Distributed Complexity Theory

Roger Wattenhofer

Thanks to my (former) students…


Fabian Kuhn


Thomas Moscibroda


Yvonne Anne Pignolet

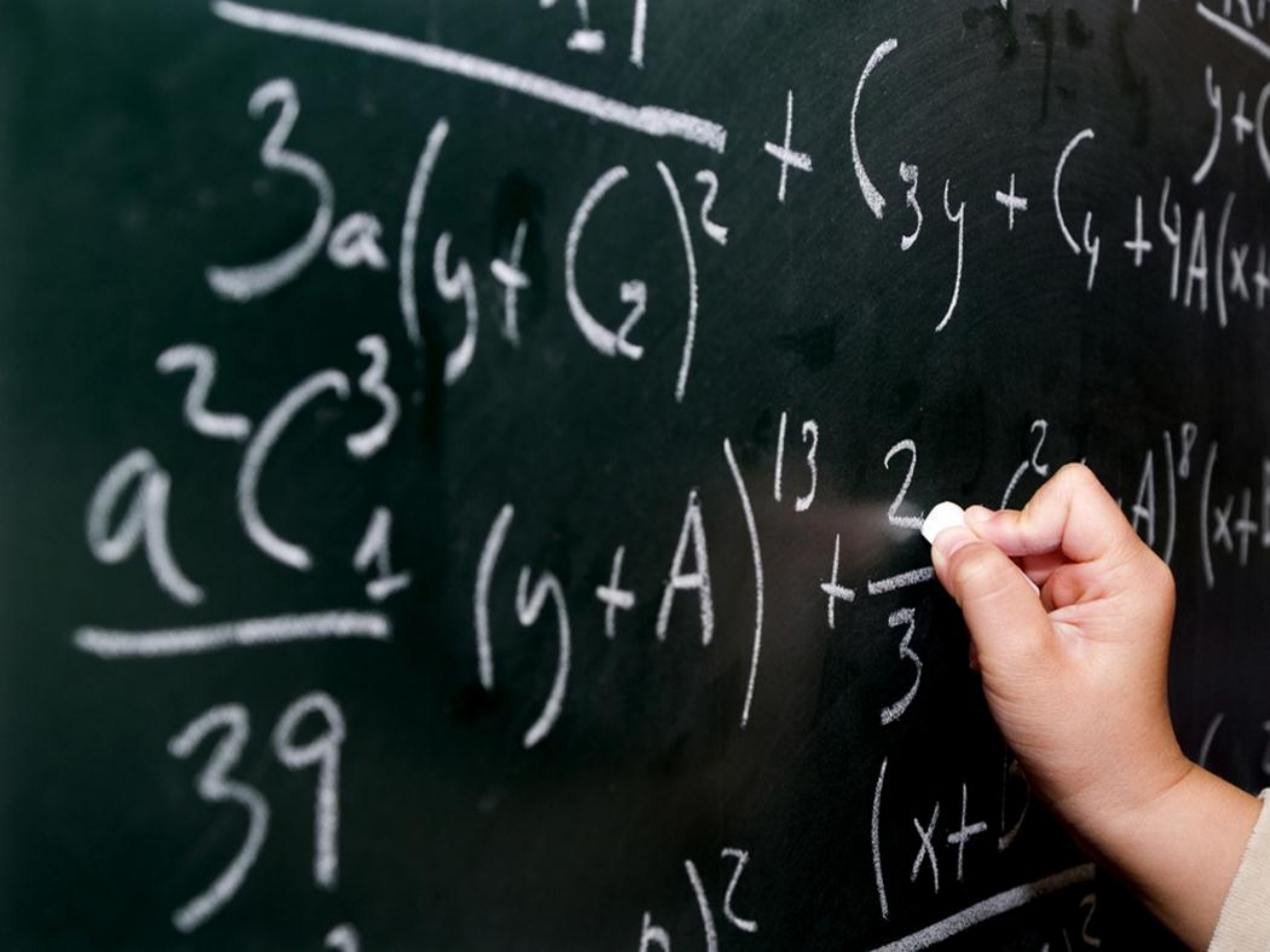

Christoph Lenzen

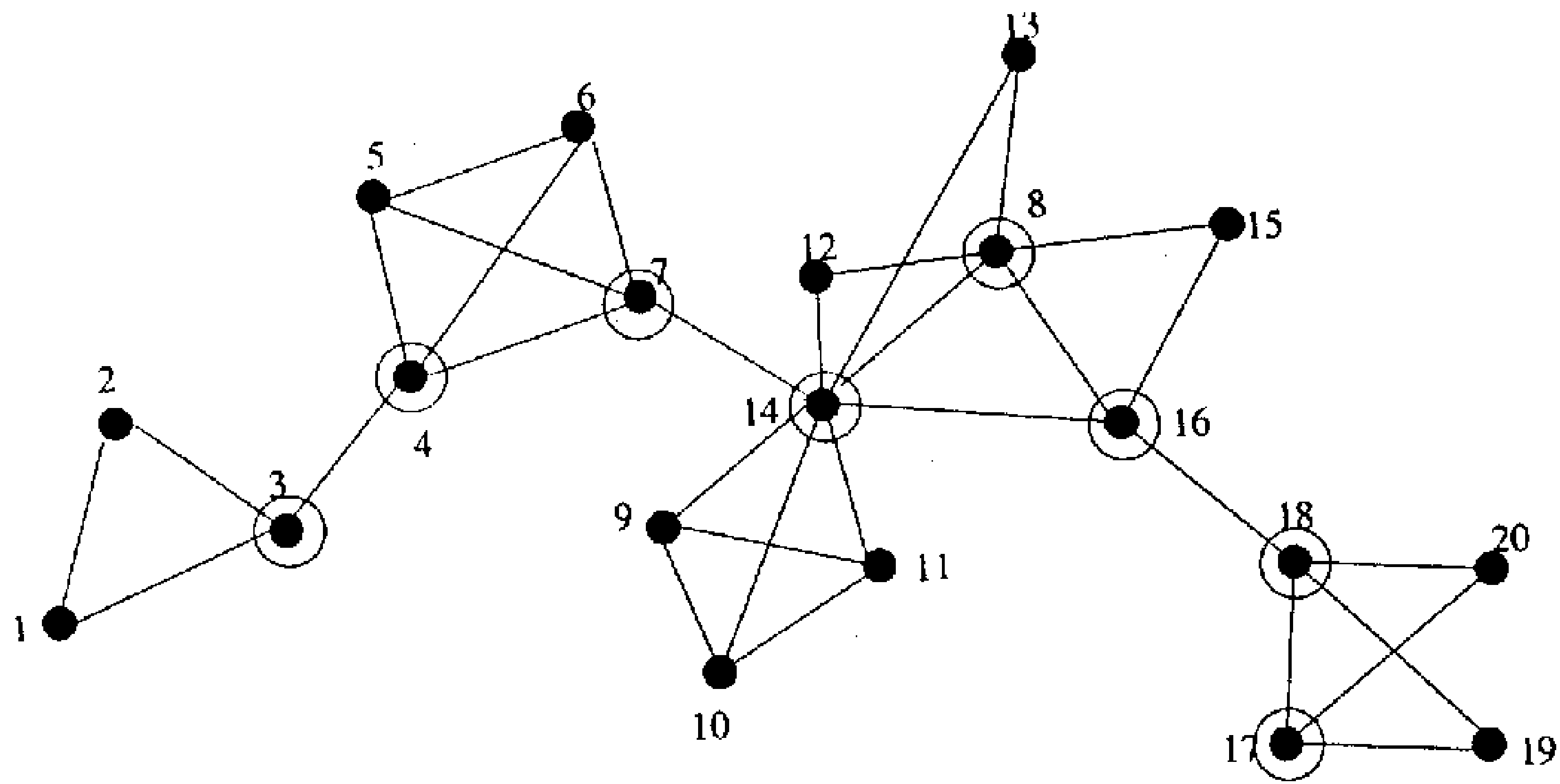

Johannes Schneider


Stephan Holzer

… and colleagues for pushing the area!

“Innovation”?





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Distributed Complexity Theory

Complexity Theory

Can a Computer Solve Problem *P* in Time *t*?

Distributed ↓ Complexity Theory

Can a ~~Computer~~ Network Solve Problem *P* in Time *t*?

Distributed

Complexity Theory

Can a ~~Computer~~ Network Solve Problem *P* in Time *t*?

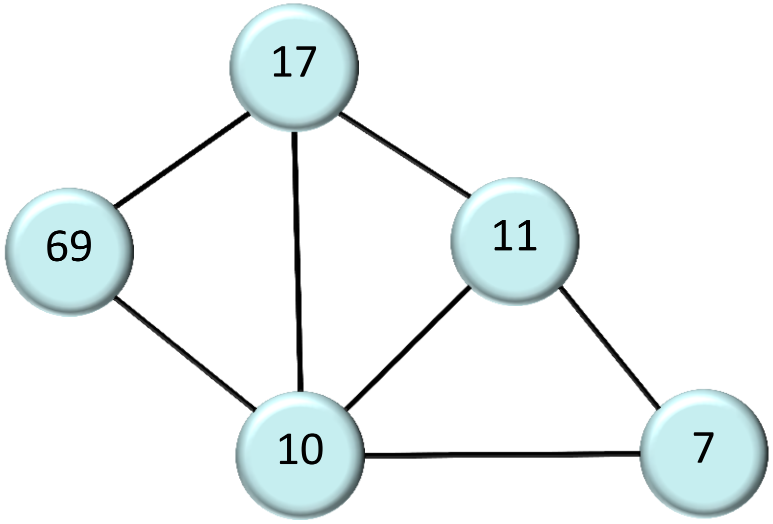
P: Approximation of Minimum Dominating Set

t: Arbitrary parameter, e.g. constant

[Kuhn, W, PODC 2003]

Distributed (Message-Passing) Algorithms

- Nodes are agents with unique ID's that can communicate with neighbors by sending messages. In each synchronous round, every node can send a (different) message to each neighbor.



