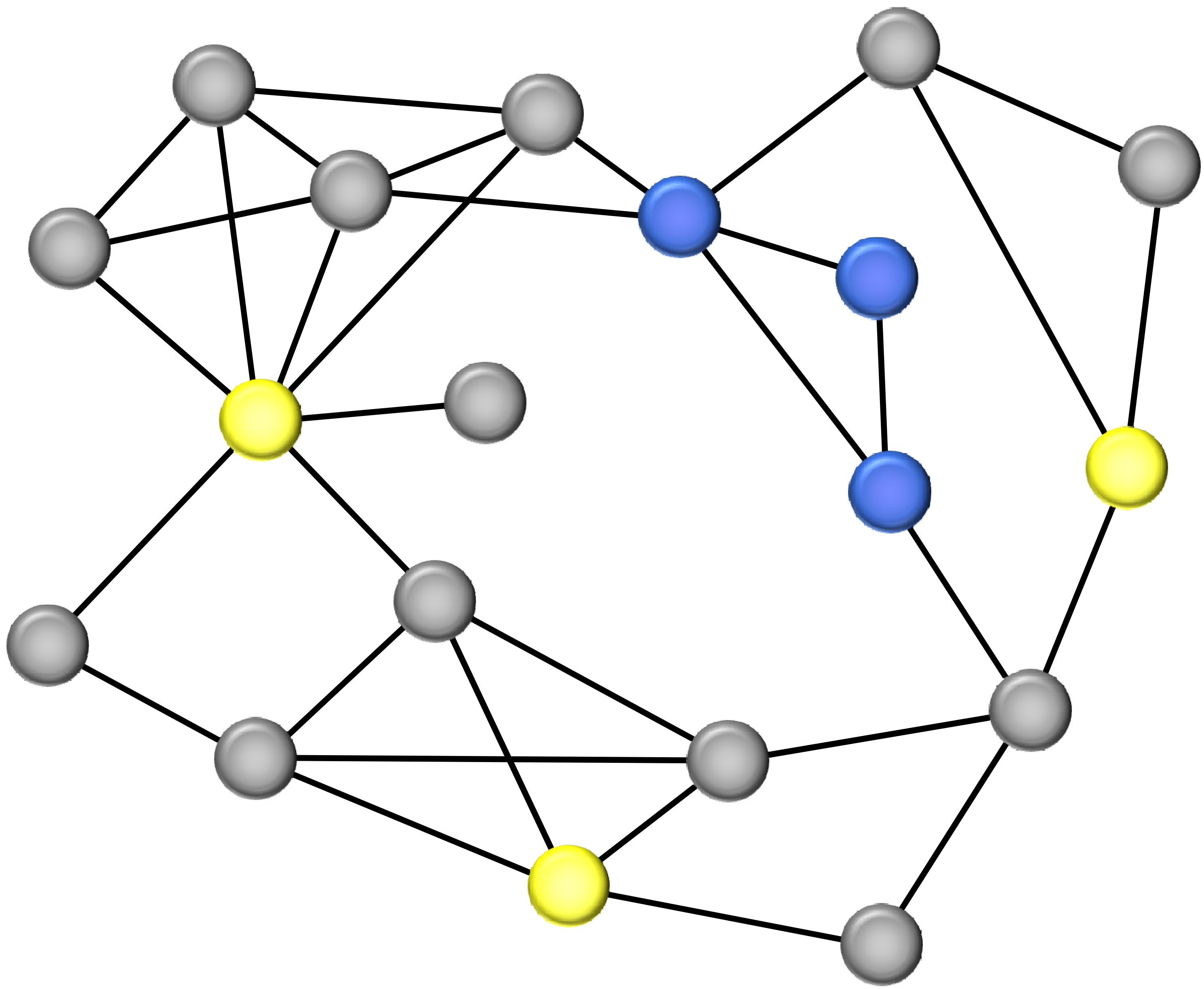
Motivation by Example



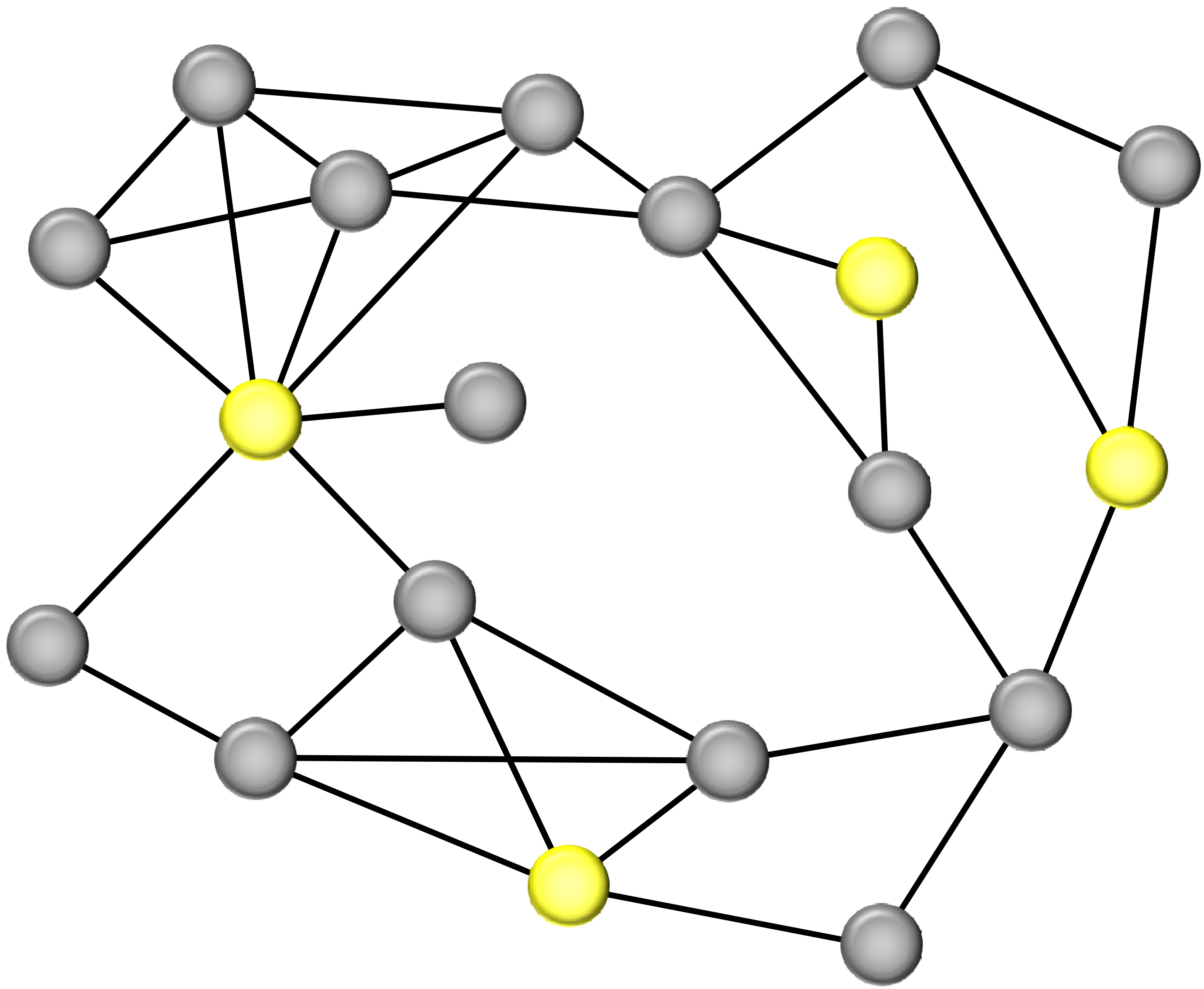
Motivation by Example



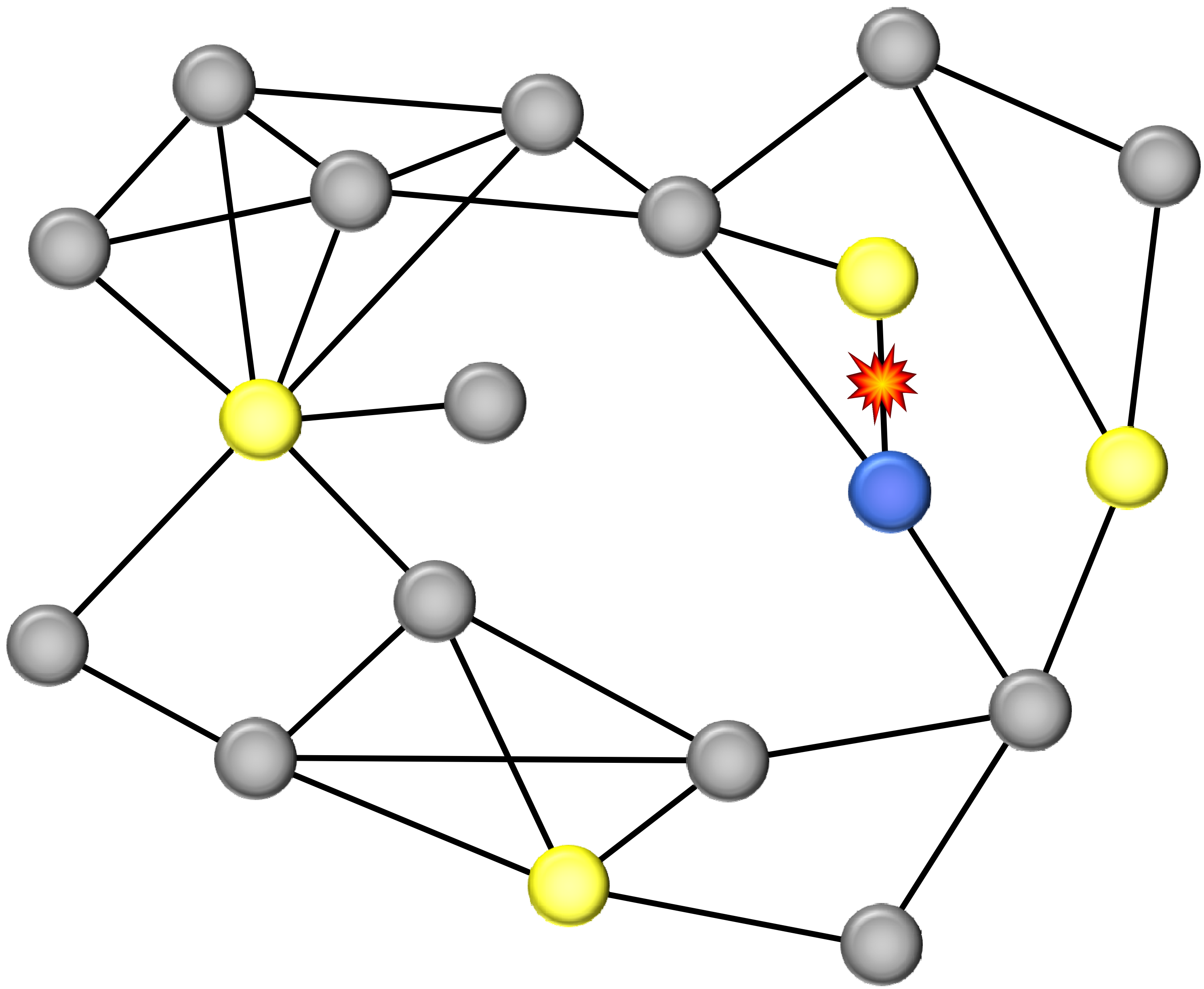
Motivation by Example



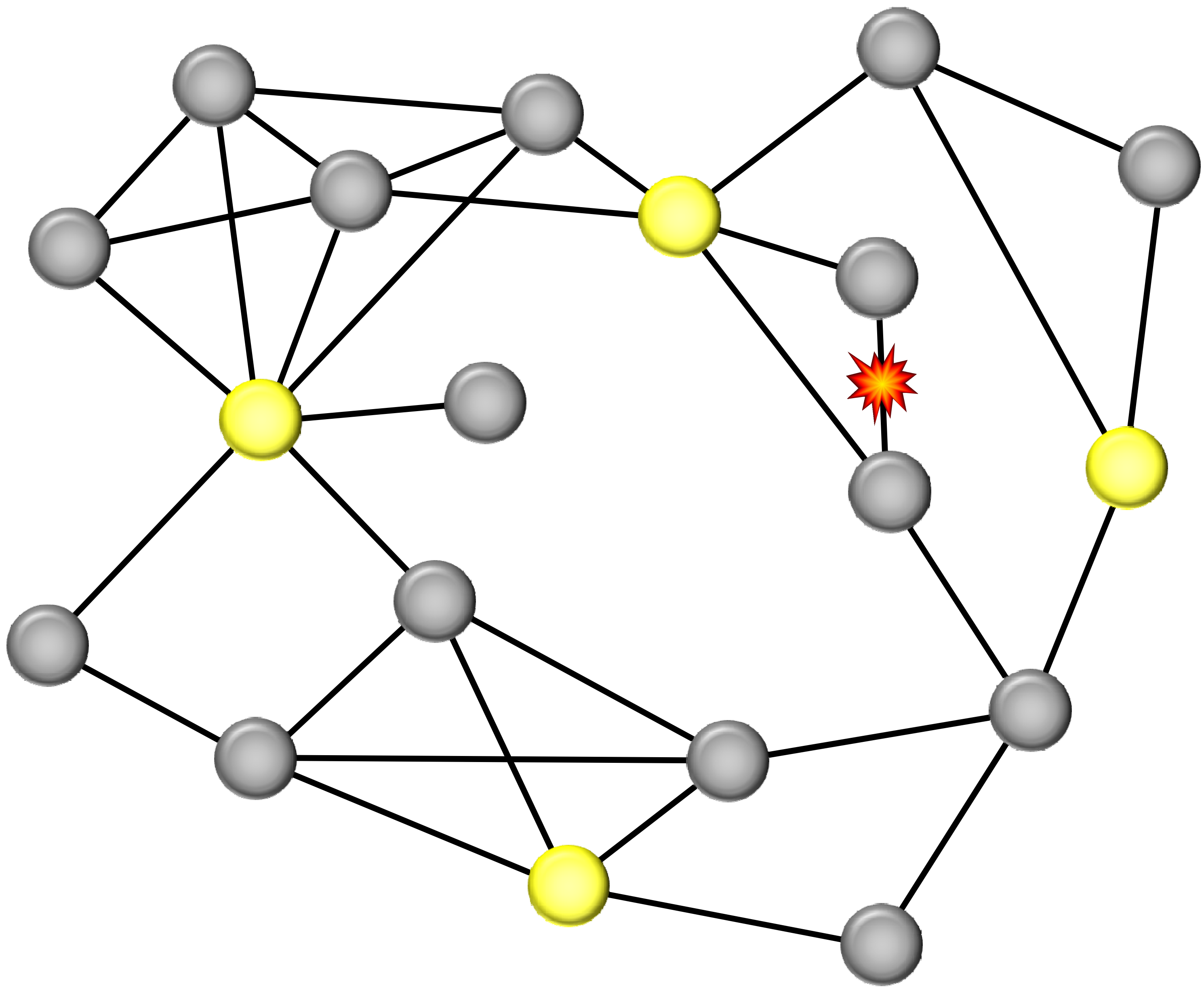
