Computing the Best Policy That Survives a Vote

Andrei Constantinescu
Roger Wattenhofer

Distributed Computing Group
ETH zürich

A Board of Directors

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A Board of Directors

Issue 1: Increase salaries?

A Board of Directors

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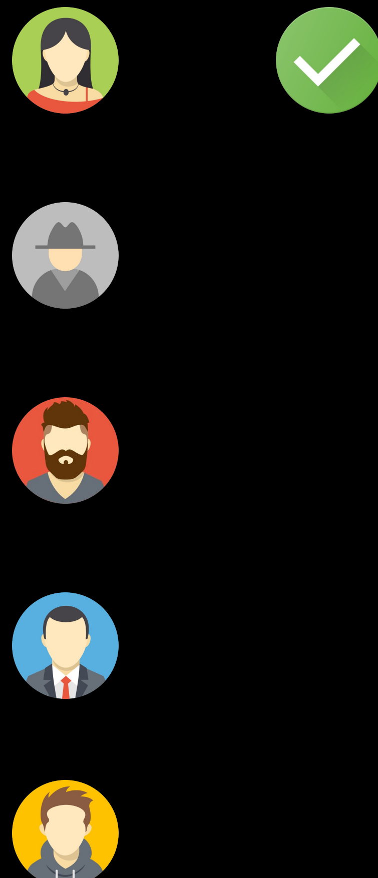
Assume

Binary Issues

A Board of Directors

Issue 1: Increase salaries?



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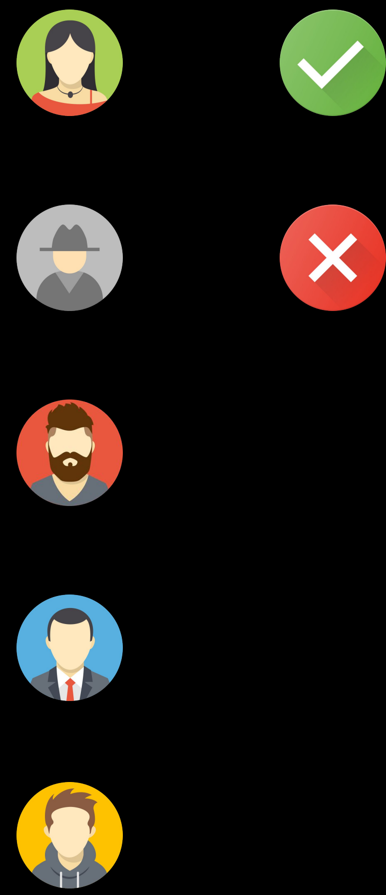

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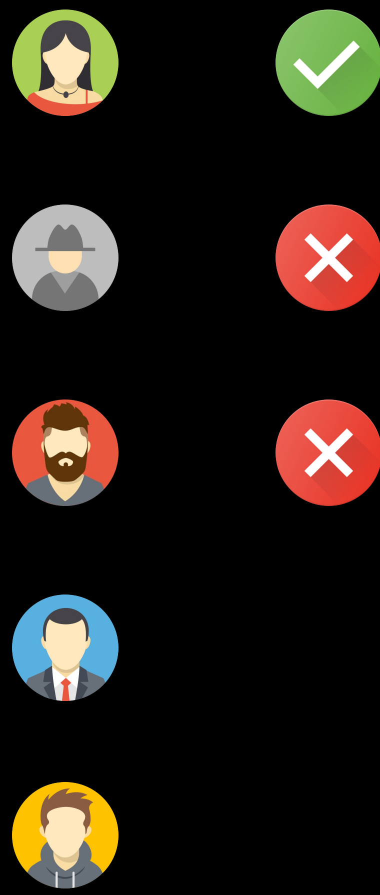

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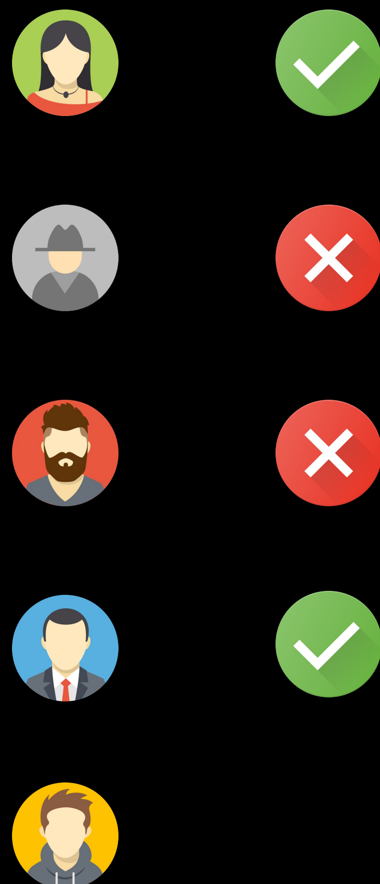

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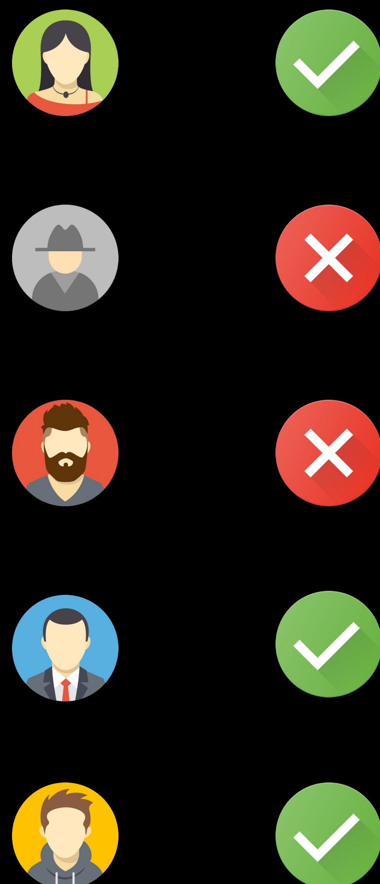

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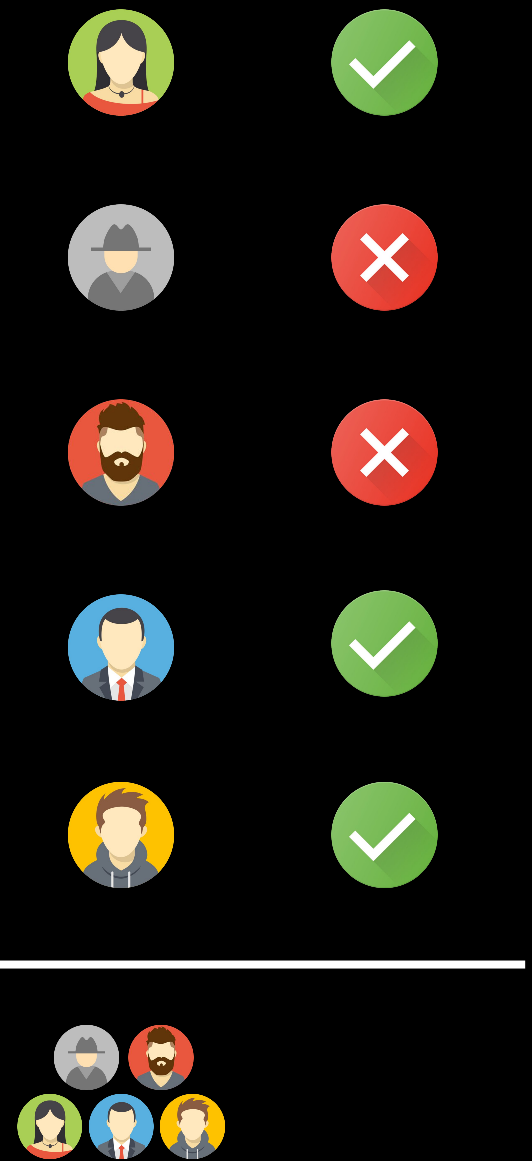

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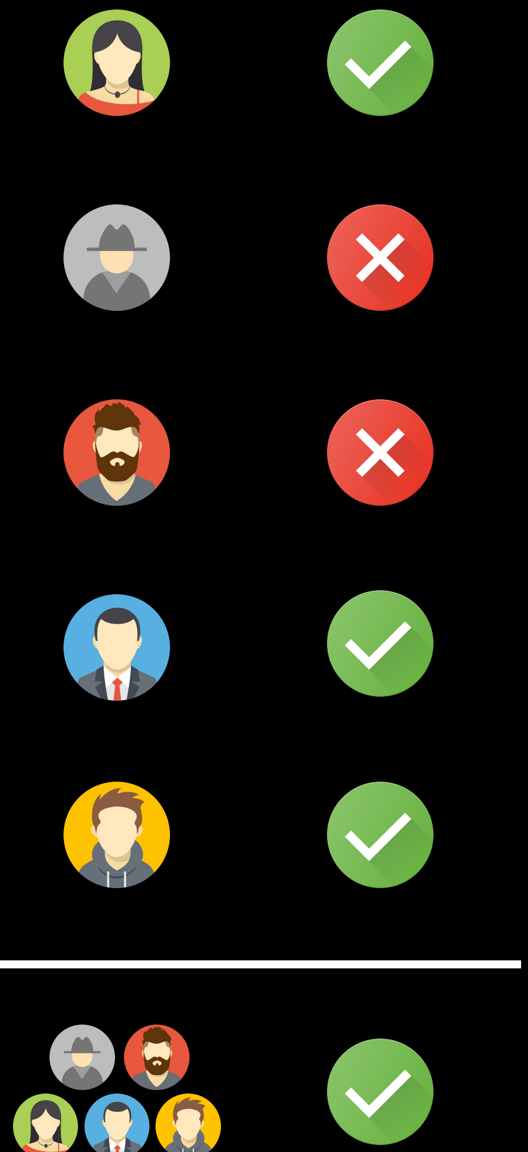

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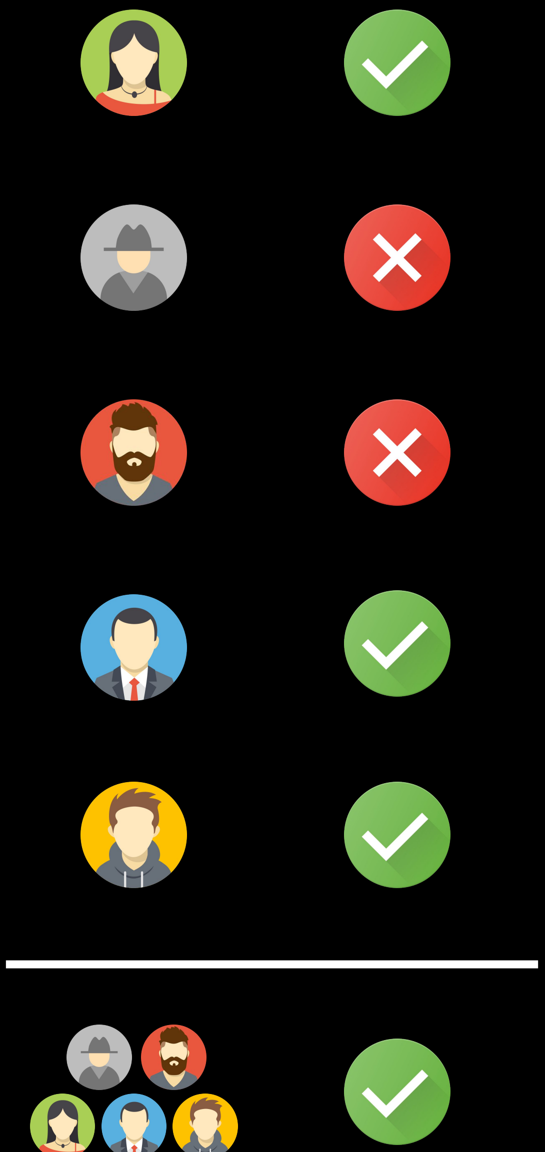
Issue 1: Increase salaries?

Issue 2: Start an advertising campaign?