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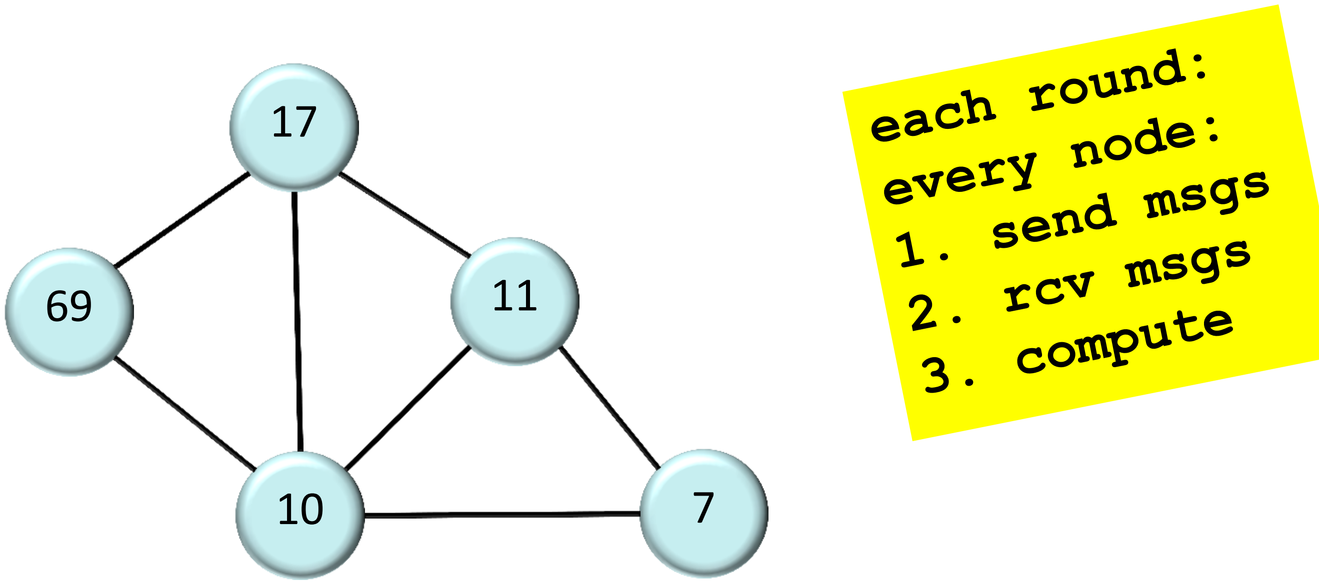
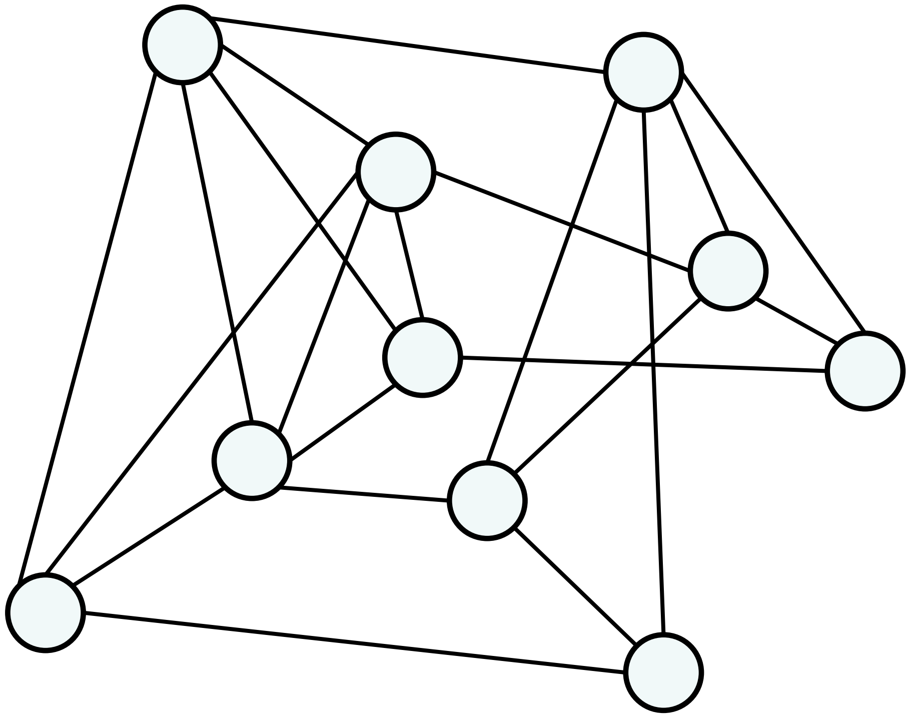


- Distributed (Time) Complexity: How many rounds does problem take?

An Example

```
each round:
every node:
1. send msgs
2. rcv msgs
3. compute
```

How Many Nodes in Network?



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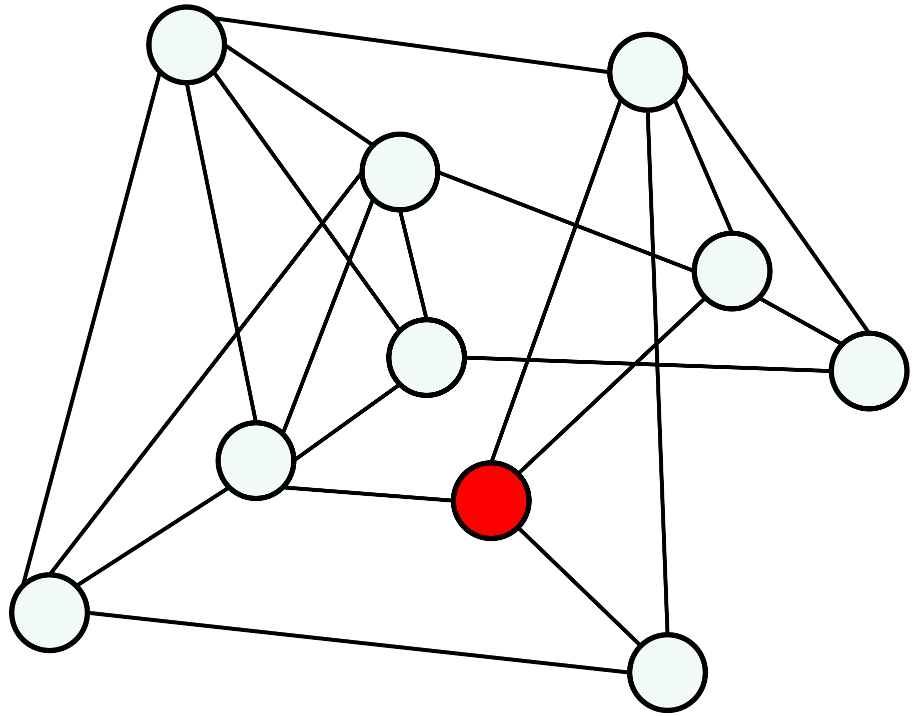

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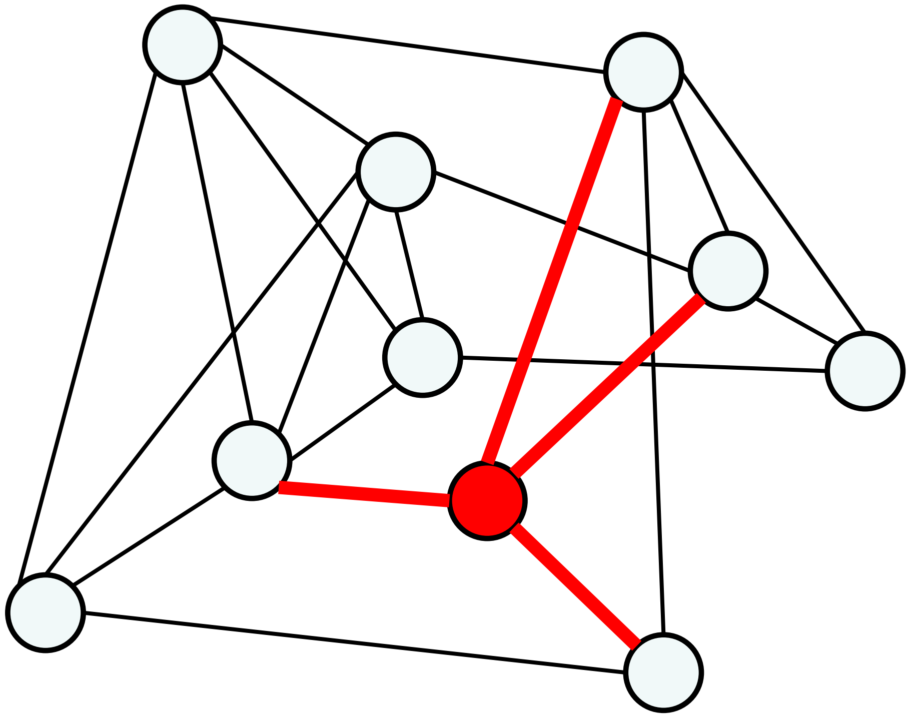