Motivation by Example



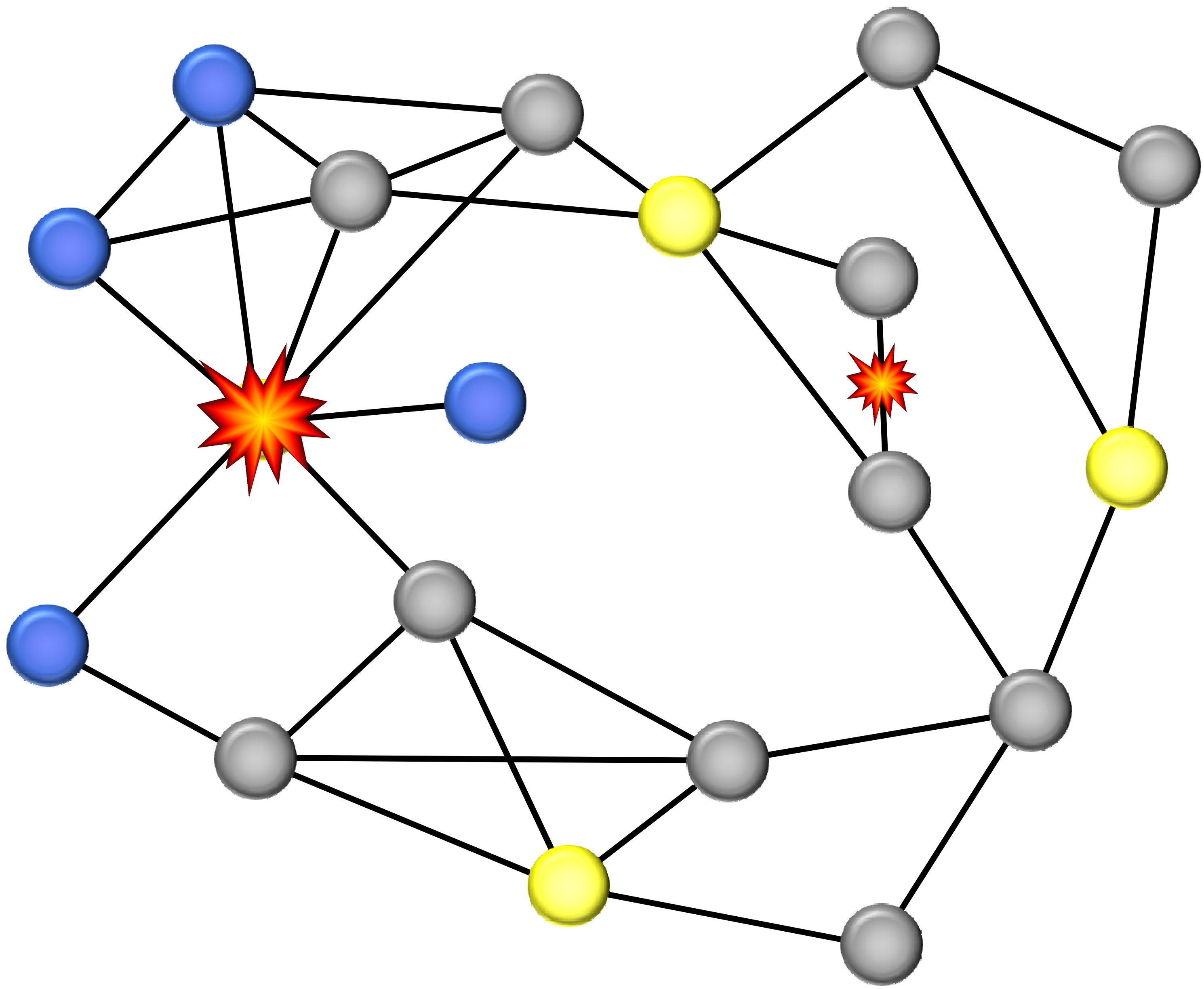
Motivation by Example



Motivation by Example

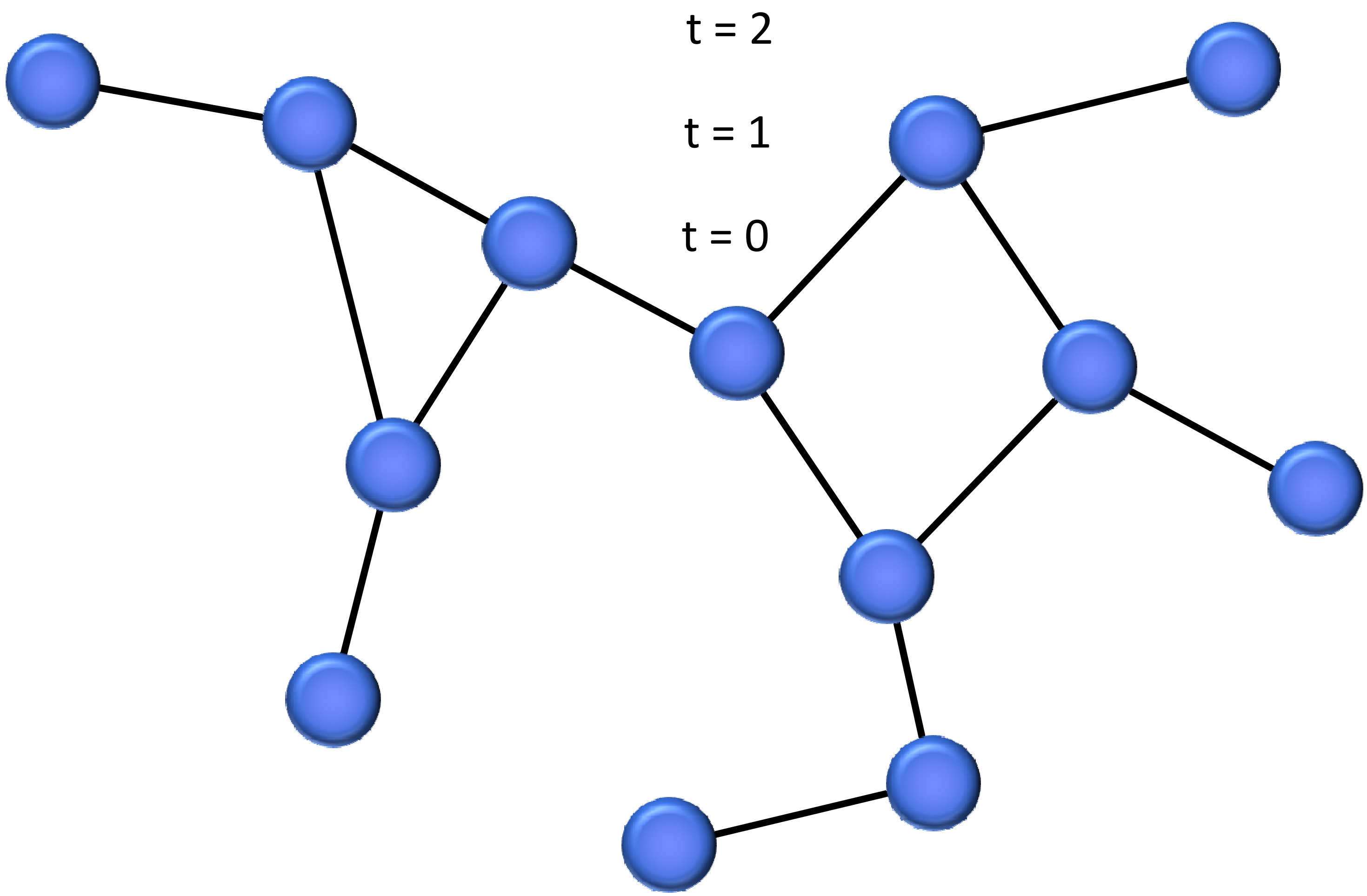


Motivation by Example



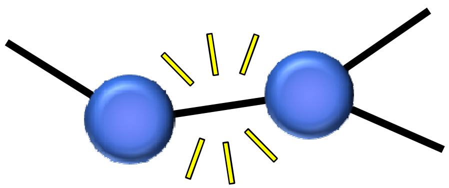
Model

- Neighborhood model