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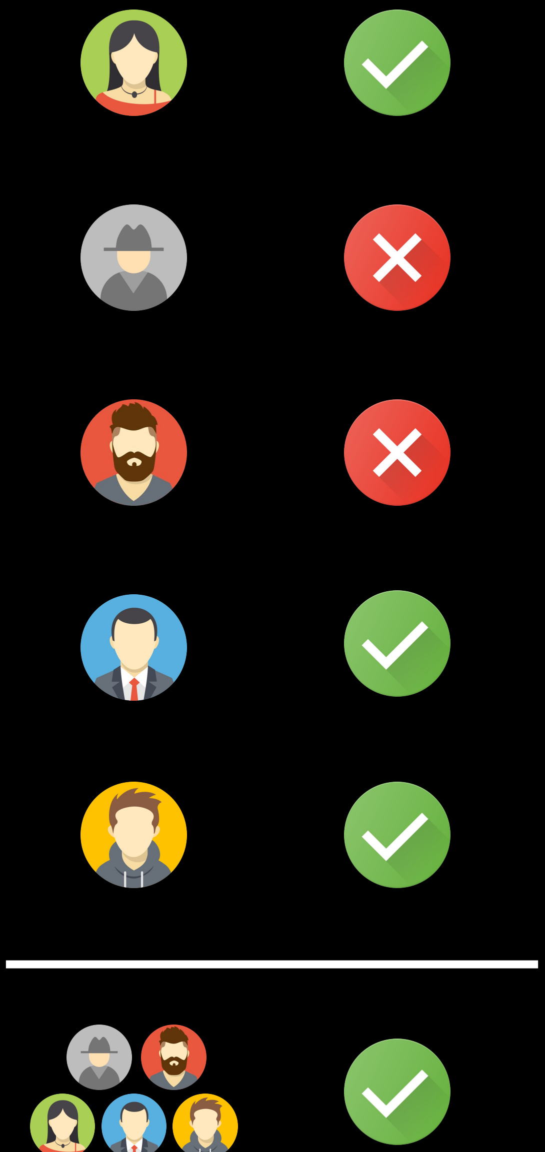

Assume Independent Binary Issues

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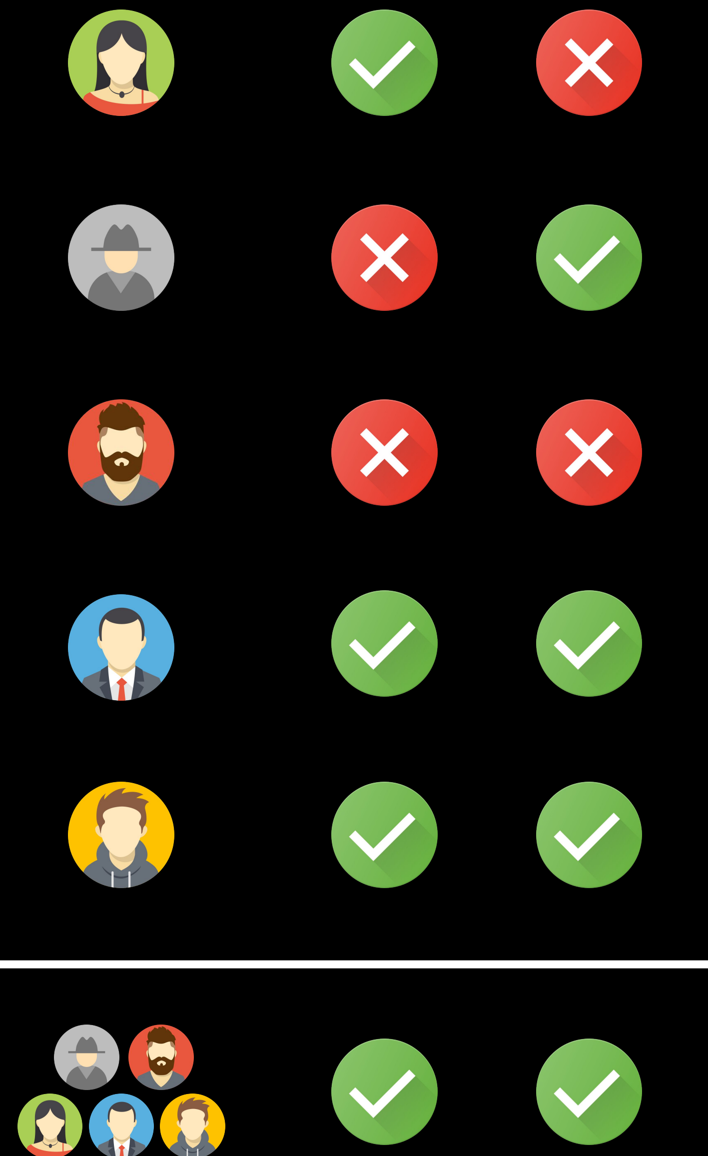

Assume Independent Binary Issues

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Issue 1: Increase salaries?

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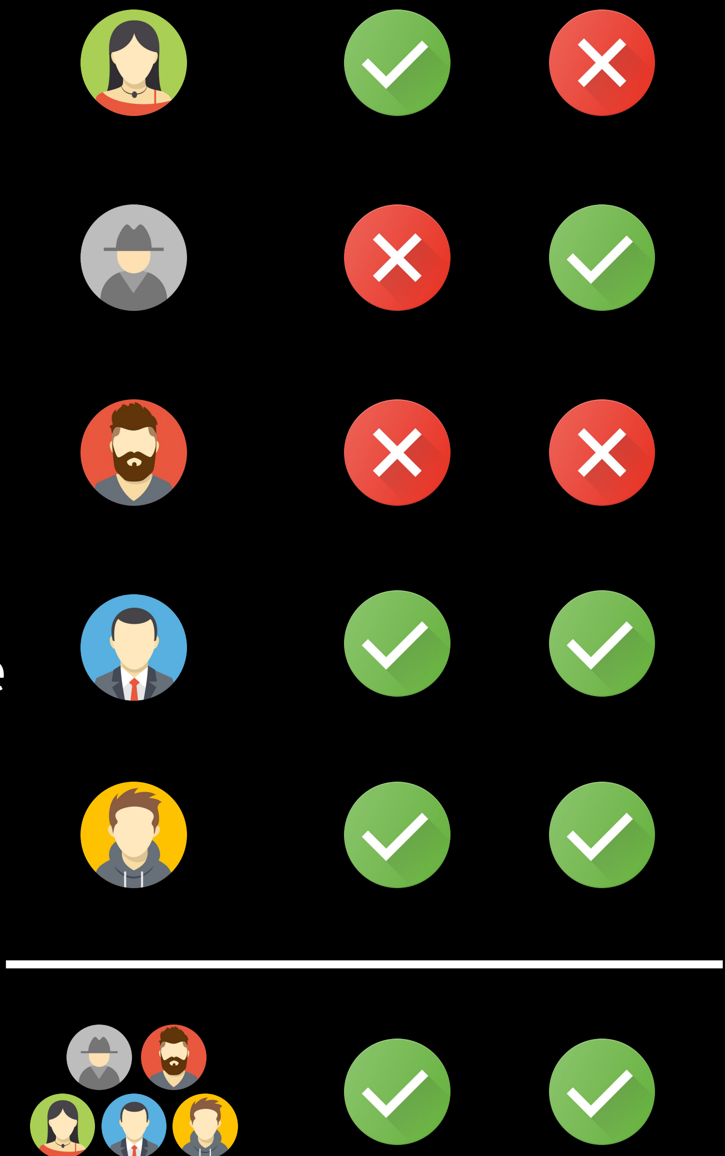
Issue 3: Hire more researchers?



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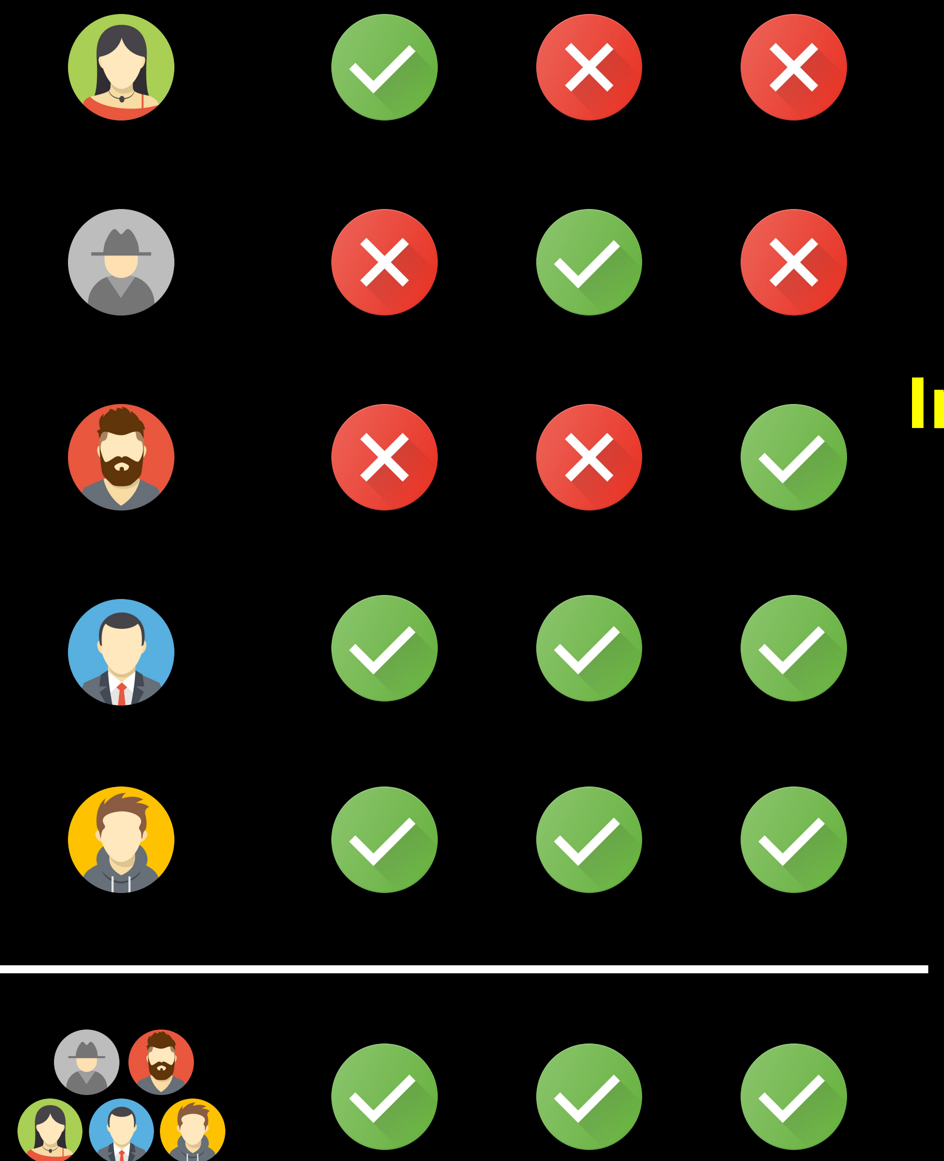

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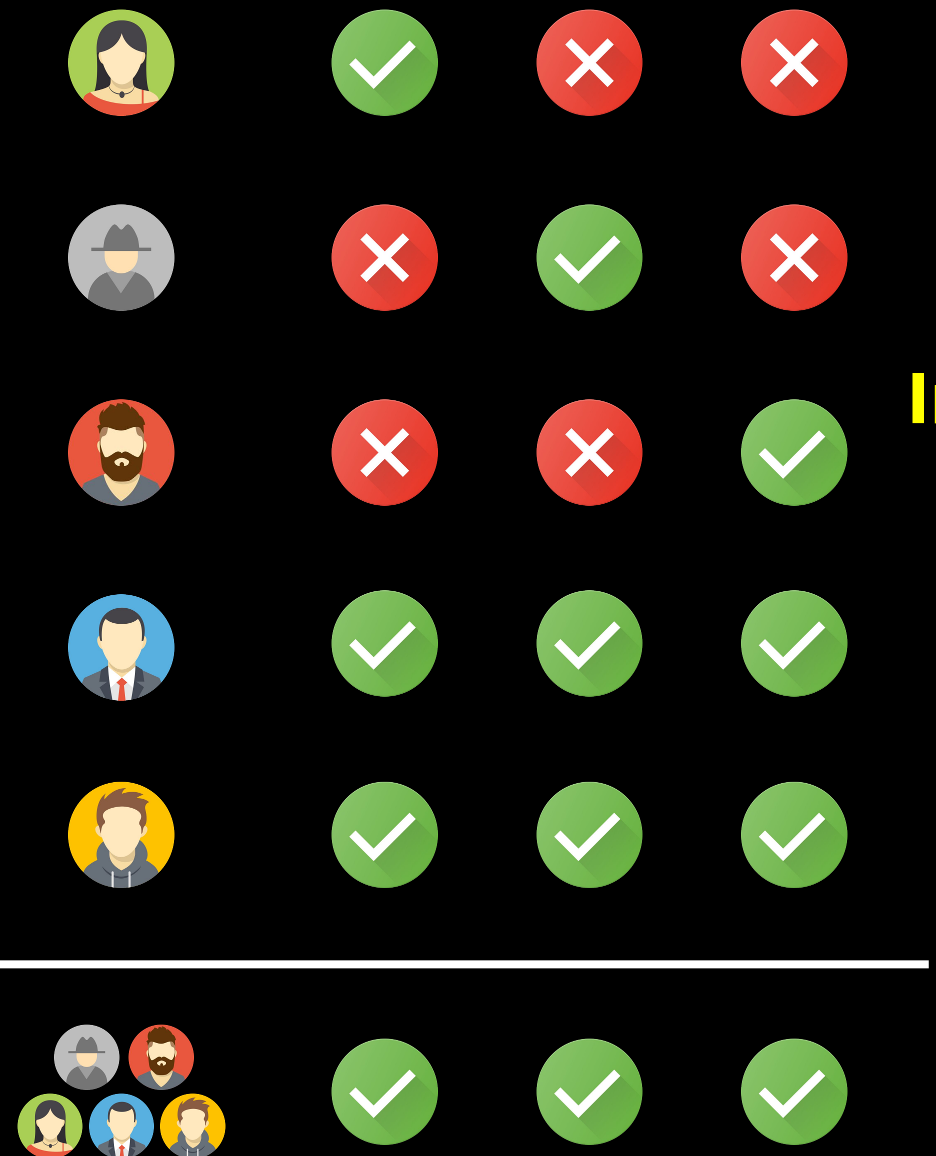