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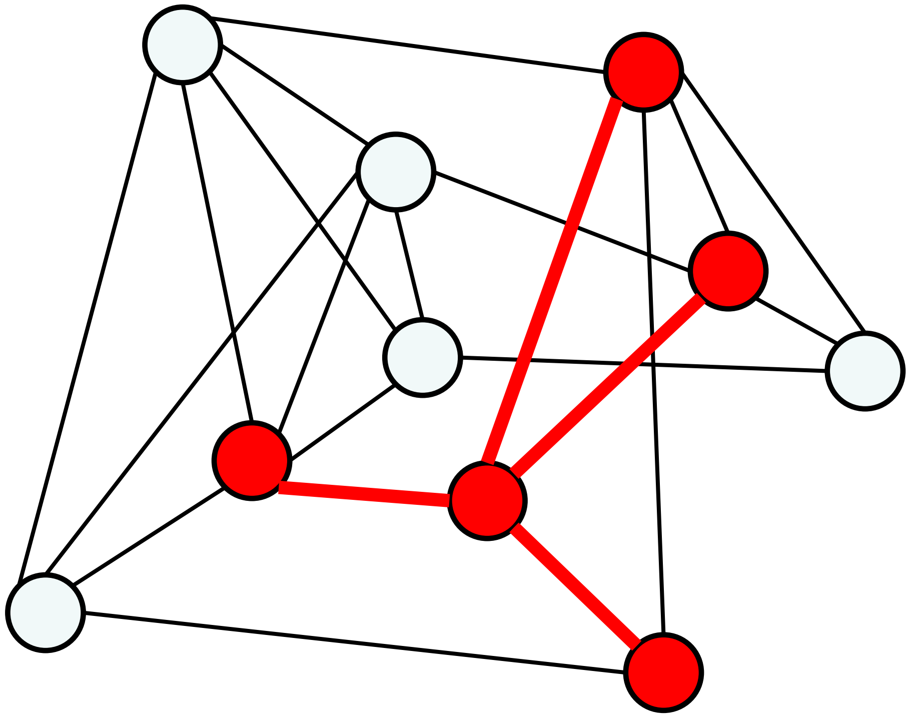

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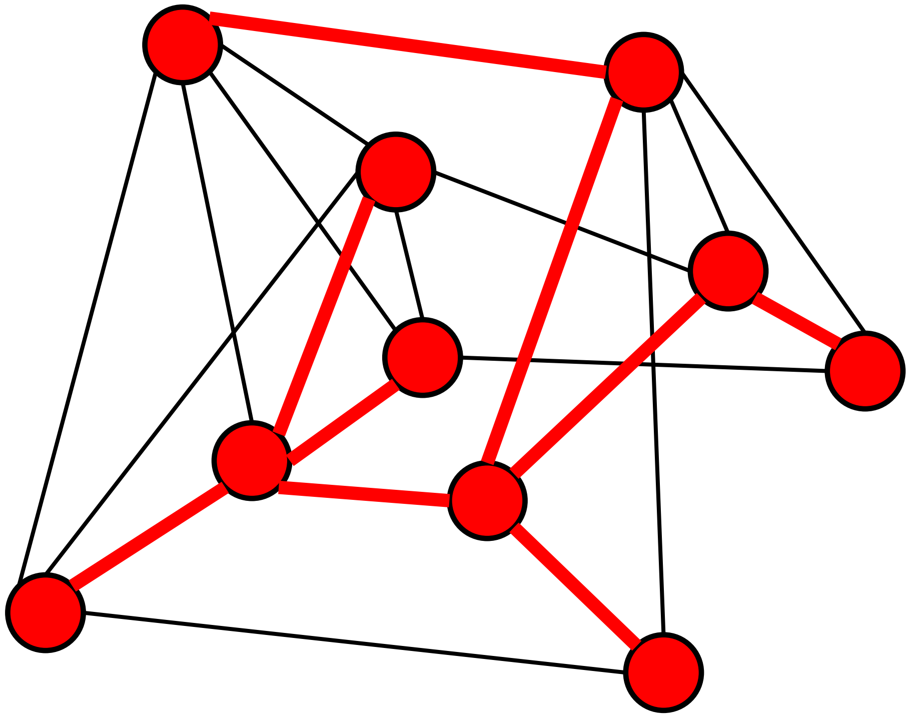

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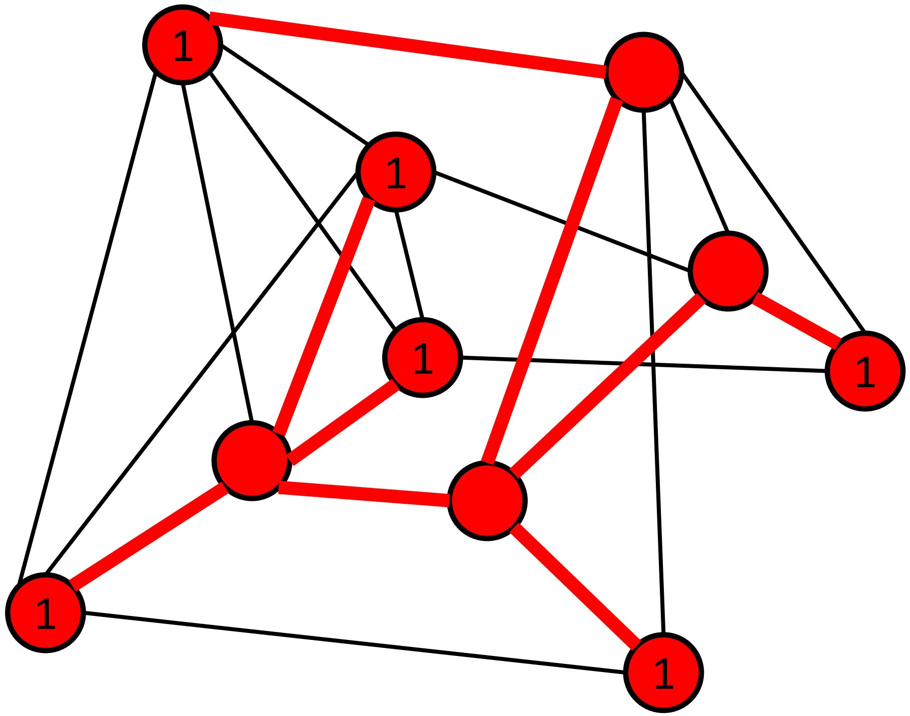

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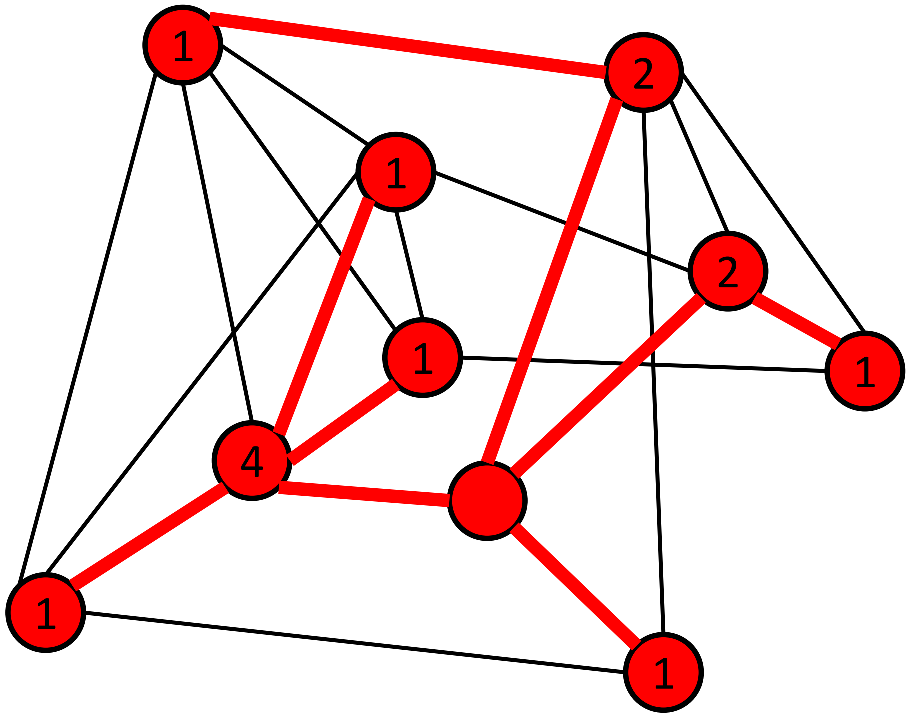

