Computing the Best Policy That Survives a Vote

Andrei Constantinescu
Roger Wattenhofer

Distributed Computing Group

ETH Zürich
A Board of Directors
A Board of Directors
A Board of Directors

**Issue 1**: Increase salaries?
A Board of Directors

Issue 1: Increase salaries?

Issue 2: Start an advertising campaign?

Issue 3: Hire more researchers?

Assume

Binary Issues
A Board of Directors

**Issue 1:** Increase salaries?

**Issue 2:** Start an advertising campaign?

**Issue 3:** Hire more researchers?

Assume

Binary Issues
A Board of Directors

**Issue 1:** Increase salaries?

- [ ] Yes
- [x] No

**Issue 2:** Start an advertising campaign?

- [x] Yes

**Issue 3:** Hire more researchers?

- [ ] Yes

Assume Binary Issues
A Board of Directors

**Issue 1**: Increase salaries?

- [ ]
- [x]
- [x]
- [x]
- [x]

Assume Independent Binary Issues
A Board of Directors

Issue 1: Increase salaries?

- Person 1: Yes
- Person 2: No
- Person 3: No
- Person 4: Yes

Assume Binary Issues
A Board of Directors

Issue 1: Increase salaries?

Assume

Binary Issues
A Board of Directors

**Issue 1:** Increase salaries?

- [ ] Yes
- [x] No
- [ ] No
- [ ] No
- [ ] Yes
- [ ] Yes
- [ ] Yes
- [ ] Yes

Assume Independent Binary Issues
A Board of Directors

**Issue 1:** Increase salaries?

- Person A: Yes
- Person B: No
- Person C: No
- Person D: No
- Person E: No
- Group: Yes
A Board of Directors

**Issue 1**: Increase salaries?
- **Checkmark**: Yes
- **X**: No

**Issue 2**: Start an advertising campaign?
- **Checkmark**: No
- **X**: No

**Assume Binary Issues**

---

**Assume Checkmark**: Yes
A Board of Directors

Issue 1: Increase salaries?

Issue 2: Start an advertising campaign?

Assume Independent Binary Issues
A Board of Directors

**Issue 1**: Increase salaries?
- [ ] Yes
- [ ] No

**Issue 2**: Start an advertising campaign?
- [ ] Yes
- [ ] No

**Issue 3**: Hire more researchers?
- [ ] Yes
- [ ] No

Assume Independent Binary Issues
A Board of Directors

**Issue 1:** Increase salaries?

- Assume
- Independent
- Binary

**Issue 2:** Start an advertising campaign?

- Assume
- Independent
- Binary

**Issue 3:** Hire more researchers?

- Assume
- Independent
- Binary
## A Board of Directors

### Issue 1: Increase salaries?
- [ ] Approve
- [x] Reject
- [x] Abstain

### Issue 2: Start an advertising campaign?
- [ ] Approve
- [ ] Reject
- [x] Abstain

### Issue 3: Hire more researchers?
- [x] Approve
- [x] Reject
- [x] Abstain

Assume Independent Binary Issues
A Board of Directors

Issue 1: Increase salaries?
Issue 2: Start an advertising campaign?
Issue 3: Hire more researchers?

Assume Independent Binary Issues

Issue-Wise-Majority (IWM)
# A Board of Directors

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**Issue-Wise-Majority (IWM)**: ![Checkmark 1]

![Checkmark 2]

![Checkmark 3]

![Checkmark 4]
A Board of Directors

Issue-Wise-Majority (IWM)

3 issues agree
A Board of Directors

Issue-Wise-Majority (IWM)

1 issue agrees

3 issues agree

A Board of Directors

Issue-Wise-Majority (IWM)
A Board of Directors

How about?

1 issue agrees

3 issues agree

Issue-Wise-Majority (IWM)
A Board of Directors

How about?

NO!

3 issues agree

1 issue agrees

Issue-Wise-Majority (IWM)
A Board of Directors

How about?

Yes

Yes

Yes

NO!

NO!

NO!

1 issue agrees

3 issues agree

Issue-Wise-Majority (IWM)
A Board of Directors

How about?

Yes

Yes

NO!

NO!

Yes

Yes

NO!

NO!

1 issue agrees

3 issues agree

Issue-Wise-Majority (IWM)
A Board of Directors

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[Image of a smaller board with four members: two with green checks, one with a red x, and one missing.]
A Board of Directors

2 issues agree with IWM
A Board of Directors

2 issues agree

2 issues agree
with IWM
# A Board of Directors

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2 issues agree with IWM
A Board of Directors

- 0 issues agree with IWM
- 2 issues agree
A Board of Directors

How about?