  - Unbounded message sizes
  - Synchronous rounds

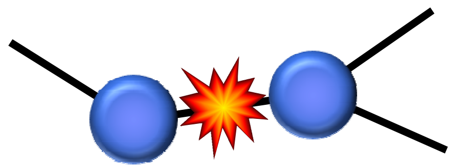


Model (cont.)

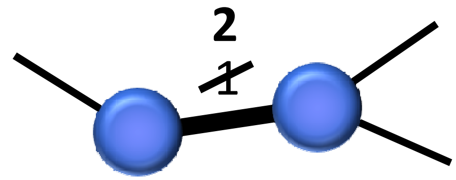
- We say a problem can be ***fixed locally*** if any solution can be fixed within O(1) rounds after a graph change.



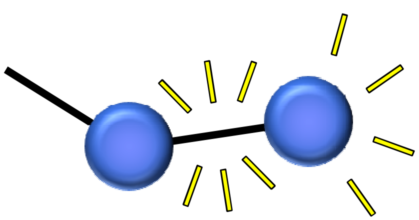
(+e)
Edge Insertion



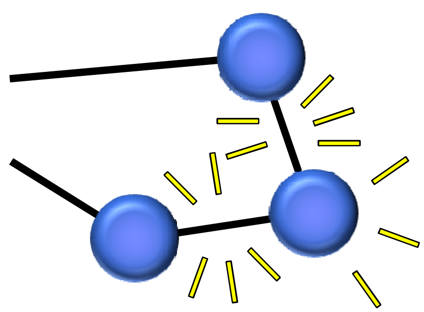
(-e)
Edge Deletion



(w → w')
Weight Change

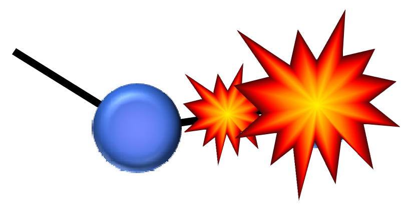


($+v_1$)