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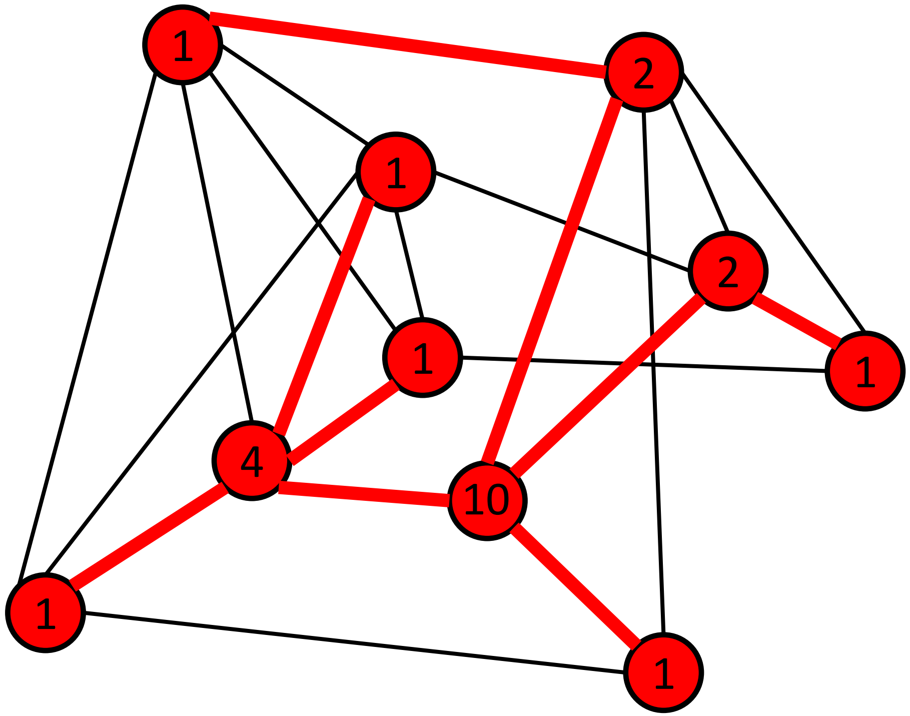
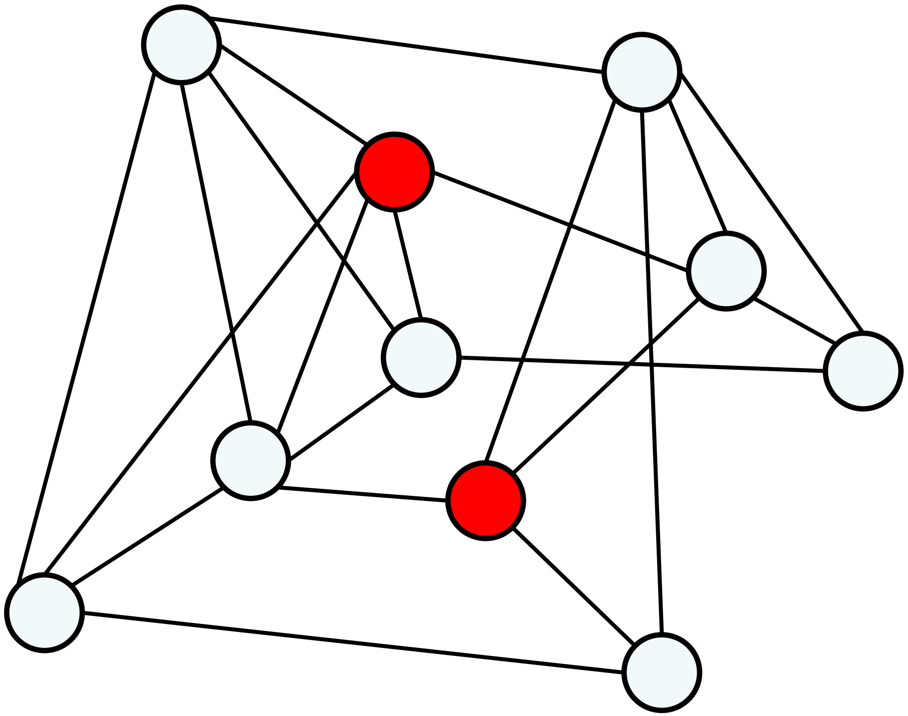


With a simple flooding/echo process, a network can find the number of nodes in time $O(D)$, where D is the diameter (size) of the network.

Diameter of Network?



- Distance between two nodes = Number of hops of shortest path

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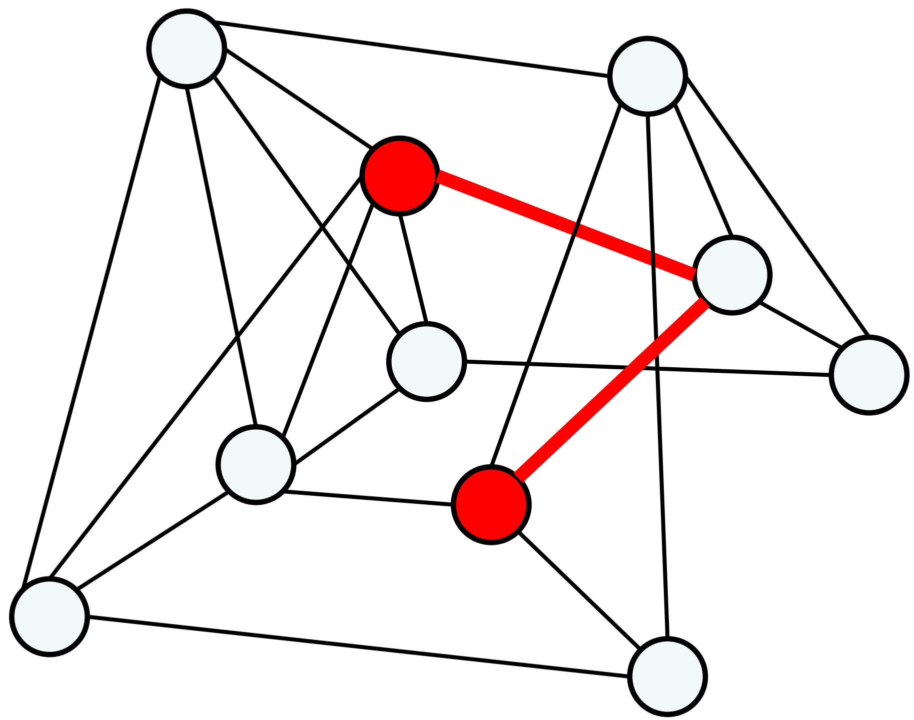

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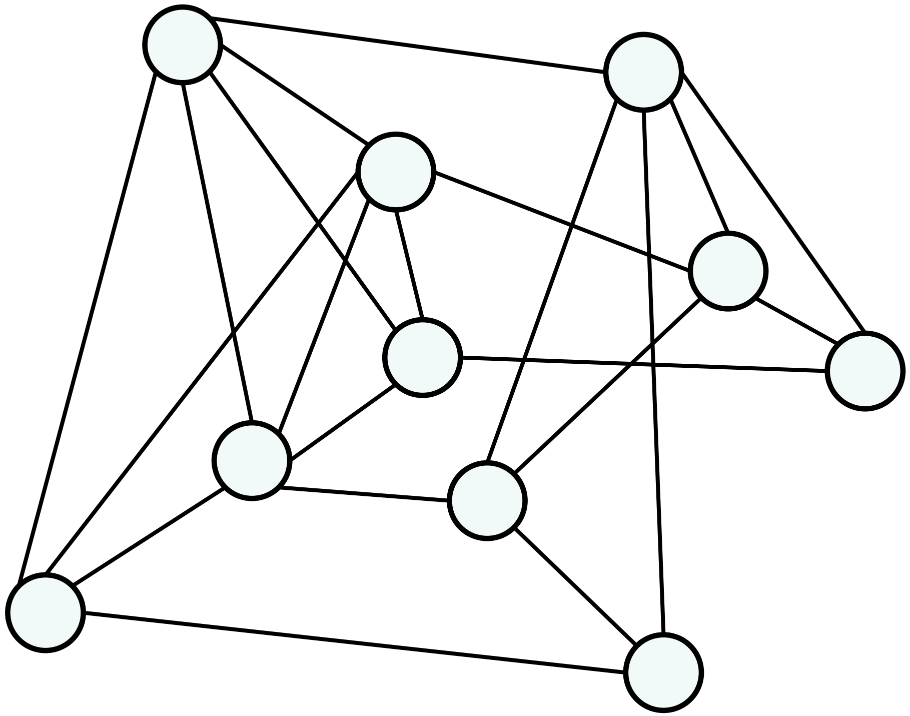
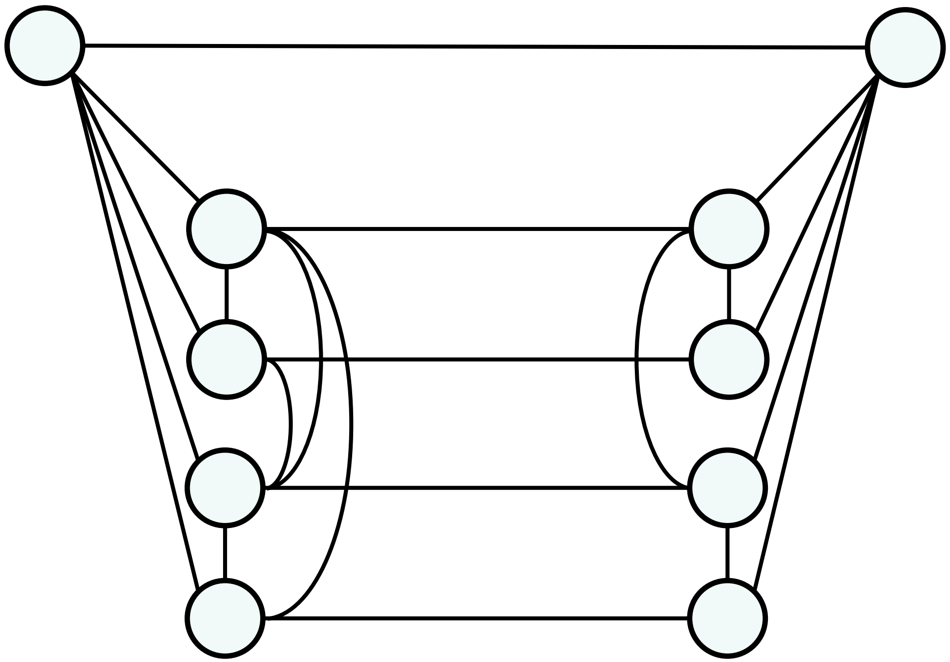


- Distance between two nodes = Number of hops of shortest path
- Diameter of network = Maximum distance, between any two nodes

Diameter of Network?