Issue-Wise-Majority (IWM)

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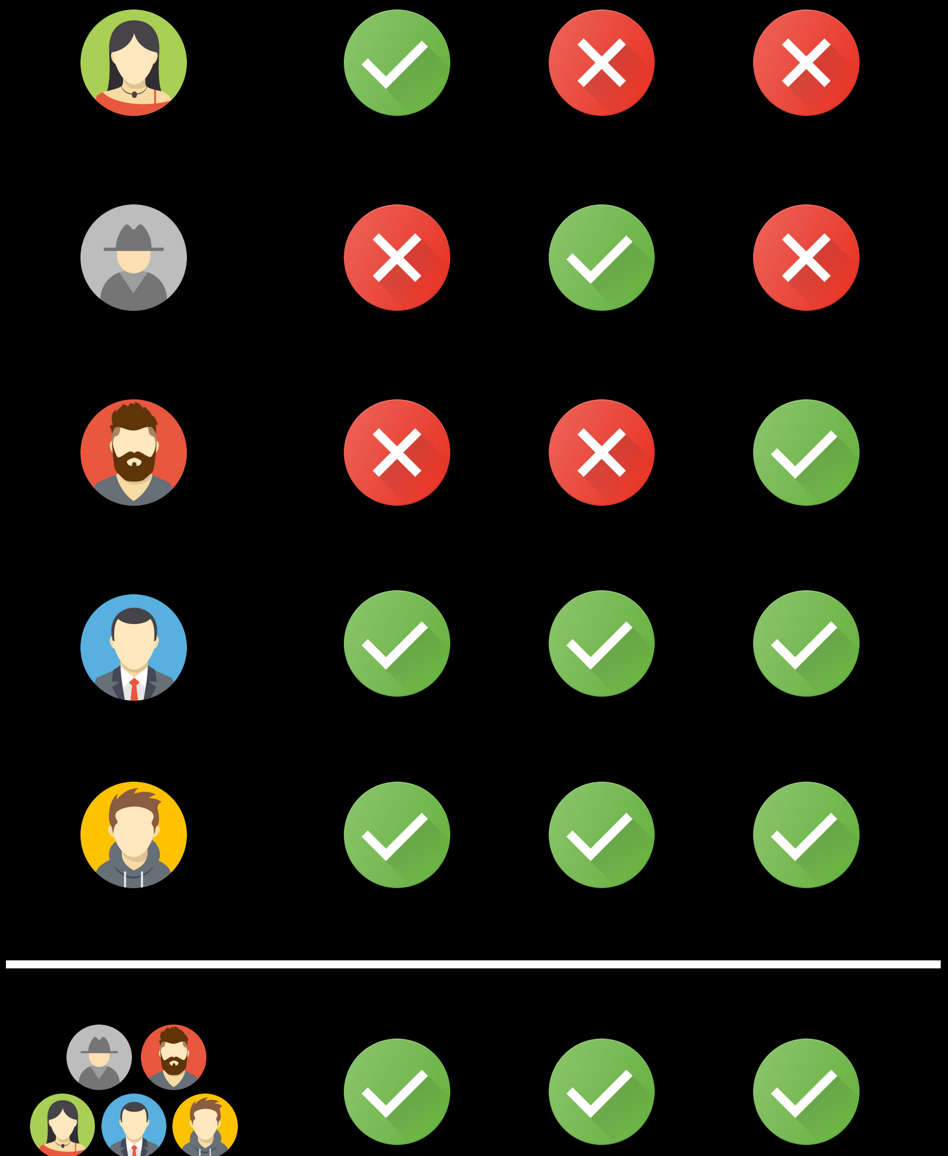

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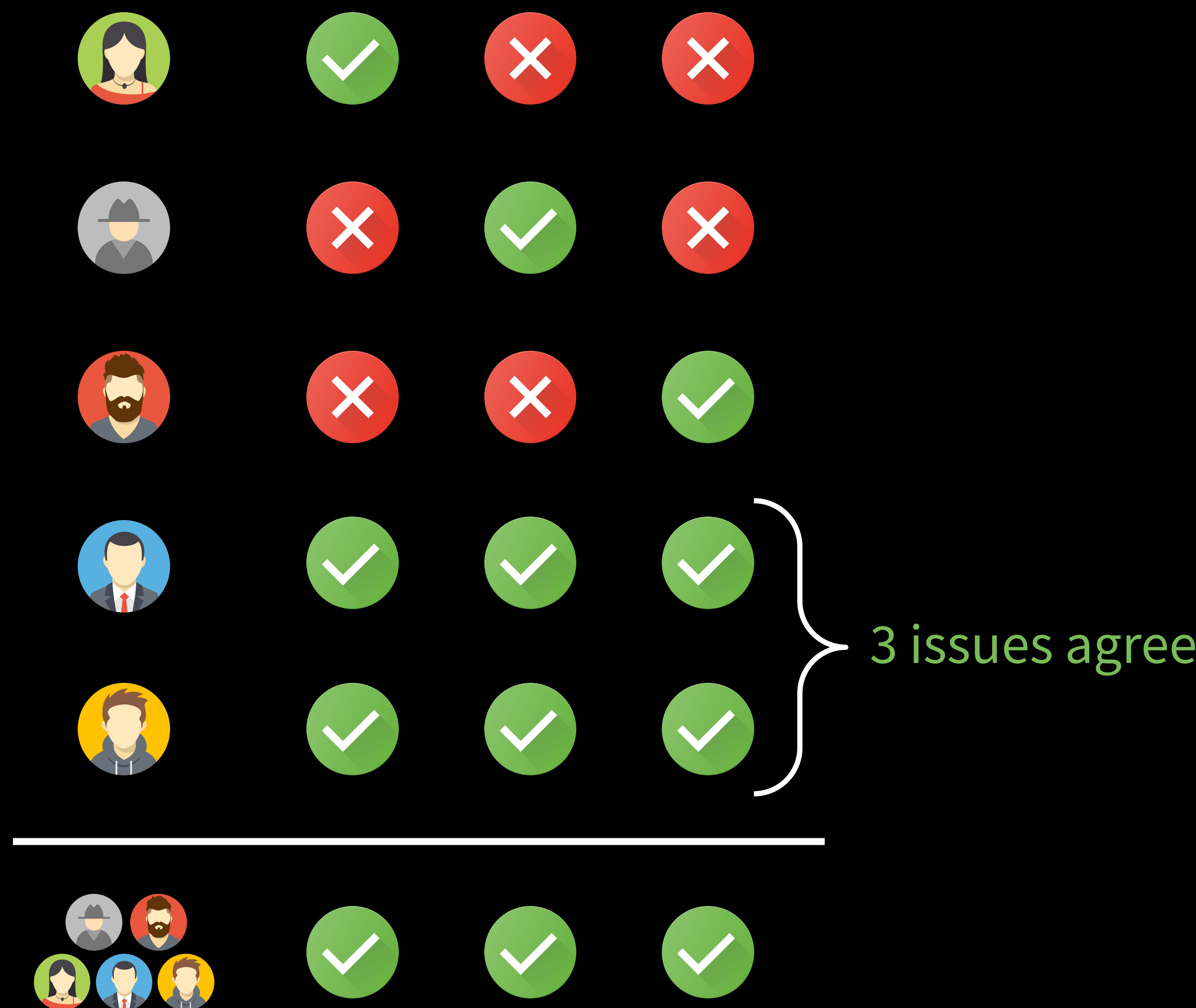

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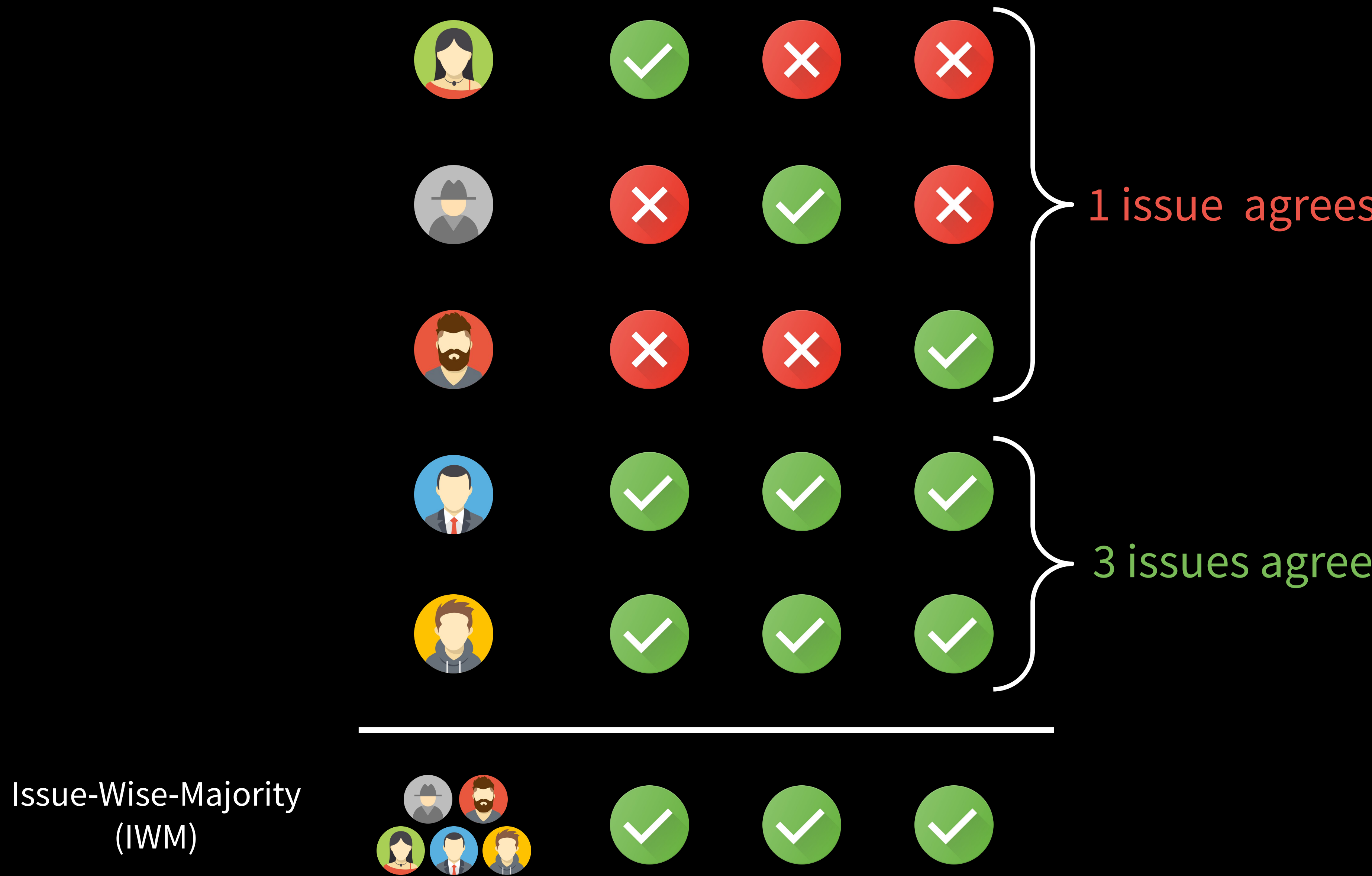