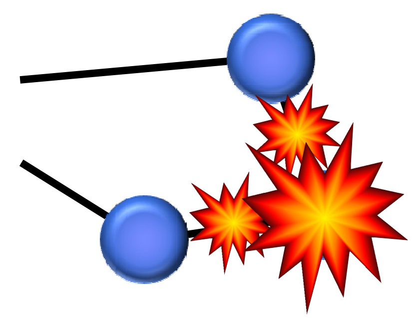


($+v_*$)

Node Insertion



($-v_1$)

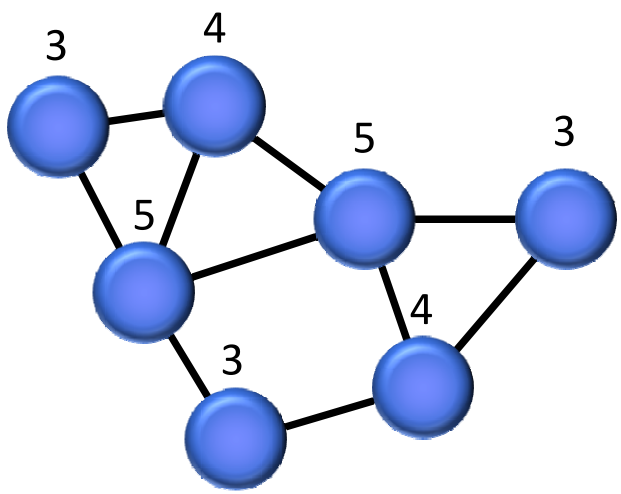


($-v_*$)

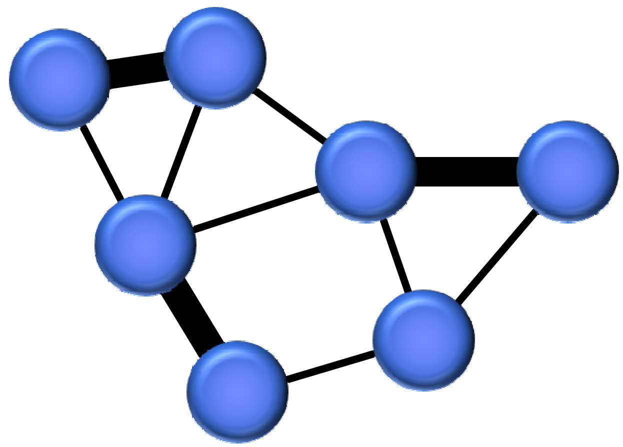
Node Deletion

Computation vs. Fixing

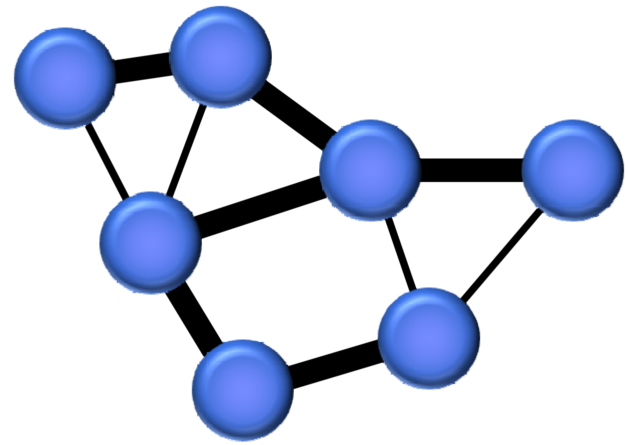
- Computing solutions is very well-studied
- Different "complexity classes" have been defined:



Γ_1-Count
"local"



Maximal Matching
"polylog"



Spanning Tree
"global"

- Previous work on local fixing
  - 1995: Maximal Independent Sets (MIS), by Kutten and Peleg
  - 2007: O(1)-Maximum Weighted Matchings, by Lotker, Patt-Shamir and Rosén

Problems

	LBound	UBound
Γ_1-Count	Ω(1)	O(1)
o(n)-MDS	Ω(1)	O(1)
MIS	Ω(√log(n))	O(log(n))
O(1)-MWM	Ω(√log(n))	O(log(n))
MM	Ω(√log(n))	O(log(n))
2-MVC	Ω(√log(n))	O(log(n))
$\Gamma_{\log(n)}$-Count	Ω(log(n))	O(log(n))
ST	Ω(D)	O(D)
MST	Ω(D)	O(D)
SPT	Ω(D)	O(D)
Flow	Ω(D)	O(D)
Leader	Ω(D)	O(D)
Count	Ω(D)	O(D)

Problems

	LBound	UBound	+e	-e	$w \to w'$	$+v_1$	$-v_1$	$+v_*$	$-v_*$
Γ_1-Count	$\Omega(1)$	$O(1)$	✓	✓		✓	✓	✓	✓
o(n)-MDS	$\Omega(1)$	$O(1)$	✓	✓		✓	✓	✓	✓
MIS	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
O(1)-MWM	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓	✓	✓	✓	✓	✓
MM	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
2-MVC	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
$\Gamma_{\log(n)}$-Count	$\Omega(\log(n))$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
ST	$\Omega(D)$	$O(D)$	✓	✓		✓	✓	✓	✓
MST	$\Omega(D)$	$O(D)$	✓	✓	✓	✓	✓	✓	✓
SPT	$\Omega(D)$	$O(D)$	✓	✓	✓	✓	✓	✓	✓
Flow	$\Omega(D)$	$O(D)$	✓	✓	✓	✓	✓	✓	✓
Leader	$\Omega(D)$	$O(D)$	✓	✓		✓	✓	✓	✓
Count	$\Omega(D)$	$O(D)$	✓	✓		✓	✓	✓	✓