0 issues agree

2 issues agree

2 issues agree with IWM
A Board of Directors

How about?

- Yes
- NO!
- NO!
- NO!

2 issues agree with IWM
A Board of Directors

How about?

0 issues agree

2 issues agree

2 issues agree with IWM
The Problem, Formally

- $N$ voters, $T$ issues (topics, motions, laws, etc.)
  - Issues are binary and independent.
  - In this talk: $N$ and $T$ are odd.
- Voters' preferences are $T$-bit vectors.
- Proposals are $T$-bit vectors.
  - e.g., Issue-Wise-Majority (IWM) proposal.
- A voter with preference vector $v$ supports a proposal $p$ iff $v$ agrees with $p$ in $> T/2$ bits.

Problem: Find proposal agreeing with IWM in as many bits such that $> N/2$ voters support it.

[Fritsch and Wattenhofer, AAMAS'22]
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[Fritsch and Wattenhofer, AAMAS’22]
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*[Fritsch and Wattenhofer, AAMAS’22]*
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\{supports \iff prefers \( p \) to opposite of \( p \}\}
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  \{	ext{supports} \iff \text{prefers} p \text{ to opposite of } p\}\}

○ Problem: Find proposal agreeing with IWM in as many bits as possible such that $> N/2$ voters support it.
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◎ A voter with preference vector \( \mathbf{v} \) **supports** a proposal \( \mathbf{p} \) iff \( \mathbf{v} \) agrees with \( \mathbf{p} \) in \( > \frac{T}{2} \) bits (else they **oppose** it).
  
  \{ supports ⇔ prefers \( \mathbf{p} \) to opposite of \( \mathbf{p} \) \}

◎ **Problem**: Find proposal agreeing with IWM in as many bits as possible such that \( > \frac{N}{2} \) voters **support** it.

[Fritsch and Wattenhofer, AAMAS’22]
How Bad Can It Get?
<table>
<thead>
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<th>How Bad Can It Get?</th>
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<tbody>
<tr>
<td>![Image of checkboxes with some checked and some not checked]</td>
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How Bad Can It Get?

\[ \begin{array}{cccccc}
\textbf{v}_1 & \checkmark & \times & \times & \times & \times \\
\textbf{v}_2 & \times & \checkmark & \times & \times & \times \\
\textbf{v}_3 & \times & \times & \checkmark & \times & \times \\
\textbf{v}_4 & \times & \times & \times & \checkmark & \times \\
\textbf{v}_5 & \times & \times & \times & \times & \checkmark \\
\textbf{v}_6 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\textbf{v}_7 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\textbf{v}_8 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\textbf{v}_9 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\end{array} \]
How Bad Can It Get?

N = 9
T = 5
How Bad Can It Get?

N = 9
T = 5
How Bad Can It Get?

N = 9
T = 5

Prop. p

v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
How Bad Can It Get?

N = 9  
T = 5

Prop. p

\[ \begin{array}{cccccc}
\text{v}_1 & \checkmark & \times & \times & \times & \times \\
\text{v}_2 & \times & \checkmark & \times & \times & \times \\
\text{v}_3 & \times & \times & \checkmark & \times & \times \\
\text{v}_4 & \times & \times & \times & \checkmark & \times \\
\text{v}_5 & \times & \times & \times & \times & \checkmark \\
\text{v}_6 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\text{v}_7 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\text{v}_8 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\text{v}_9 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\end{array} \]
How Bad Can It Get?

\[ N = 9 \]
\[ T = 5 \]

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\( \text{Prop. } p \) \[ ✓ \] \[ ✓ \] \[ ✓ \] \[ ✓ \] \[ ✓ \]
How Bad Can It Get?

N = 9
T = 5

Prop. p

Opp. p

Supp. p

v₁  ✅  ❌  ❌  ❌  ❌  ❌
v₂  ❌  ✅  ❌  ❌  ❌  ❌
v₃  ❌  ❌  ✅  ❌  ❌  ❌
v₄  ❌  ❌  ❌  ✅  ❌  ❌
v₅  ❌  ❌  ❌  ❌  ✅  ❌
v₆  ✅  ✅  ✅  ✅  ✅  ✅
v₇  ✅  ✅  ✅  ✅  ✅  ✅
v₈  ✅  ✅  ✅  ✅  ✅  ✅
v₉  ✅  ✅  ✅  ✅  ✅  ✅
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**Opp. p**

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T = 5
How Bad Can It Get?