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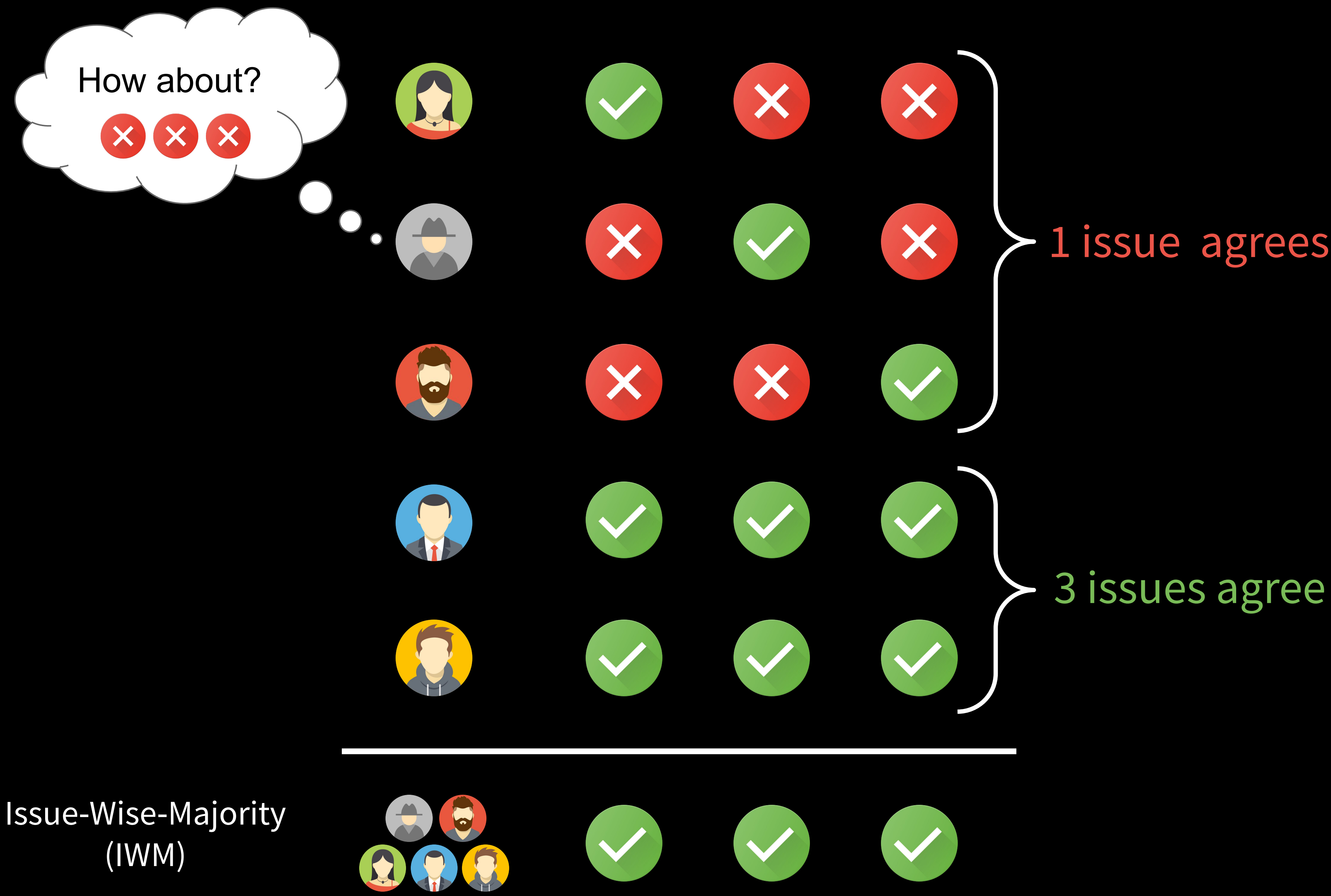

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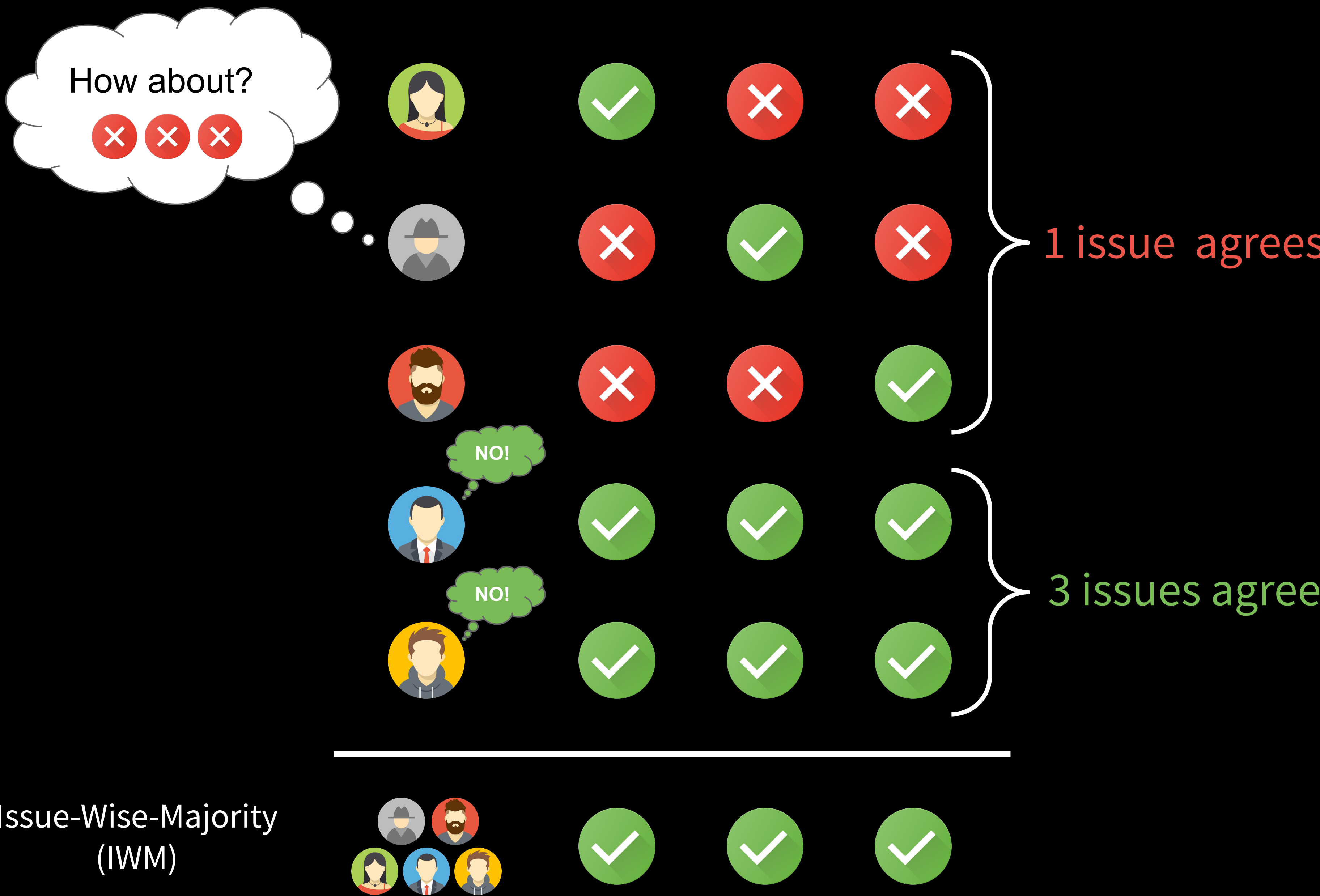

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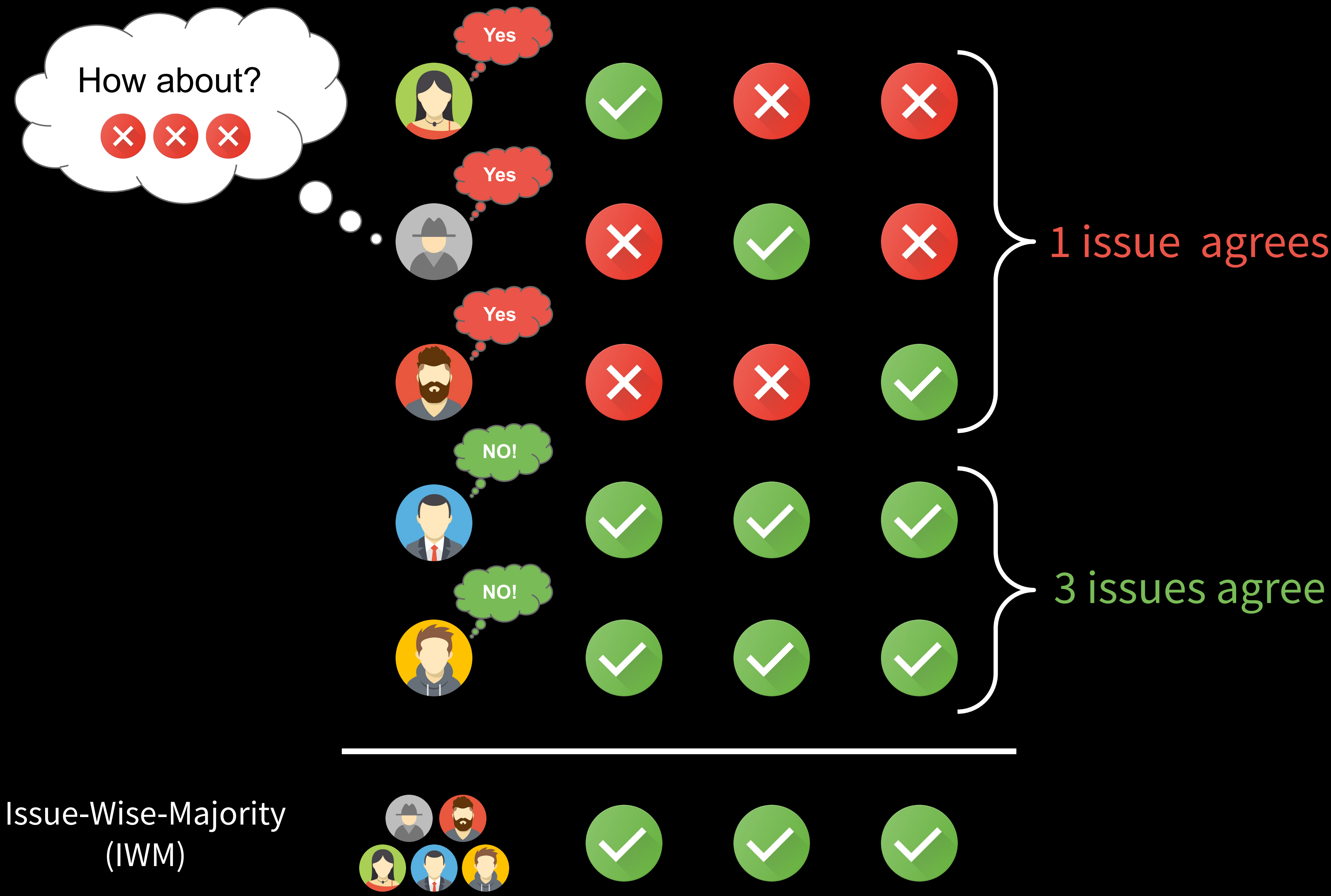

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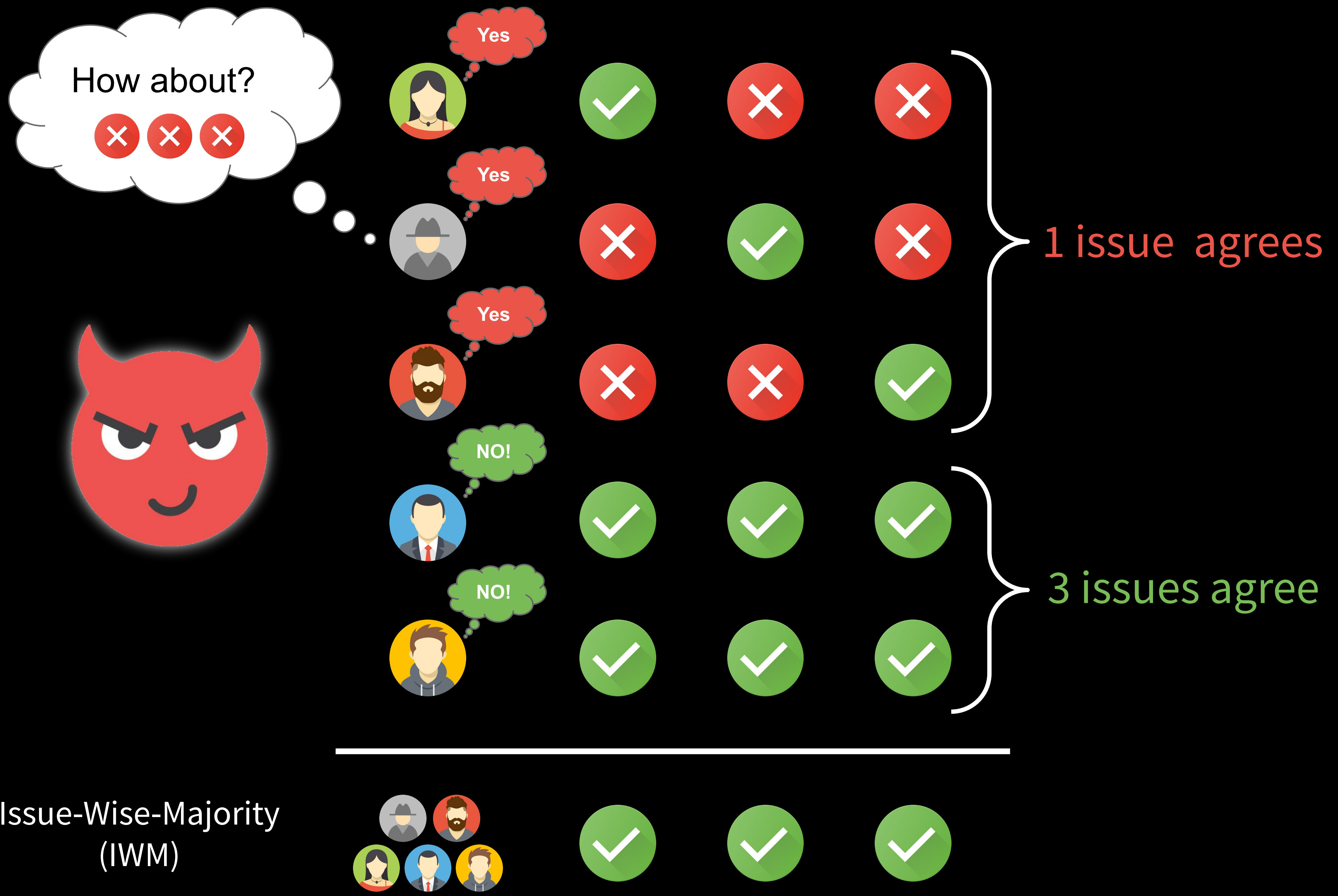