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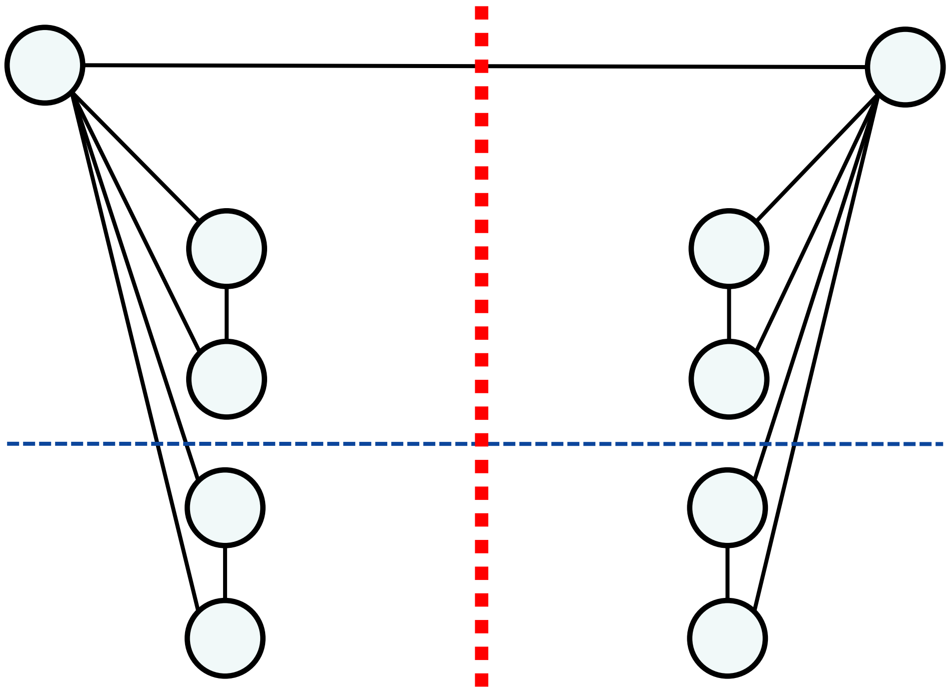

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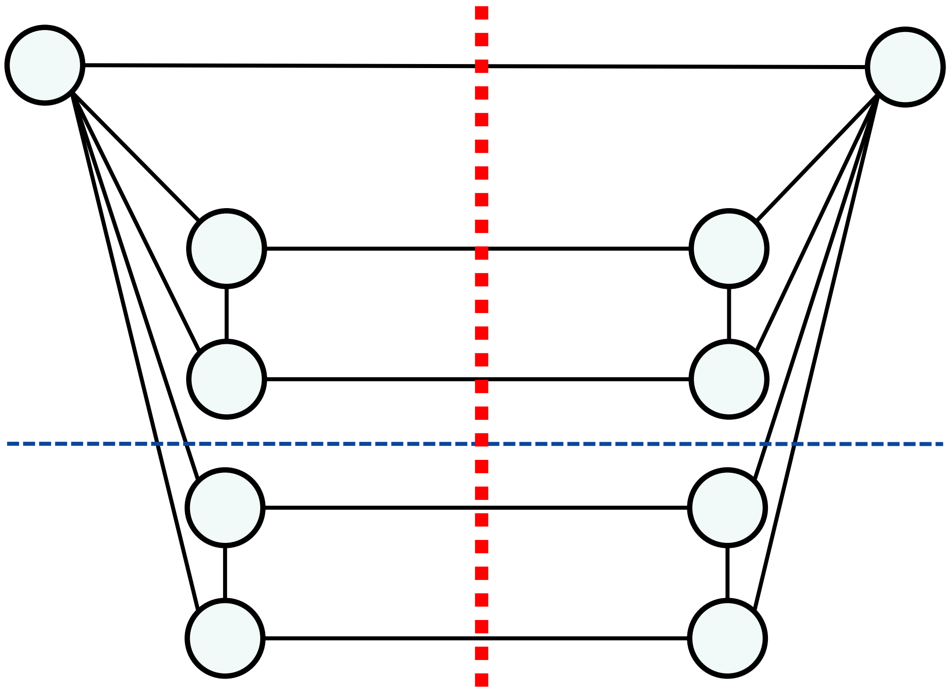

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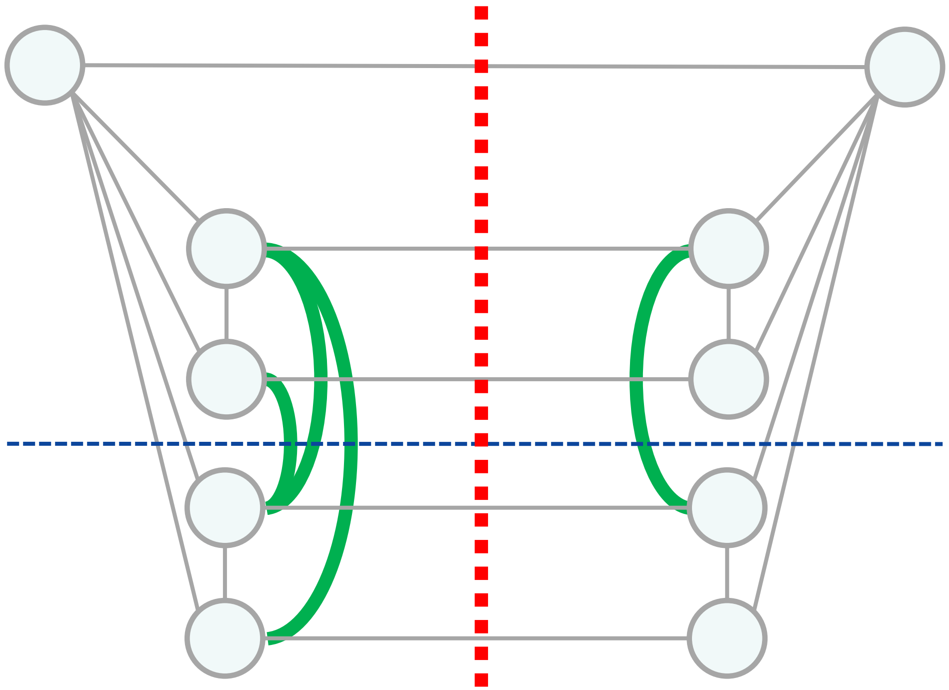

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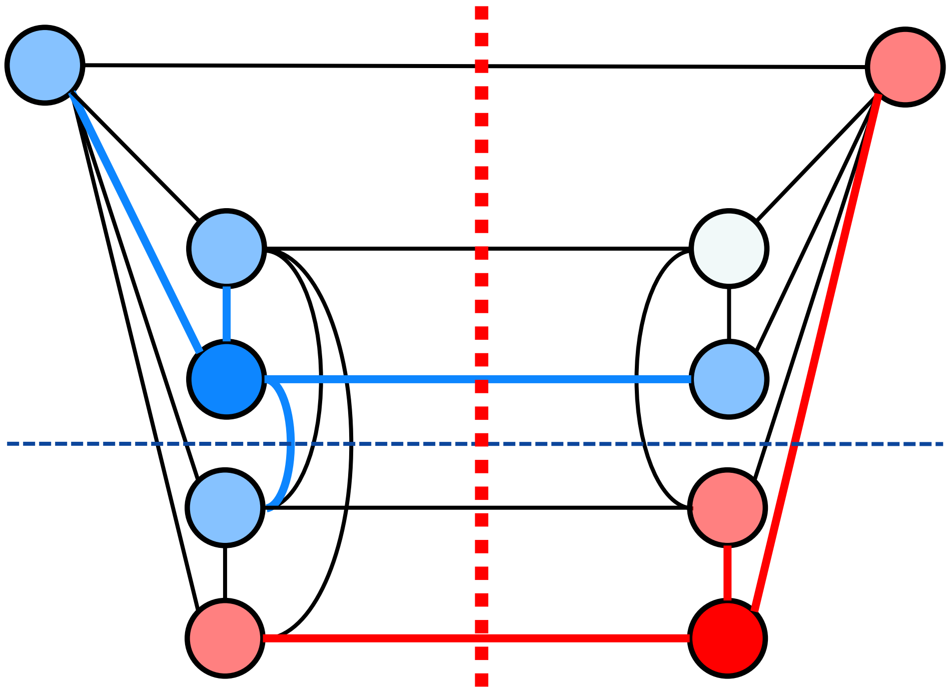

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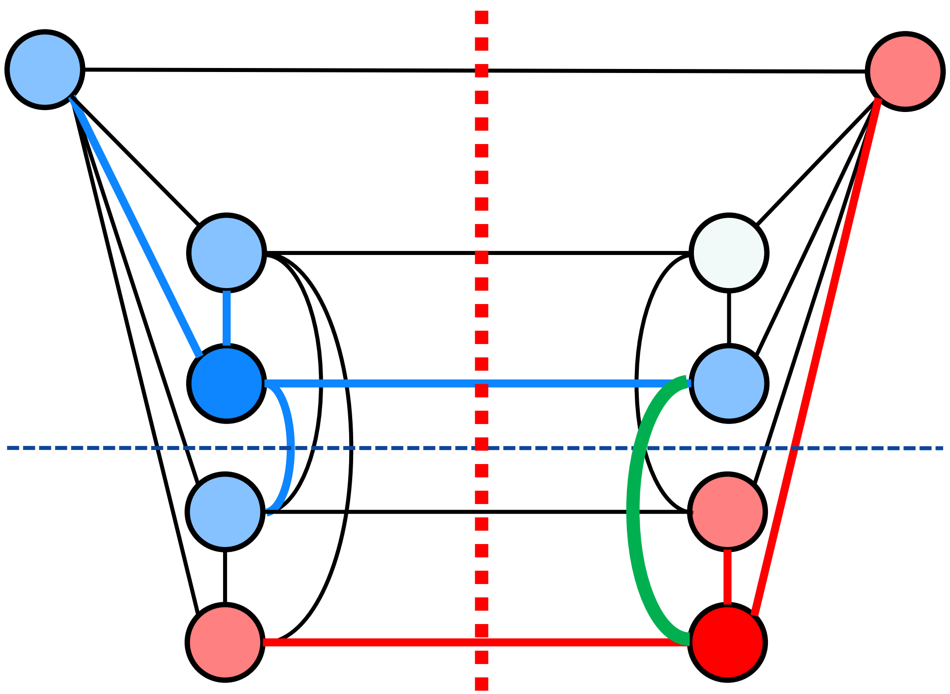