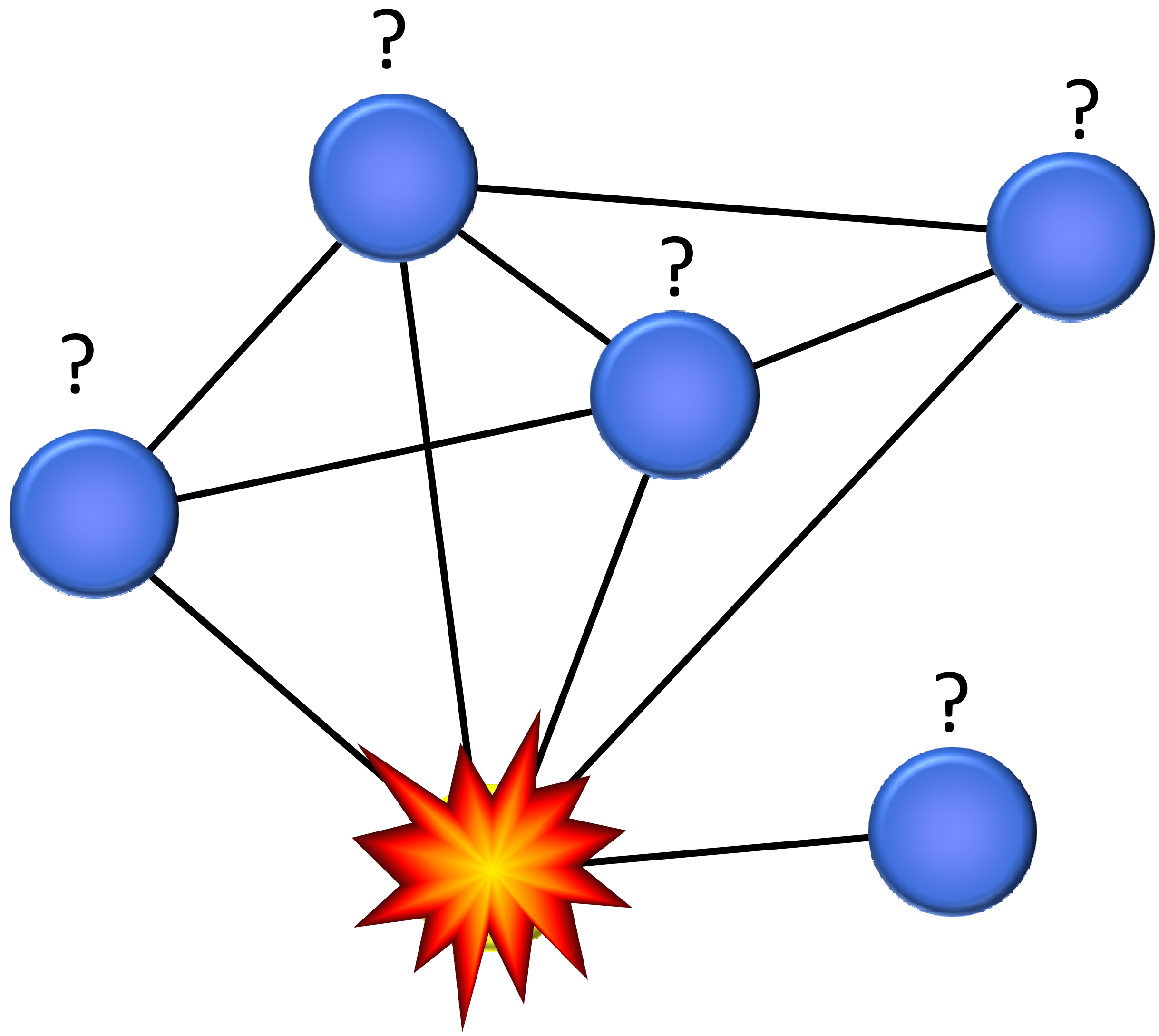
Problems

	LBound	UBound	+e	-e	$w \to w'$	$+v_1$	$-v_1$	$+v_*$	$-v_*$
Γ_1-Count	$\Omega(1)$	$O(1)$	✓	✗		✓	✗	✓	✗
$o(n)$-MDS	$\Omega(1)$	$O(1)$	✗	✓		✗	✓	✗	✓
MIS	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✗		✓	✗	✓	✗
$O(1)$-MWM	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✗	✓	✗	✗	✓	✗	✓
MM	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✗		✓	✗	✓	✗
2-MVC	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✗	✓		✗	✓	✗	✓
$\Gamma_{\log(n)}$-Count	$\Omega(\log(n))$	$O(\log(n))$	✓	✗		✓	✗	✓	✗
ST	$\Omega(D)$	$O(D)$	✓	✗		✓	✗	✓	✗
MST	$\Omega(D)$	$O(D)$	✗	✓	✗	✗	✓	✗	✓
SPT	$\Omega(D)$	$O(D)$	✓	✗	✓	✓	✗	✓	✗
Flow	$\Omega(D)$	$O(D)$	✗	✓	✗	✗	✓	✗	✓
Leader	$\Omega(D)$	$O(D)$	✓	✗		✓	✗	✓	✗
Count	$\Omega(D)$	$O(D)$	✗	✓		✗	✓	✗	✓