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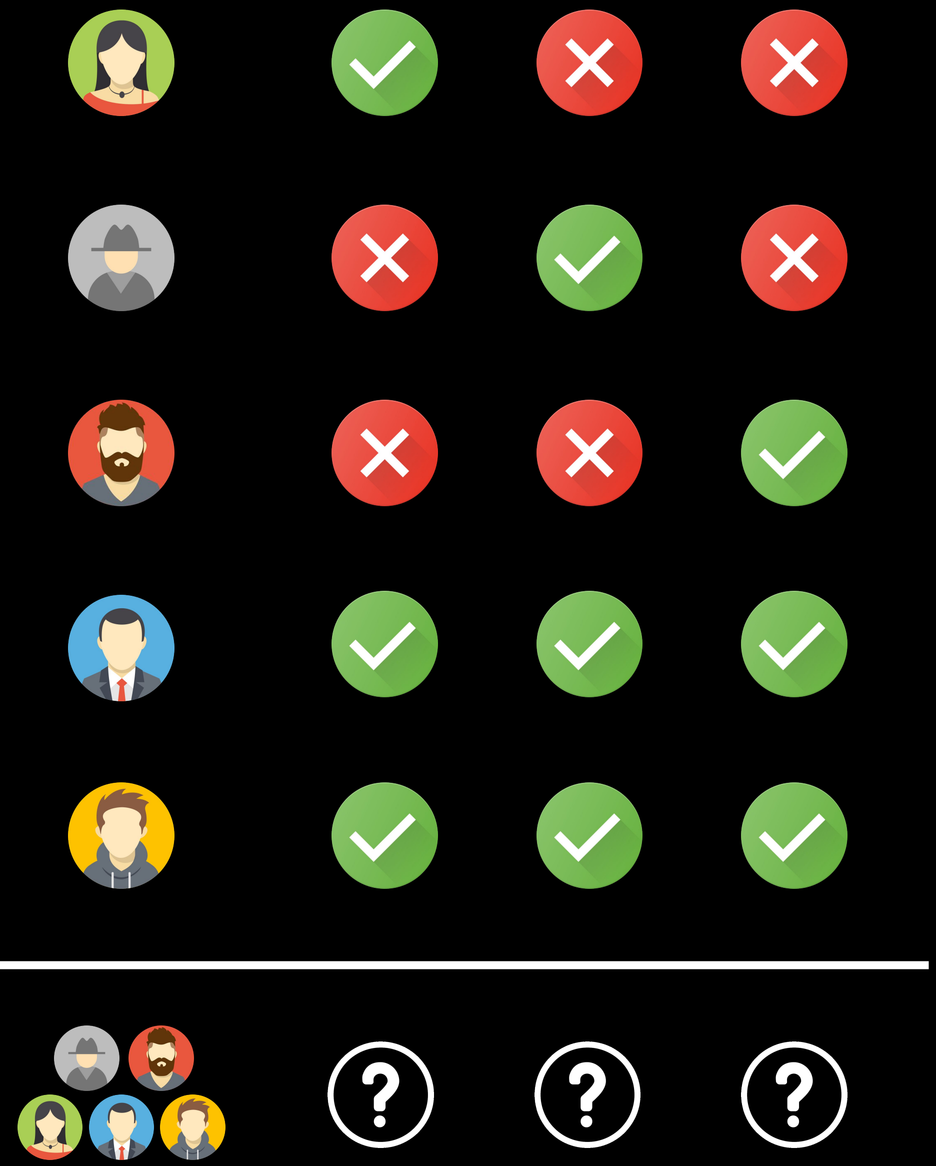

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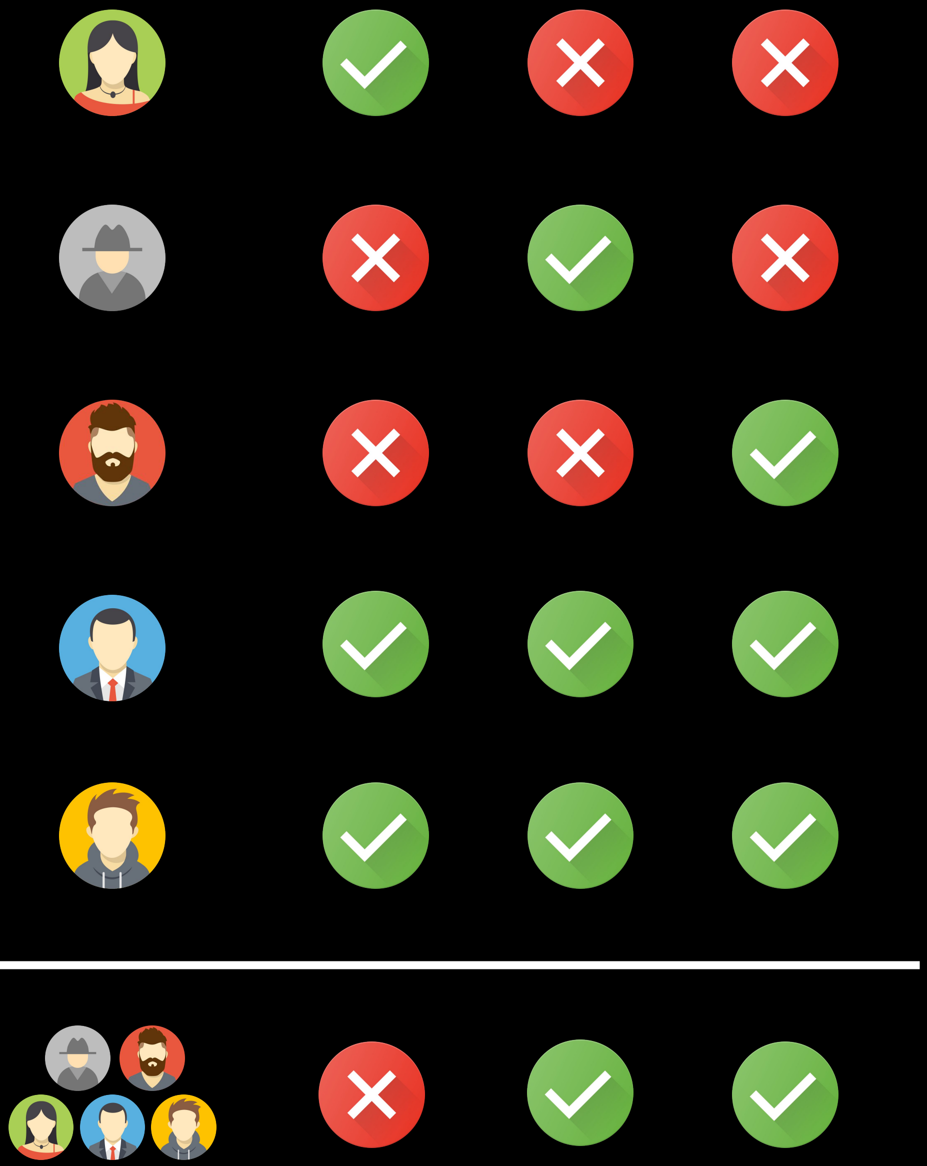

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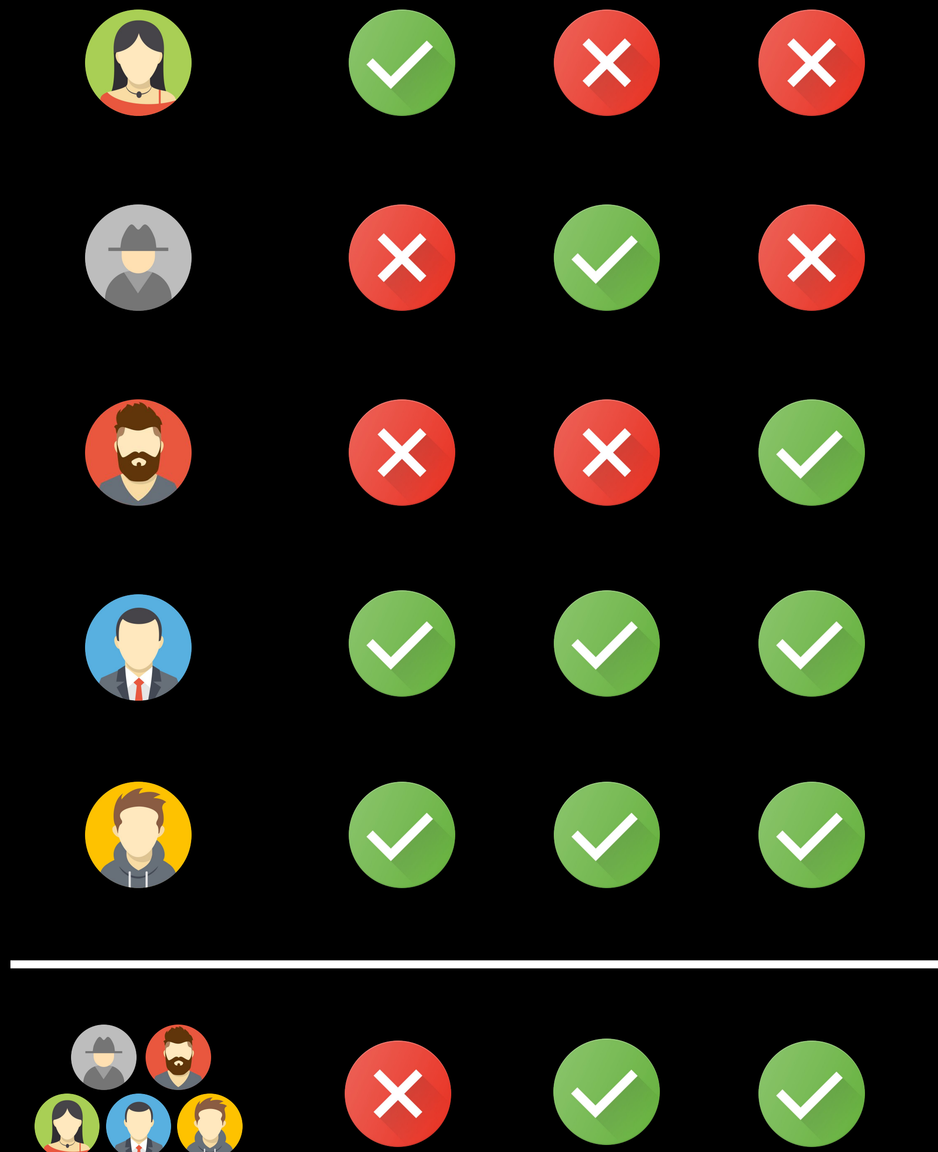