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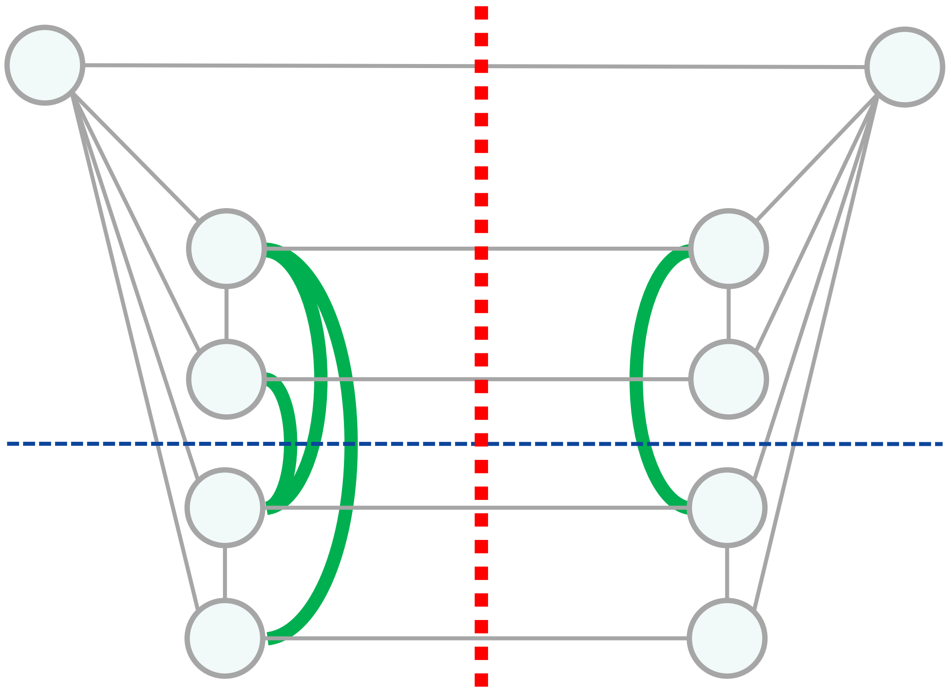
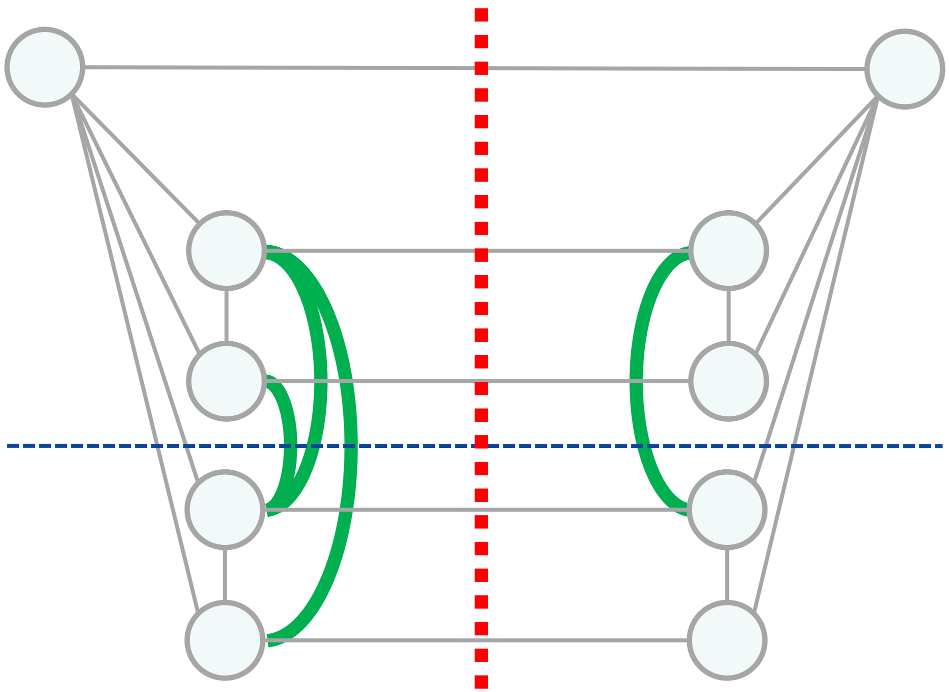


Networks Cannot Compute Their Diameter in Sublinear Time!

(even if diameter is just a small constant)



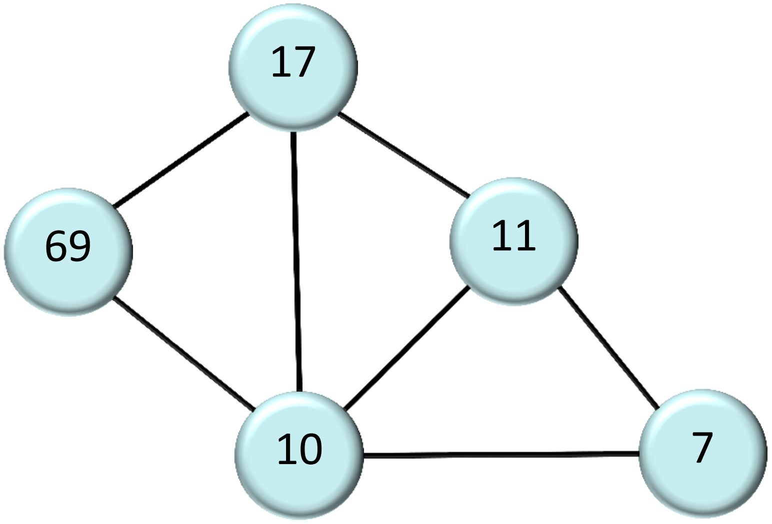
Pair of rows connected neither left nor right? Communication complexity:
Transmit $\Theta(n^2)$ information over $O(n)$ edges → $\Omega(n)$ time!

[Frischknecht, Holzer, W, 2012]

What about a “local” task?

Example: Minimum Vertex Cover (MVC)

- Given a network with *n* nodes, nodes have unique IDs.
- Find a Minimum Vertex Cover (MVC)
  - a minimum set of nodes such that all edges are adjacent to node in MVC



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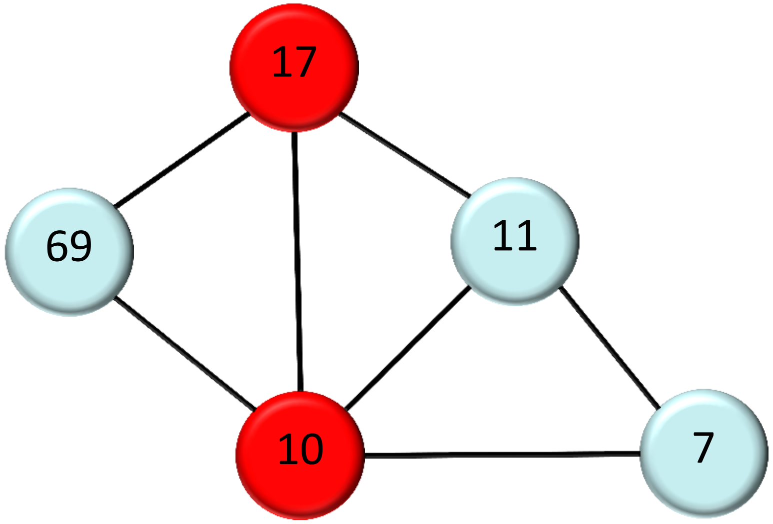

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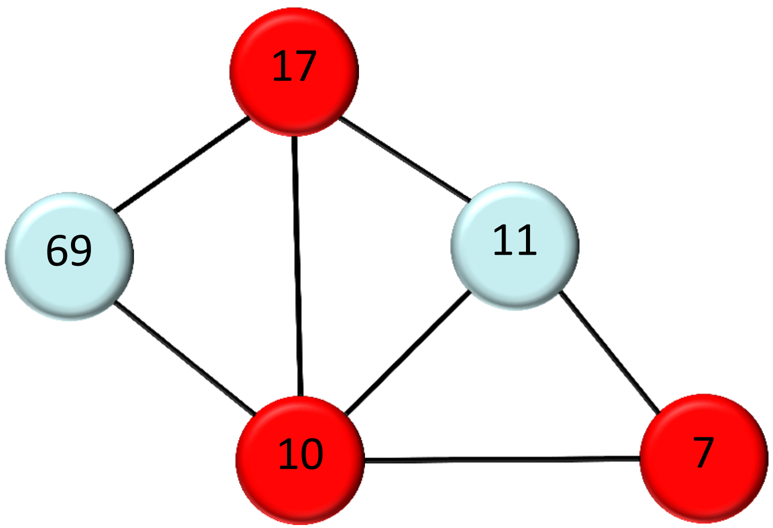