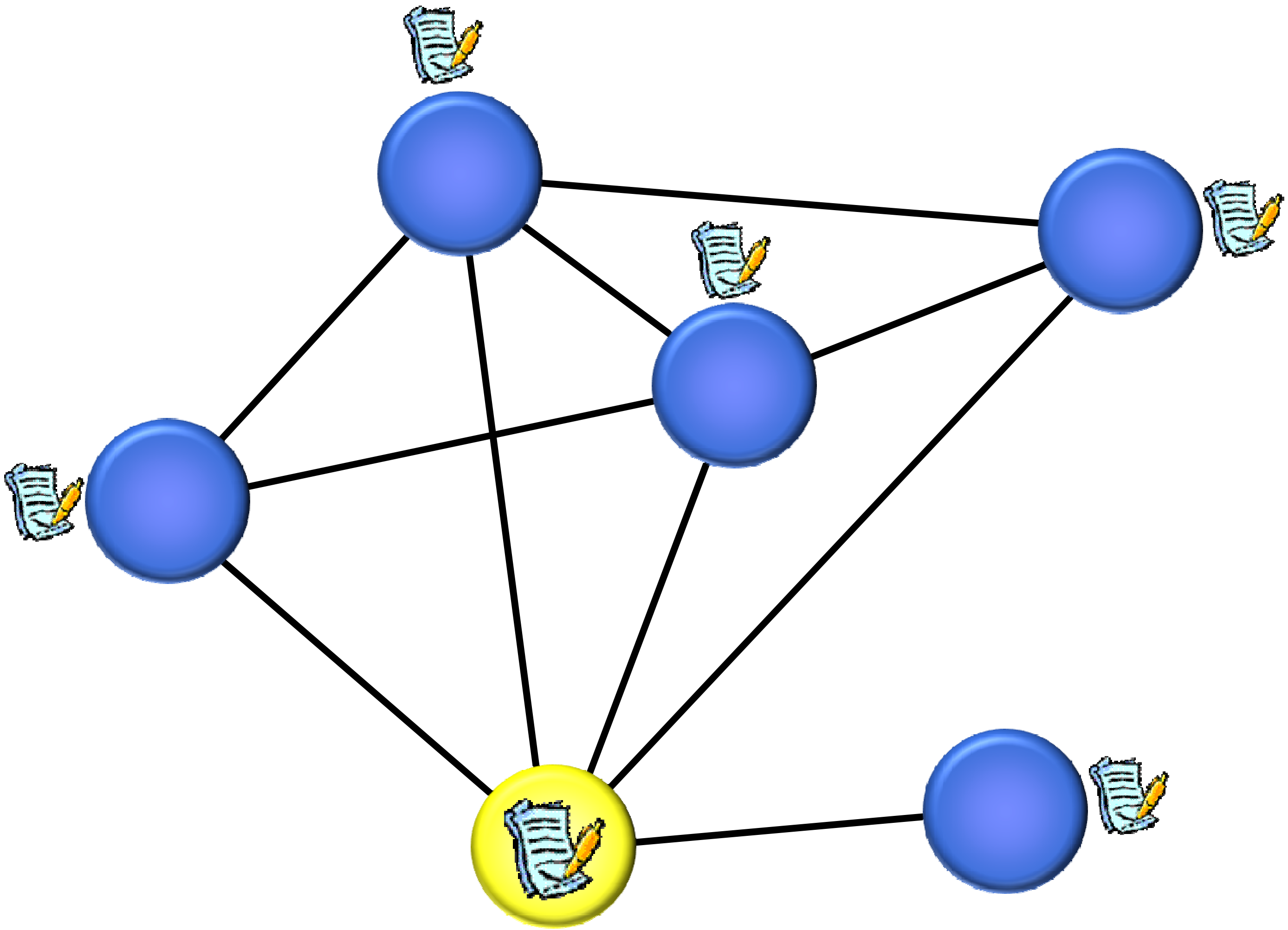
Results

	LBound	UBound	+e	-e	$w \rightarrow w'$	$+v_1$	$-v_1$	$+v_*$	$-v_*$
Γ_1-Count	$\Omega(1)$	$O(1)$	✓	✓		✓	✓	✓	✓
$o(n)$-MDS	$\Omega(1)$	$O(1)$	✗	✗		✗	✗	✗	✗
MIS	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
$O(1)$-MWM	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓	✓	✓	✓	✓	✓
MM	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
2-MVC	$\Omega(\sqrt{\log(n)})$	$O(\log(n))$	✓	✓		✓	✓	✓	✓
$\Gamma_{\log(n)}$-Count	$\Omega(\log(n))$	$O(\log(n))$	✗	✗		✗	✗	✗	✗
ST	$\Omega(D)$	$O(D)$	✓	✗		✓	✓	✓	✗
MST	$\Omega(D)$	$O(D)$	✗	✗	✗	✓	✓	✗	✗
SPT	$\Omega(D)$	$O(D)$	✗	✗	✗	✓	✓	✗	✗
Flow	$\Omega(D)$	$O(D)$	✗	✗	✗	✓	✓	✗	✗
Leader	$\Omega(D)$	$O(D)$	✓	✓		✓	✓	✓	✓
Count	$\Omega(D)$	$O(D)$	✓	✓		✗	✗	✗	✗

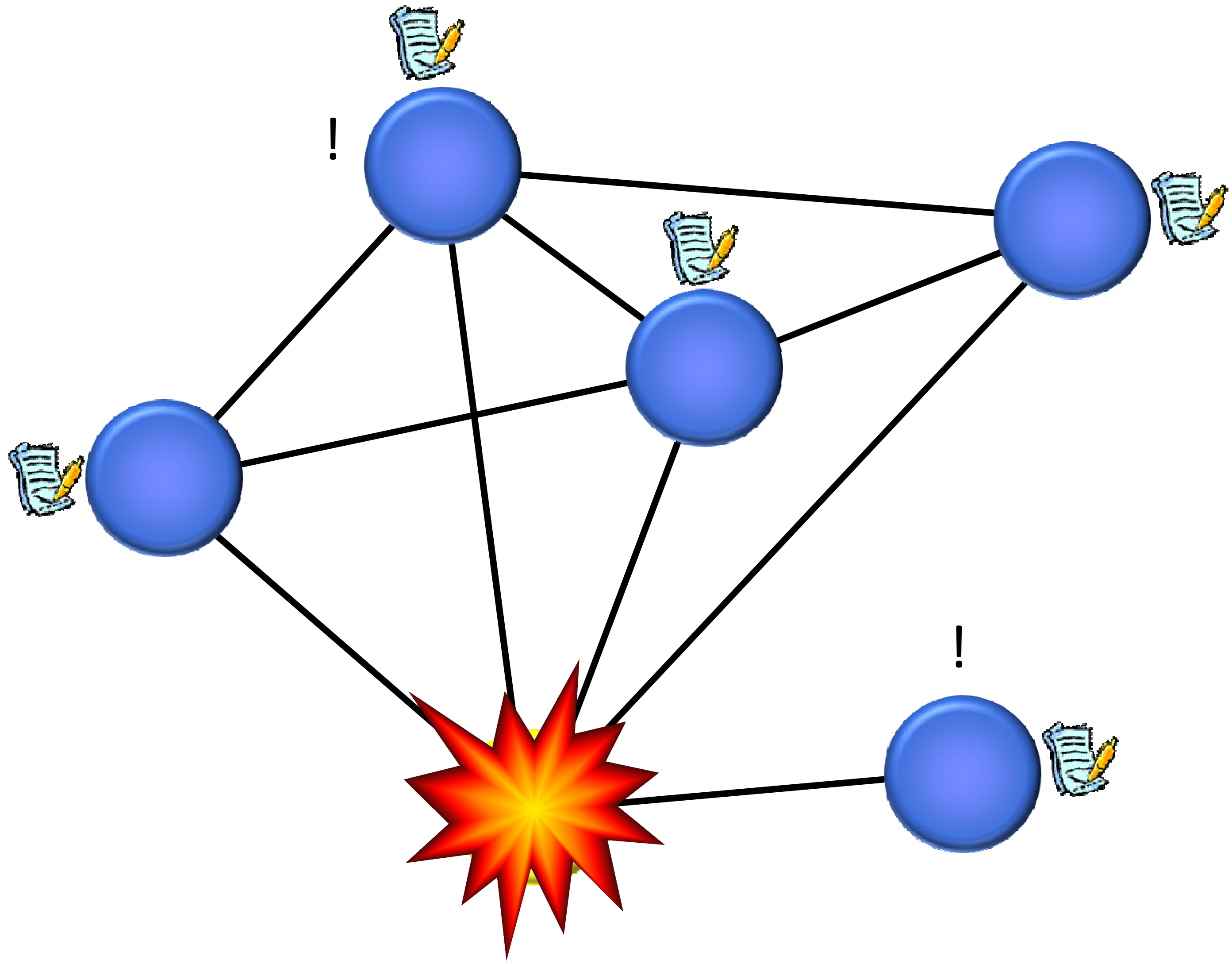
Maximal Independent Set (MIS)