2 issues agree
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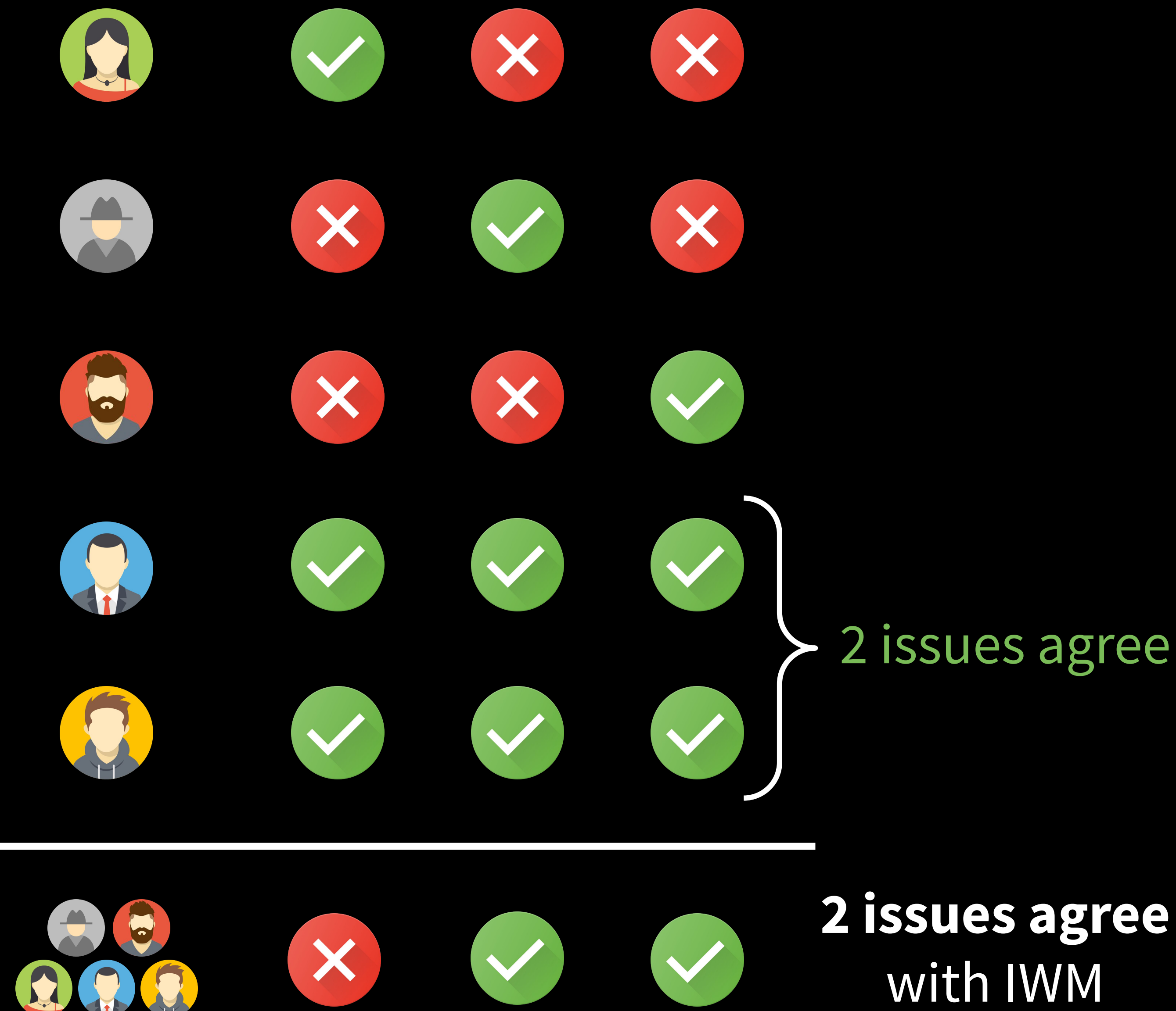

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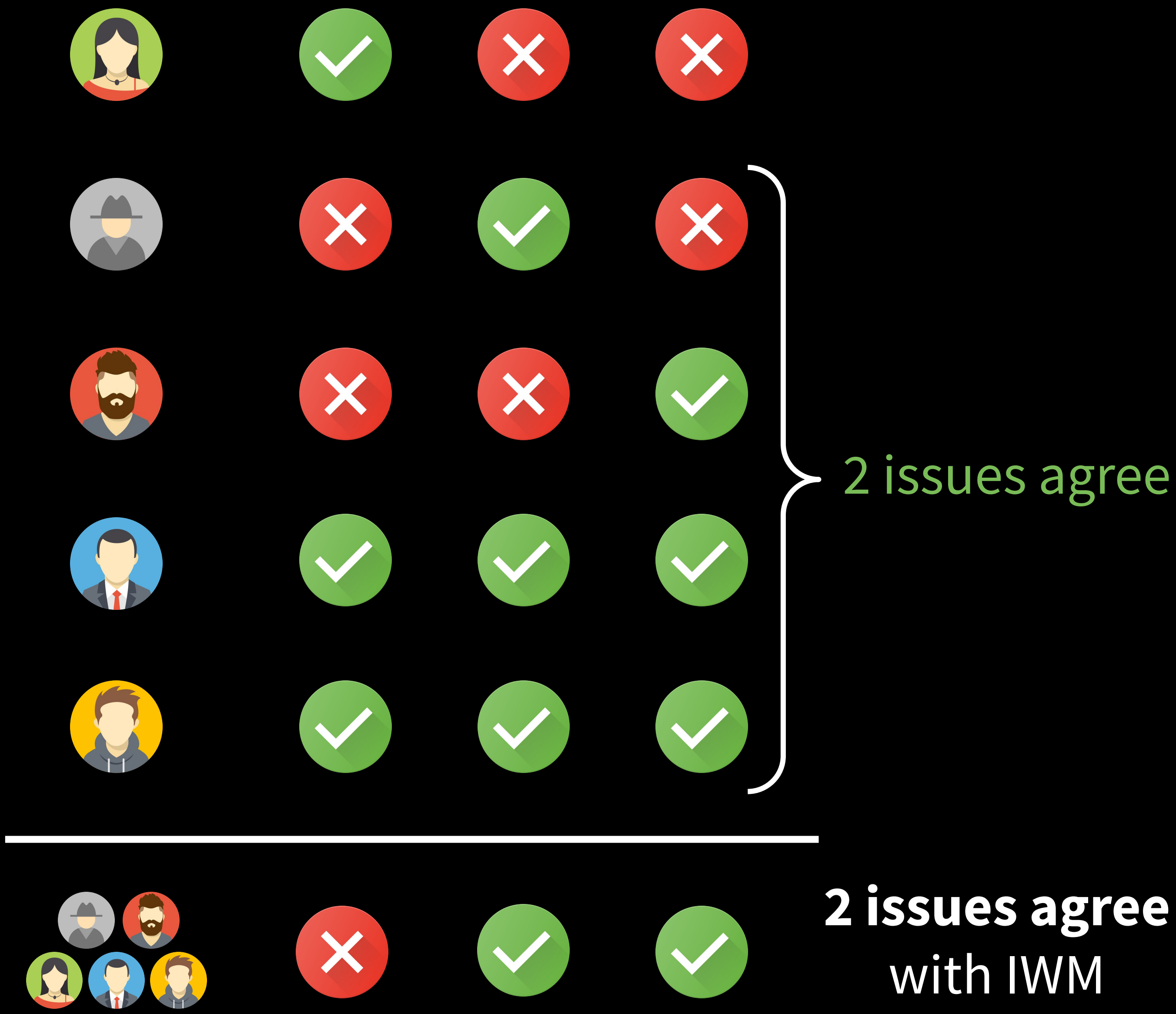

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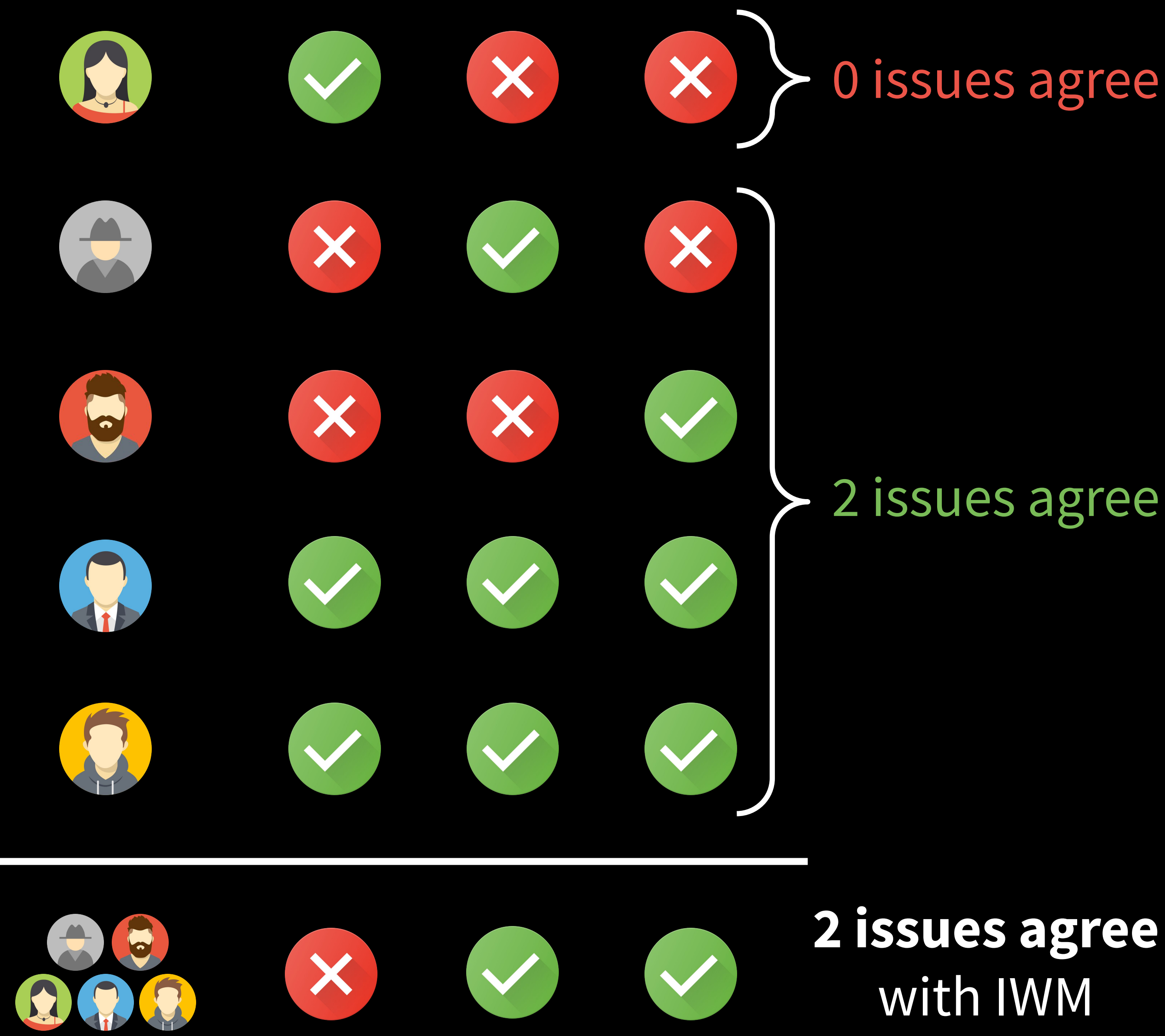

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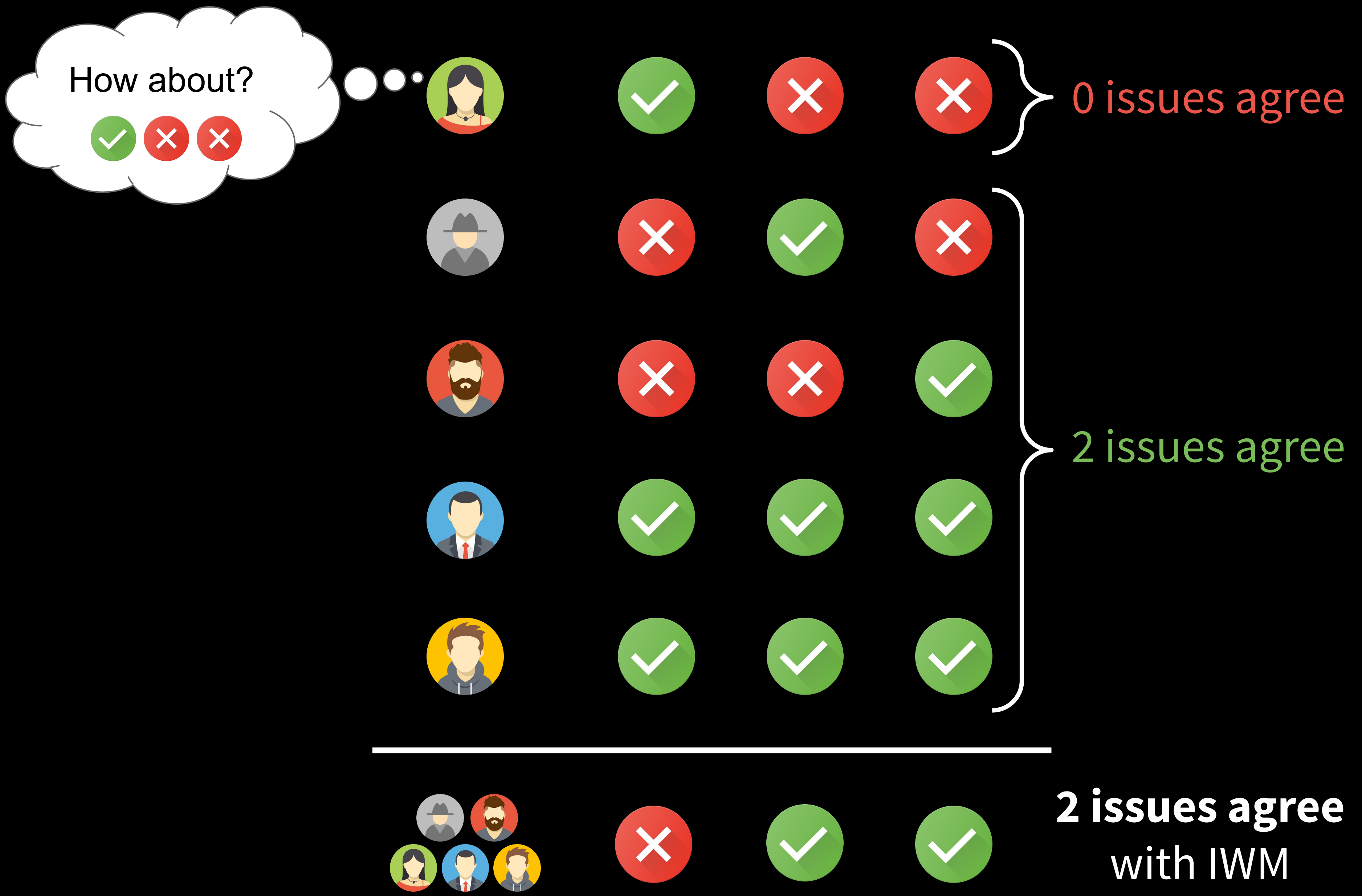