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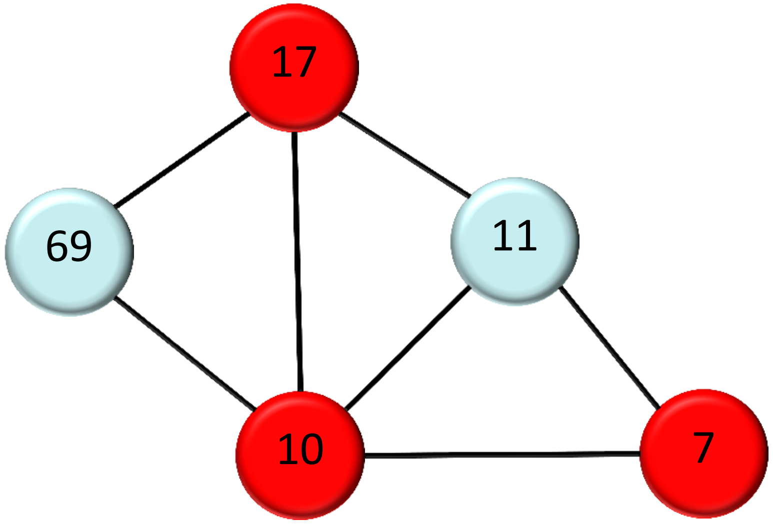
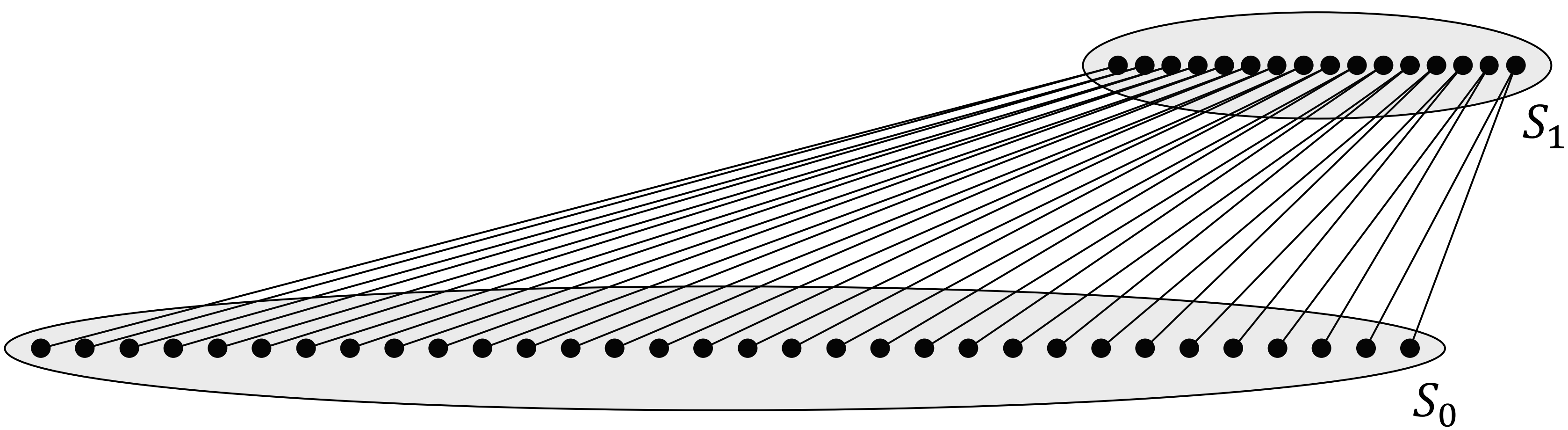


- Various simple (non-distributed) 2-approximations exist!
- What about distributed algorithms?!?

Finding the MVC (by Distributed Algorithm)

- Given the following bipartite graph with $|S_0| = \delta\ |S_1|$
- The MVC is just all the nodes in S_1
- Distributed Algorithm…



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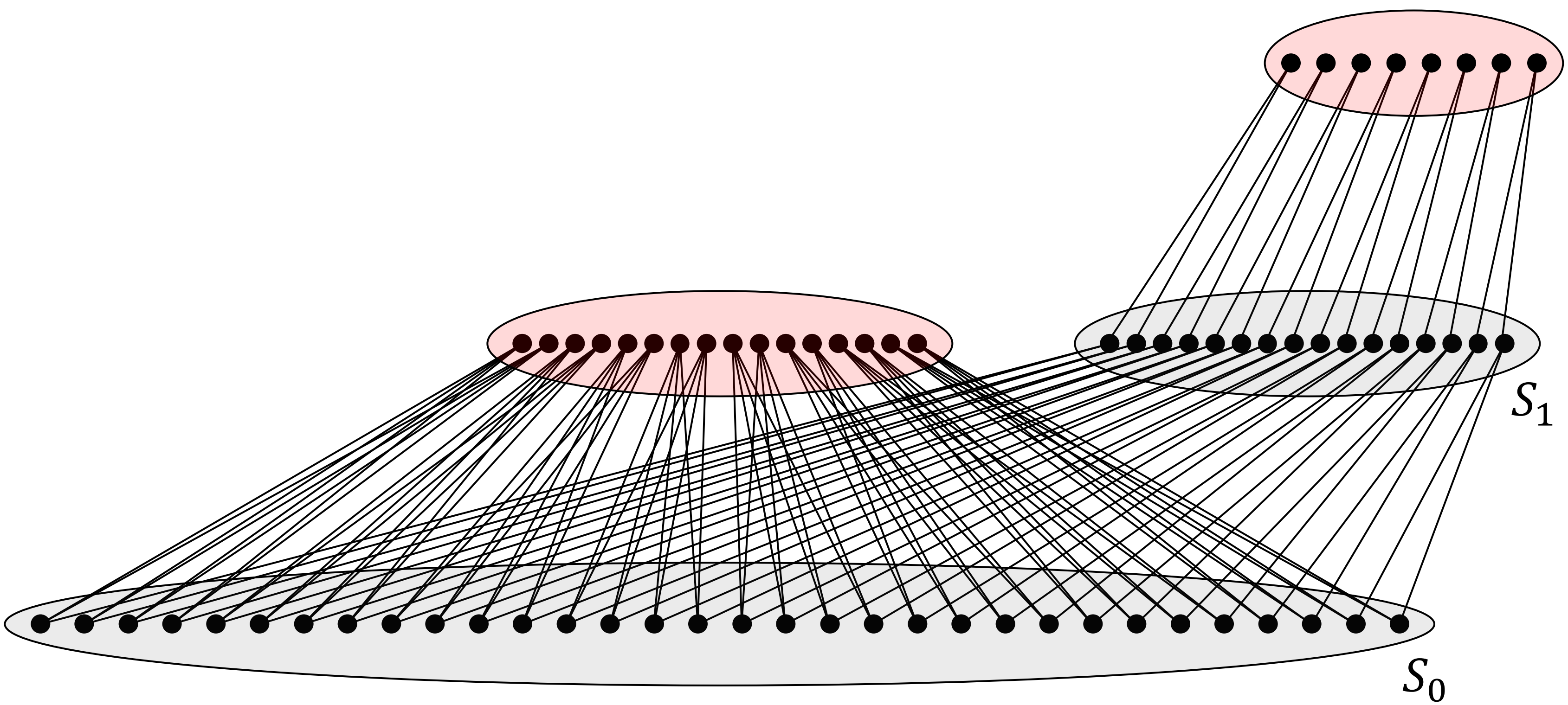