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V_2 \quad \times \quad \checkmark \quad \times \quad \times \quad \times \\
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V_5 \quad \times \quad \times \quad \times \quad \times \quad \checkmark \\
V_6 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\
V_7 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\
V_8 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\
V_9 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\

\text{Opp. } p \\
\text{Supp. } p \\

\text{Prop. } p
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N = 9  
T = 5

3 issues agree with IWM
What Was Known
What Was Known

say $T = 2k + 1$
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What Was Known

say $T = 2k + 1$
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**What Was Known**

Say \( T = 2k + 1 \)

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(by previous construction)
## What Was Known

say $T = 2k + 1$

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(consider proposal with $k + 1$ ones, its opposite has $k$ ones, one has more support)

(by previous construction)
What Was Known

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say $T = 2k + 1$
### What Was Known

Say $T = 2k + 1$

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[Fritsch and Wattenhofer, AAMAS’22]
What Was Known  

\[ T = 2k + 1 \]

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[Fritsch and Wattenhofer, AAMAS’22]
- nonconstructive
What Was Known

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[Fritsch and Wattenhofer, AAMAS’22]
- nonconstructive (and a bit magic)
What Was Known

**Lemma A.1.** For $l = 0, \ldots, t$,

$$
\sum_{k=\lfloor t/2 \rfloor}^{t} (2k - t) s_{k,l} = t \binom{t-1}{\lfloor t/2 \rfloor}.
$$

**Proof.** Let

$$
f(l) = \sum_{k=\lfloor t/2 \rfloor}^{t} (2k - t) s_{k,l}.
$$

Note that we use the convention that $\binom{k}{n} = 0$ for $k > n$ and $k < 0$. Hence, the upper summation bound in the formula for $s_{k,l}$ from Lemma 4.4 can be omitted. Inserting this formula yields

$$
f(l) = \sum_{k=\lfloor t/2 \rfloor}^{t} \sum_{x=\lfloor (k+l-\lfloor t/2 \rfloor)/2 \rfloor}^{\infty} \binom{l}{x} \binom{t-l}{k-x} (2k - t)
$$

$$
= \sum_{x=\lfloor (l+1)/2 \rfloor}^{\infty} \binom{l}{x} \sum_{k=\lfloor t/2 \rfloor}^{\infty} \binom{t-l}{k-x} (2k - t)
$$

$$
= \sum_{x=\lfloor (l+1)/2 \rfloor}^{\infty} \binom{l}{x} \sum_{y=\lfloor t/2 \rfloor-x}^{\infty} \binom{t-1-([t/2]-x)}{y} (2y + 2x - t).
$$

We swapped summations in the second step and substituted $y = k - x$ in the third step. Note that

$$
\binom{t-l}{y} (2y + 2x - t) + \binom{t-l}{t-l-y} (2(t-l-y) + 2x - t) = 2 \binom{t-l}{t/2} (2x - t).
$$

Using this we further conclude

$$
f(l) = \sum_{x=\lfloor (l+1)/2 \rfloor}^{\infty} \binom{l}{x} \sum_{y=\lfloor t/2 \rfloor-x}^{\infty} \binom{t-1-([t/2]-x)}{y} (2x - l).
$$

In the second step, we switched the summation again. Now let $x_0 = \max(\lfloor t/2 \rfloor - y, y + l - \lfloor t/2 \rfloor)$. Then

$$
\sum_{x=x_0}^{\infty} \binom{l}{x} (2x - l) = \sum_{x=x_0}^{\infty} x \binom{l}{x} - (l - x) \binom{l}{x}
$$

$$
= \sum_{x=x_0}^{\infty} x \binom{l-1}{x-1} - l \binom{l-1}{x} = l \binom{l-1}{x_0-1}.
$$

Furthermore, the definition of $x_0$ implies

$$
\binom{l-1}{\lfloor t/2 \rfloor - y} = \binom{l-1}{y + l - \lfloor t/2 \rfloor} = \binom{l-1}{x_0 - 1}.
$$

With the previous two properties, we establish

$$
f(l) = \sum_{y=\lfloor t/2 \rfloor-l}^{\lfloor t/2 \rfloor} \binom{t-l}{y} \binom{l-1}{\lfloor t/2 \rfloor - y}
$$

$$
= l \sum_{z=0}^{l-1} \binom{t-l}{l-1} \binom{l-1}{z} = l \binom{l-1}{\lfloor t/2 \rfloor}.
$$

Here we substituted $z = \lfloor t/2 \rfloor - y$, and the last step follows from the well-known combinatorial identity $\binom{n}{k} = \sum_{j=0}^{n} \binom{i}{j} \binom{n-i}{k-j}$. □
What Was Known

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[Fritsch and Wattenhofer, AAMAS’22]
- nonconstructive
What Is New

say $T = 2k + 1$

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This paper
- probabilistic $\rightarrow$ derandomization
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This paper
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This paper | Trivial