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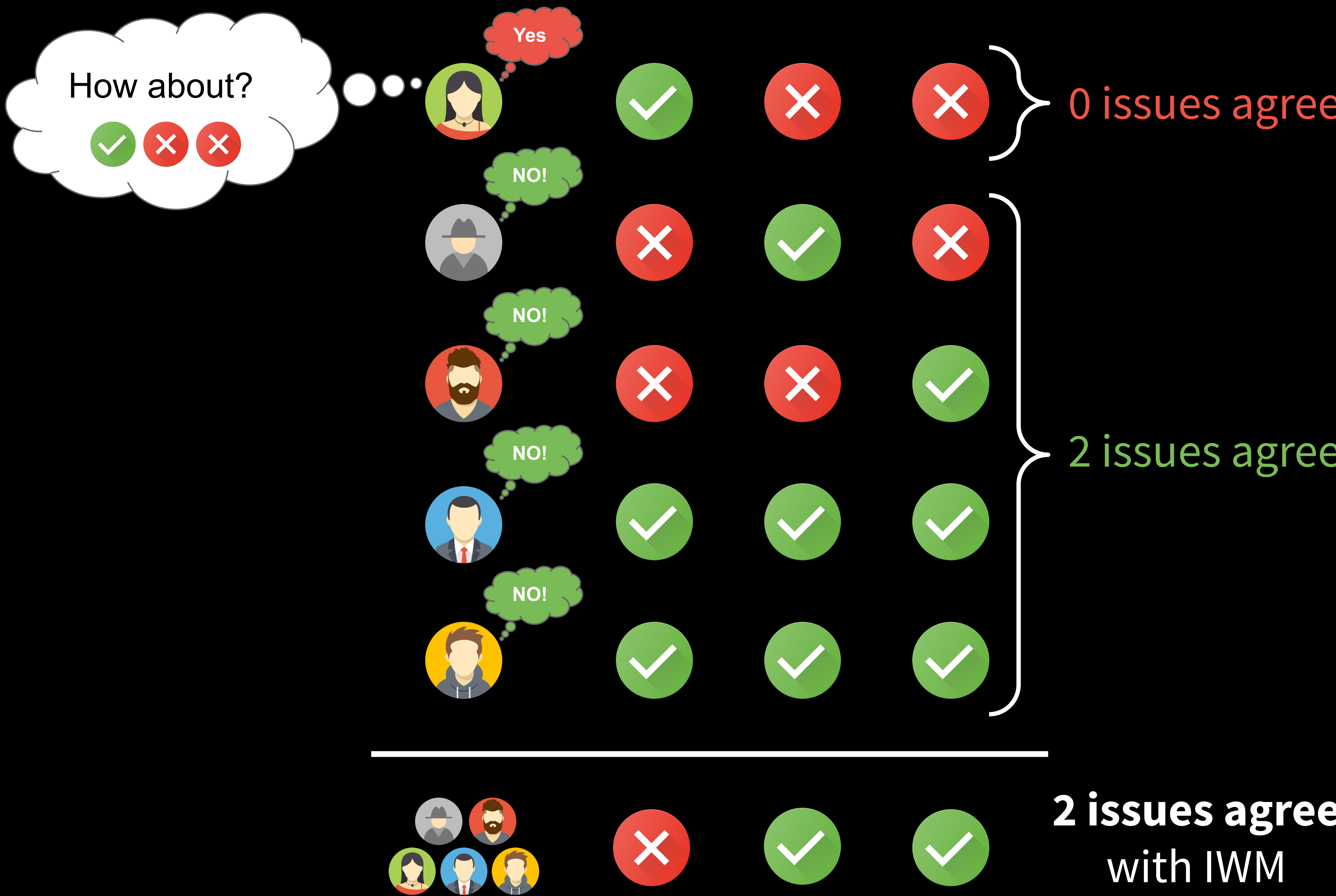

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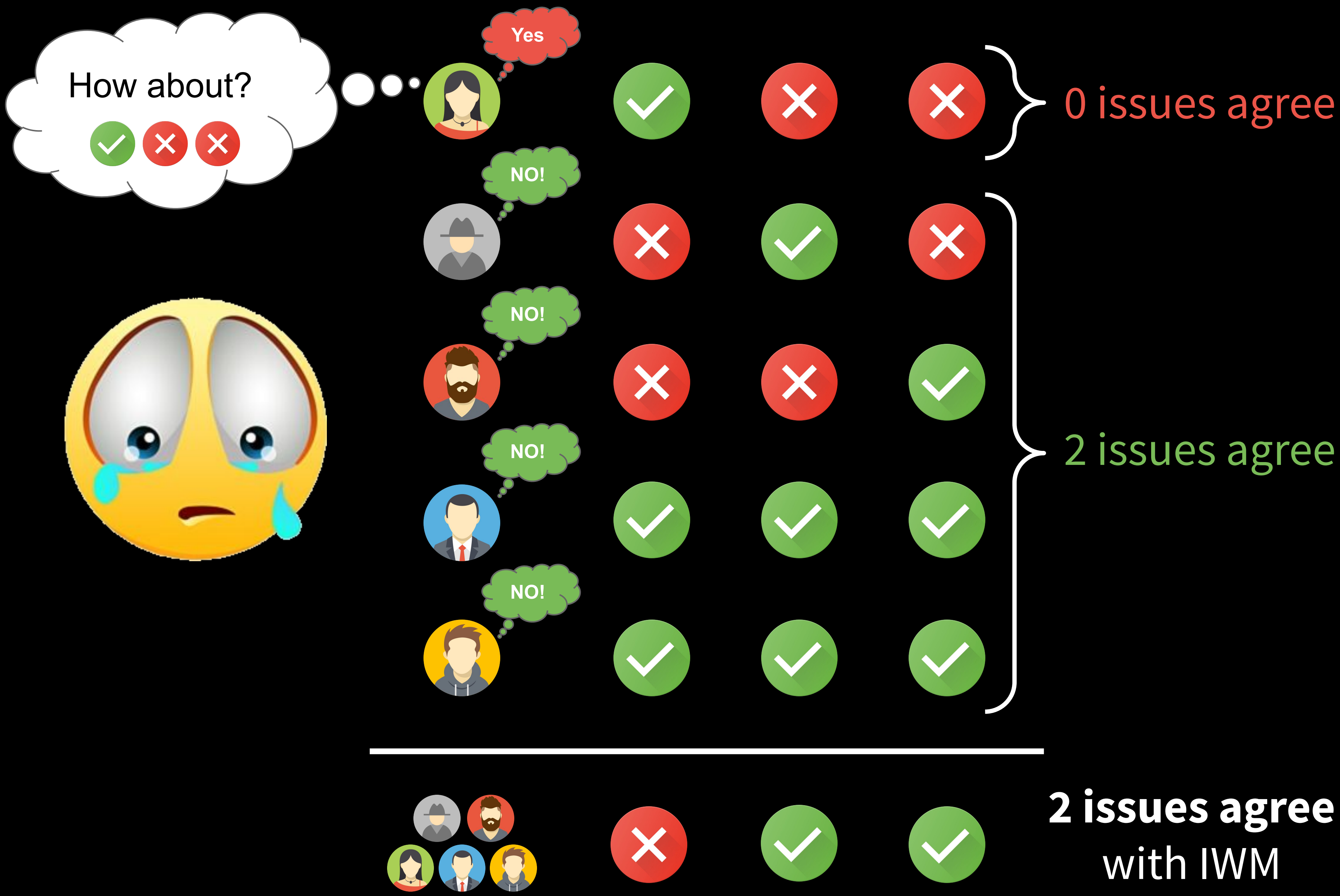

The Problem, Formally

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  [Fritsch and Wattenhofer, AAMAS'22]

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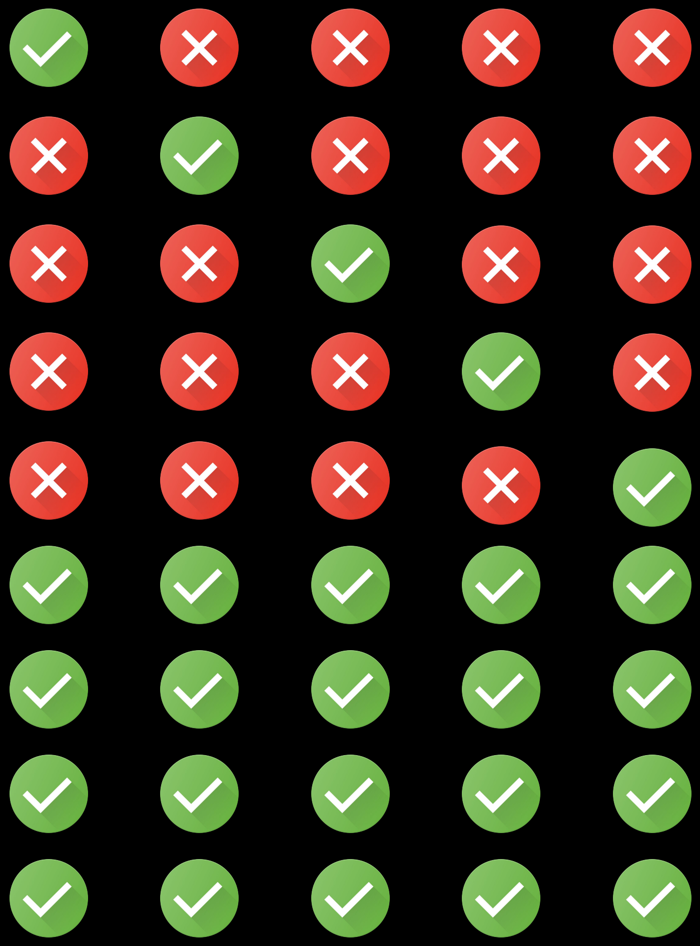

How Bad Can It Get?

v_1	✓	✗	✗	✗	✗
v_2	✗	✓	✗	✗	✗
v_3	✗	✗	✓	✗	✗
v_4	✗	✗	✗	✓	✗
v_5	✗	✗	✗	✗	✓
v_6	✓	✓	✓	✓	✓
v_7	✓	✓	✓	✓	✓
v_8	✓	✓	✓	✓	✓
v_9	✓	✓	✓	✓	✓

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N = 9

T = 5

v_1	✓	✗	✗	✗	✗
v_2	✗	✓	✗	✗	✗
v_3	✗	✗	✓	✗	✗
v_4	✗	✗	✗	✓	✗
v_5	✗	✗	✗	✗	✓
v_6	✓	✓	✓	✓	✓
v_7	✓	✓	✓	✓	✓
v_8	✓	✓	✓	✓	✓
v_9	✓	✓	✓	✓	✓

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v_2	✗	✓	✗	✗	✗
v_3	✗	✗	✓	✗	✗
v_4	✗	✗	✗	✓	✗
v_5	✗	✗	✗	✗	✓
v_6	✓	✓	✓	✓	✓
v_7	✓	✓	✓	✓	✓
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v_3	✗	✗	✓	✗	✗
v_4	✗	✗	✗	✓	✗
v_5	✗	✗	✗	✗	✓
v_6	✓	✓	✓	✓	✓
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Prop. **p**	?	?	?	?	?