“Last Will”: during settling



“Last Will”: in action



o(n)-Minimum Dominating Set

- We know algorithms which compute a o(n)-MDS in constant time [Kuhn et al., 2005]
- But we can construct o(n)-MDS solutions which cannot be fixed locally!

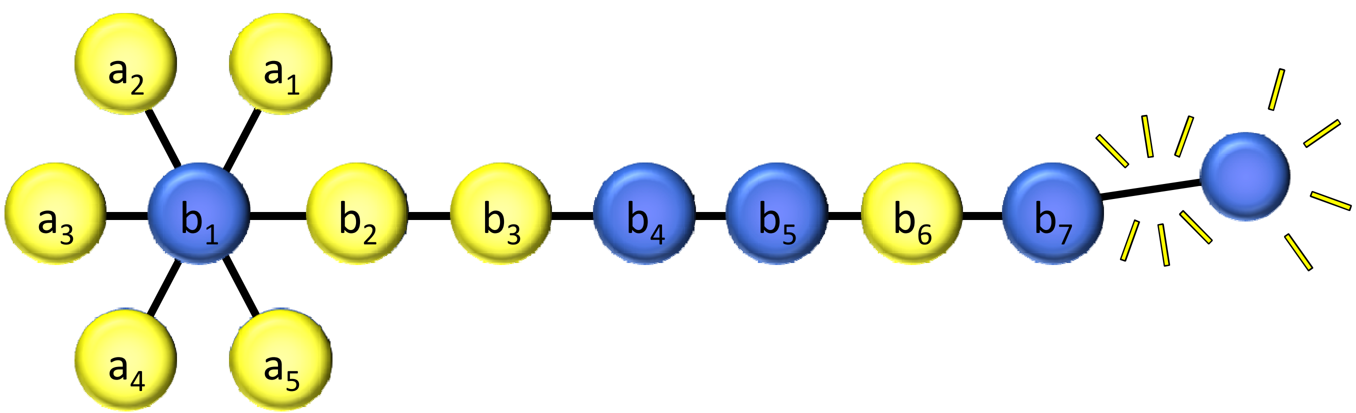
Recipe for a **k**-MDS which cannot be fixed within **c** steps:

$x = \lfloor (k - 1)(c + 1) \rfloor$, $y = 3c + 1$

$V = (a_1, a_2, \ldots, a_x, b_1, b_2, \ldots, b_y)$

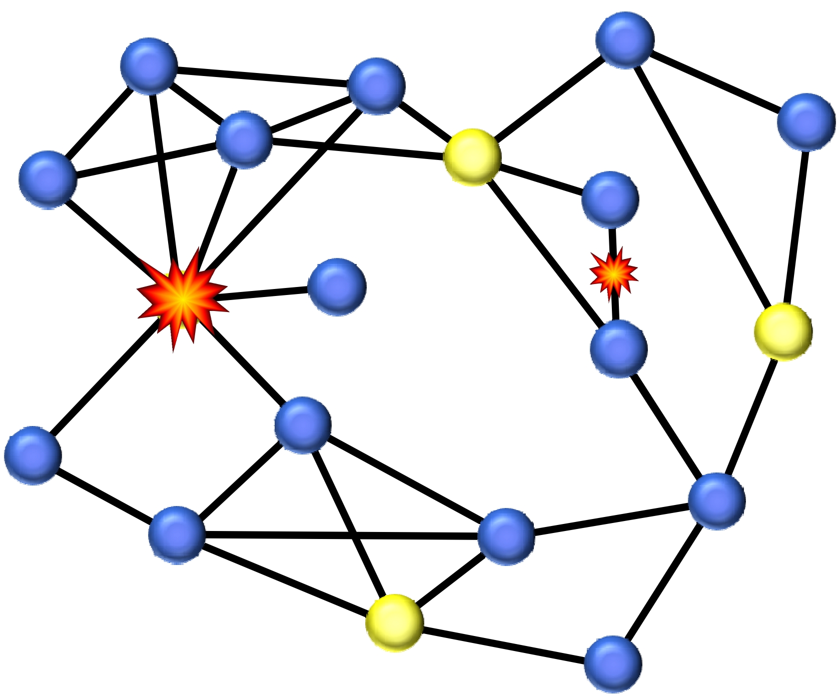
$E = \{(a_i, b_1) \mid 1 \leq i \leq x\} \cup \{(b_i, b_{i+1}) \mid 1 \leq i \leq y\}$

For k = 2.8, c = 2, x = 5 and y = 7:



Conclusion

- Traditional distributed complexity classes don't tell the whole story.
- A set of orthogonal "fixing complexity" classes may be interesting!



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$\Gamma_{\log(n)}$-Count	$\Omega(\log(n))$	$O(\log(n))$	✗	✗		✗	✗	✗	✗
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SPT	$\Omega(D)$	$O(D)$	✗	✗	✗	✓	✓	✗	✗
Flow	$\Omega(D)$	$O(D)$	✗	✗	✗	✓	✓	✗	✗
Leader	$\Omega(D)$	$O(D)$	✓	✓		✓	✓	✓	✓
Count	$\Omega(D)$	$O(D)$	✓	✓		✗	✗	✗	✗

Thanks!

Questions & Comments?