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v_4	✗	✗	✗	✓	✗
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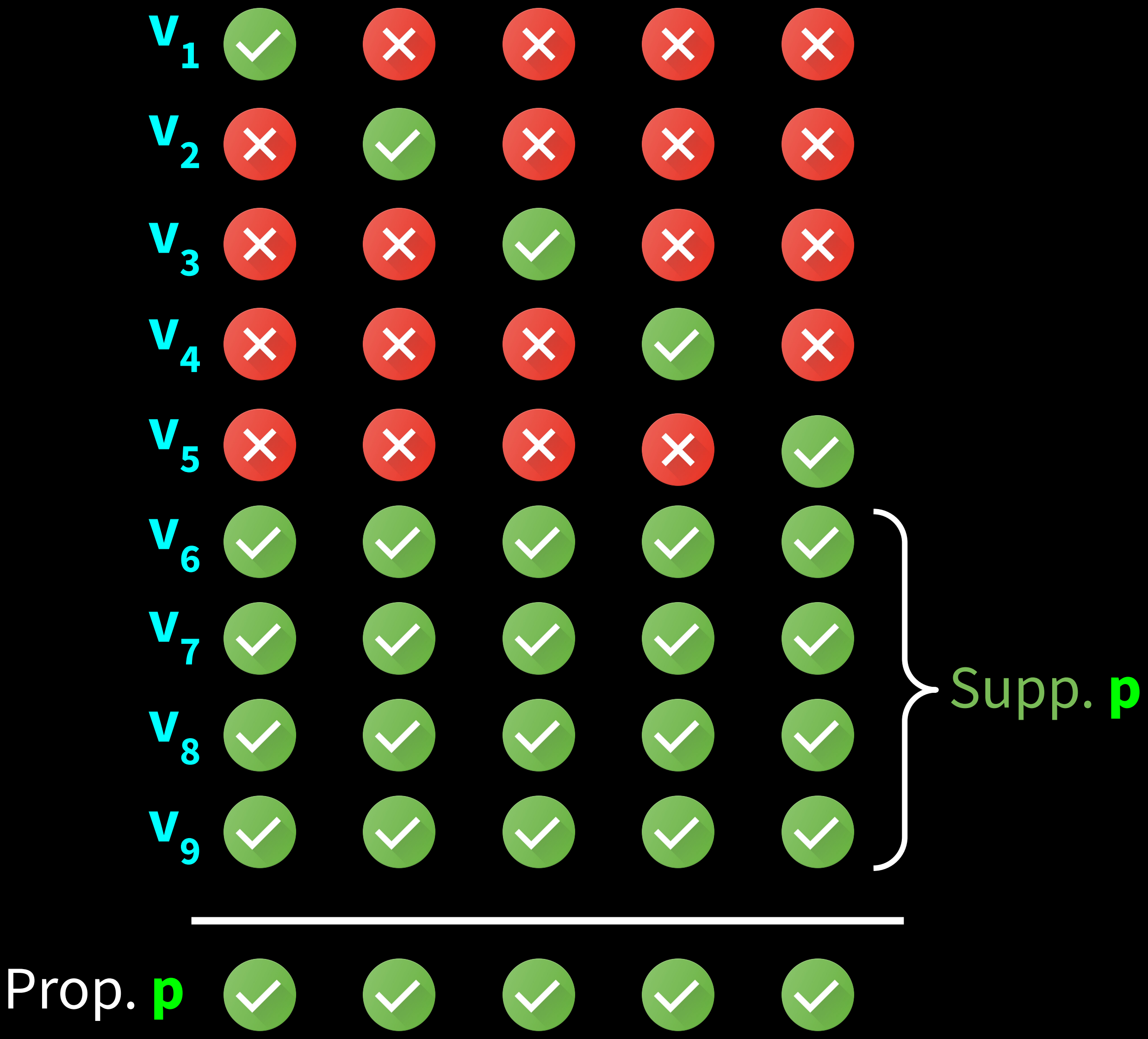

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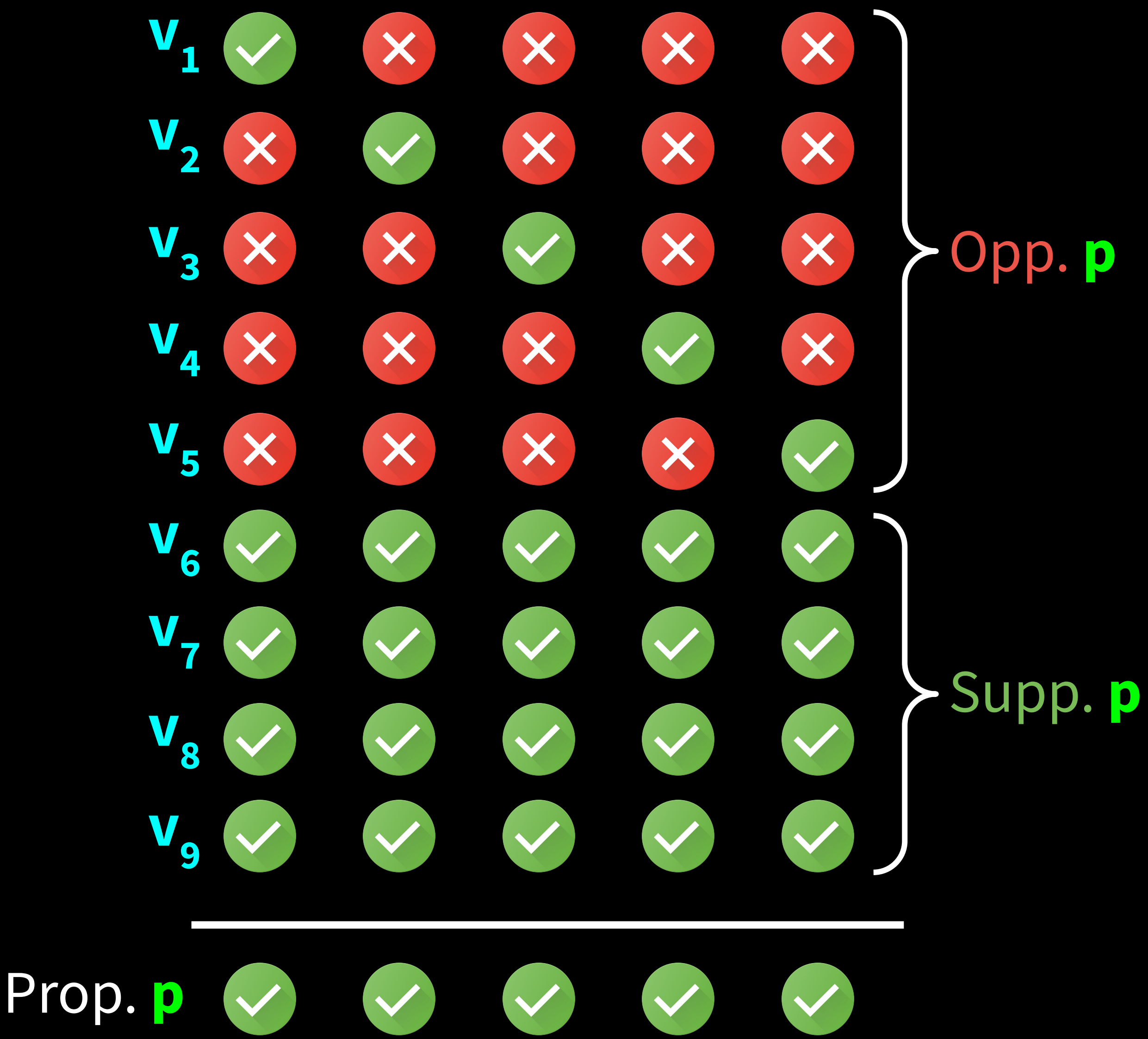

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v_1	✓	✗	✗	✗	✗	Opp. **p**
v_2	✗	✓	✗	✗	✗	
v_3	✗	✗	✓	✗	✗	
v_4	✗	✗	✗	✓	✗	
v_5	✗	✗	✗	✗	✓	
v_6	✓	✓	✓	✓	✓	Supp. **p**
v_7	✓	✓	✓	✓	✓	
v_8	✓	✓	✓	✓	✓	
v_9	✓	✓	✓	✓	✓	
Prop. **p**	✗	✓	✓	✓	✓	

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v_6	✓	✓	✓	✓	✓	
v_7	✓	✓	✓	✓	✓	
v_8	✓	✓	✓	✓	✓	
v_9	✓	✓	✓	✓	✓	
Prop. **p**	✗	✗	✓	✓	✓	

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v_7	✓	✓	✓	✓	✓	
v_8	✓	✓	✓	✓	✓	
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Prop. **p**	✗	✗	✓	✓	✓	

3 issues agree
with IWM

What Was Known

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say $T = 2k + 1$

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Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1

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Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?		…						…	

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say $T = 2k + 1$

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Always possible?		…				No	No	…	No

(by previous construction)

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Always possible?	Yes	…	Yes	Yes		No	No	…	No

(consider proposal with k + 1 ones, its opposite has k ones, one has more support)

(by previous construction)

What Was Known

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	?	No	No	…	No

What Was Known

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Always possible?	Yes	…	Yes	Yes	Yes	No	No	…	No

[Fritsch and Wattenhofer, AAMAS'22]

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Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No

[Fritsch and Wattenhofer, AAMAS'22]

- *nonconstructive*

What Was Known

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No

[Fritsch and Wattenhofer, AAMAS'22]

- *nonconstructive (and a bit magic)*

What Was Know

Agree with	0	...	k - 1	k	k +
				s	Ye

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Lemma A.1. *For $l = 0, \dots, t$,*

$$\sum_{k=\lceil t/2\rceil}^{t} (2k-t)s_{k,l} = l\binom{t-1}{\lfloor t/2\rfloor}.$$

Proof. Let

$$f(l) = \sum_{k=\lceil t/2\rceil}^{t} (2k-t)s_{k,l}.$$

Note that we use the convention that $\binom{n}{k} = 0$ for $k > n$ and $k < 0$. Hence, the upper summation bound in the formula for $s_{k,l}$ from Lemma 4.4 can be omitted. Inserting this formula yields

$$\begin{aligned} f(l) &= \sum_{k=\lceil t/2\rceil}^{t}\ \sum_{x=\lceil (k+l-\lfloor t/2\rfloor)/2\rceil}^{\infty} \binom{l}{x}\binom{t-l}{k-x}(2k-t) \\ &= \sum_{x=\lceil (l+1)/2\rceil}^{\infty} \binom{l}{x} \sum_{k=\lceil t/2\rceil}^{2x-l+\lfloor t/2\rfloor} \binom{t-l}{k-x}(2k-t) \\ &= \sum_{x=\lceil (l+1)/2\rceil}^{\infty} \binom{l}{x} \sum_{y=\lceil t/2\rceil-x}^{t-l-(\lceil t/2\rceil-x)} \binom{t-l}{y}(2y+2x-t). \end{aligned}$$

We swapped summations in the second step and substituted $y = k - x$ in the third step. Note that

$$\binom{t-l}{y}(2y+2x-t)+\binom{t-l}{t-l-y}(2(t-l-y)+2x-t) = 2\binom{t-l}{y}(2x-l).$$

Using this we further conclude

$$\begin{aligned} f(l) &= \sum_{x=\lceil (l+1)/2\rceil}^{\infty} \binom{l}{x} \sum_{y=\lceil t/2\rceil-x}^{x-l+\lfloor t/2\rfloor} \binom{t-l}{y}(2x-l) \\ &= \sum_{y=\lceil t/2\rceil-l}^{\lfloor t/2\rfloor} \binom{t-l}{y} \sum_{x=\max(\lceil t/2\rceil-y,\,y+l-\lfloor t/2\rfloor)}^{\infty} \binom{l}{x}(2x-l). \end{aligned}$$

In the second step, we switched the summation again. Now let $x_0 = \max(\lceil t/2\rceil - y, y + l - \lfloor t/2\rfloor)$. Then

$$\begin{aligned} \sum_{x=x_0}^{\infty} \binom{l}{x}(2x-l) &= \sum_{x=x_0}^{\infty} x\binom{l}{x} - (l-x)\binom{l}{x} \\ &= \sum_{x=x_0}^{\infty} l\binom{l-1}{x-1} - l\binom{l-1}{x} = l\binom{l-1}{x_0-1}. \end{aligned}$$

Furthermore, the definition of x_0 implies

$$\binom{l-1}{\lfloor t/2\rfloor - y} = \binom{l-1}{y+l-\lceil t/2\rceil} = \binom{l-1}{x_0-1}.$$

With the previous two properties, we establish

$$\begin{aligned} f(l) &= \sum_{y=\lceil t/2\rceil-l}^{\lfloor t/2\rfloor} \binom{t-l}{y} l \binom{l-1}{\lfloor t/2\rfloor - y} \\ &= l\sum_{z=0}^{l-1} \binom{t-l}{\lfloor t/2\rfloor - z}\binom{l-1}{z} = l\binom{t-1}{\lfloor t/2\rfloor}. \end{aligned}$$

Here we substituted $z = \lfloor t/2\rfloor - y$, and the last step follows from the well-known combinatorial identity $\binom{n}{k} = \sum_j \binom{i}{j}\binom{n-i}{k-j}$. $\square$

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Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No

[Fritsch and Wattenhofer, AAMAS'22]

- *nonconstructive*

What Is New

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No

This paper

- *probabilistic → derandomization*

What Is New

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No
Compute (or report “none”)	Poly	…	Poly	Poly	Poly			…	

What Is New

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No
Compute (or report “none”)	Poly	…	Poly	Poly	Poly	**NP-h**		…	

This paper

What Is New

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No
Compute (or report "none")	Poly	…	Poly	Poly	Poly	**NP-h**	Np-h	…	

This paper

What Is New

say $T = 2k + 1$

Agree with IWM in ≥ issues	0	…	k - 1	k	k + 1	k + 2	k + 3	…	2k + 1
Always possible?	Yes	…	Yes	Yes	**Yes**	No	No	…	No
Compute (or report "none")	Poly	…	Poly	Poly	Poly	**NP-h**	Np-h	…	Poly

This paper *Trivial*