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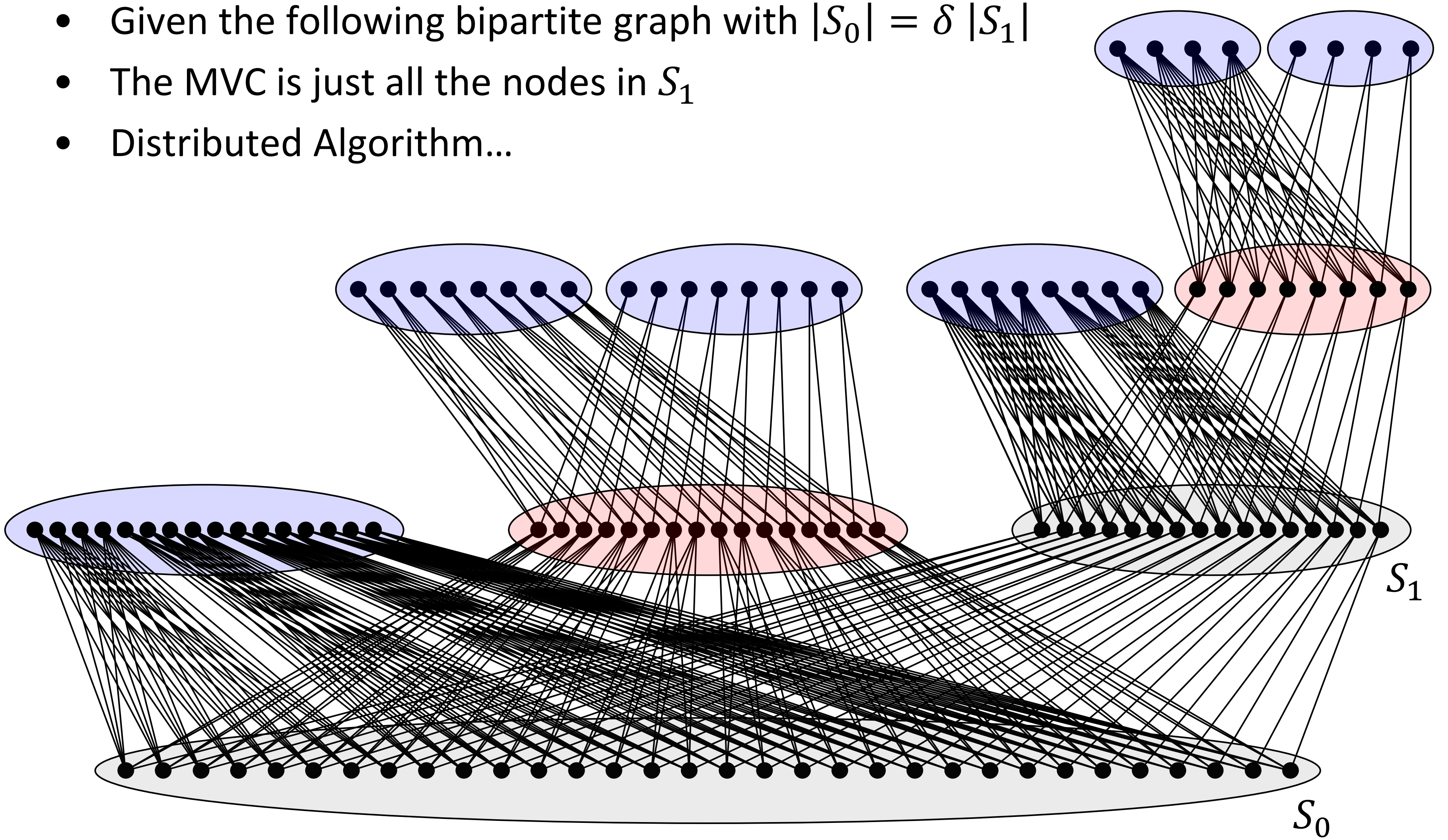
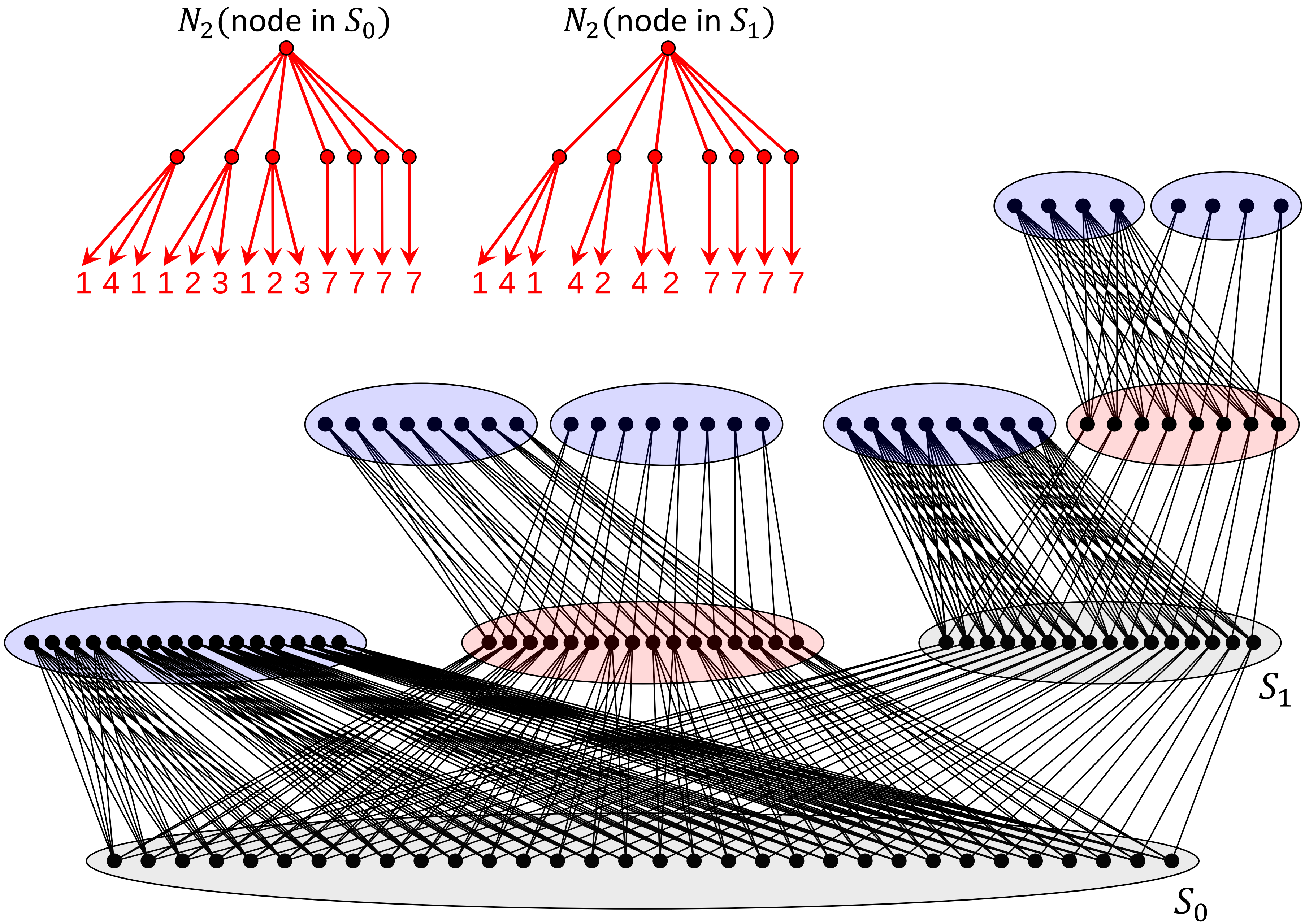
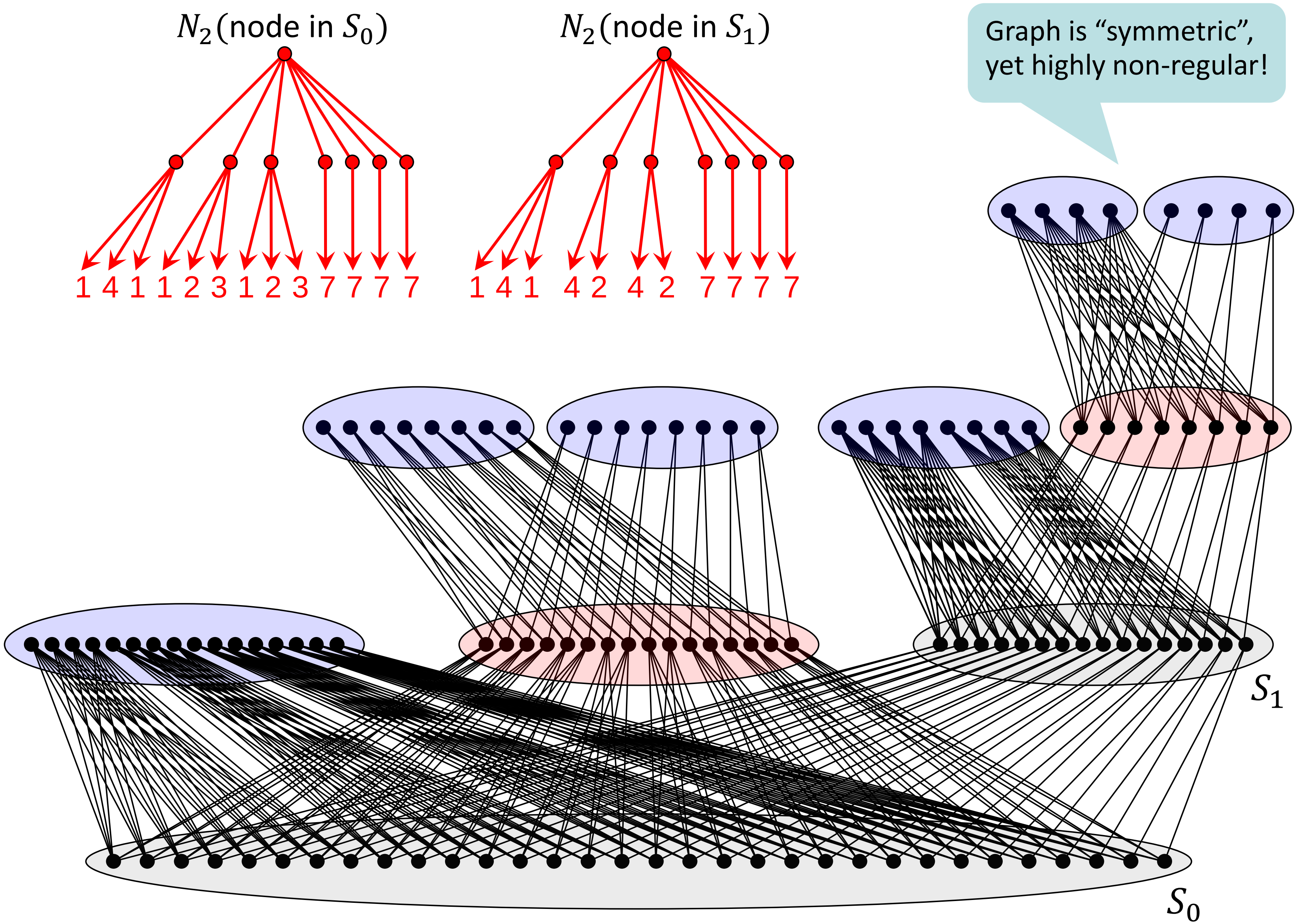




N_2(node in S_0)
1 4 1 1 2 3 1 2 3 7 7 7 7
N_2(node in S_1)
1 4 1 4 2 4 2 7 7 7 7
S_1
S_0



N_2(node in S_0)
1 4 1 1 2 3 1 2 3 7 7 7 7
N_2(node in S_1)
1 4 1 4 2 4 2 7 7 7 7
Graph is “symmetric”, yet highly non-regular!
S_1
S_0

Lower Bound: Results

- We can show that for $\epsilon > 0$, in t time, the approximation ratio is at least

$$\Omega\left(n^{\frac{1/4-\varepsilon}{t^2}}\right) \quad and \quad \Omega\left(\Delta^{\frac{1-\varepsilon}{t+1}}\right)$$

- Constant approximation needs at least $\Omega(\log\Delta)$ and $\Omega(\sqrt{\log n})$ time.
- Polylog approximation $\Omega(\log\Delta/\log\log\Delta)$ and $\Omega(\sqrt{\log n/\log\log n})$.

[Kuhn, Moscibroda, W, journal version in submission, also in arXiv]

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tight for MVC

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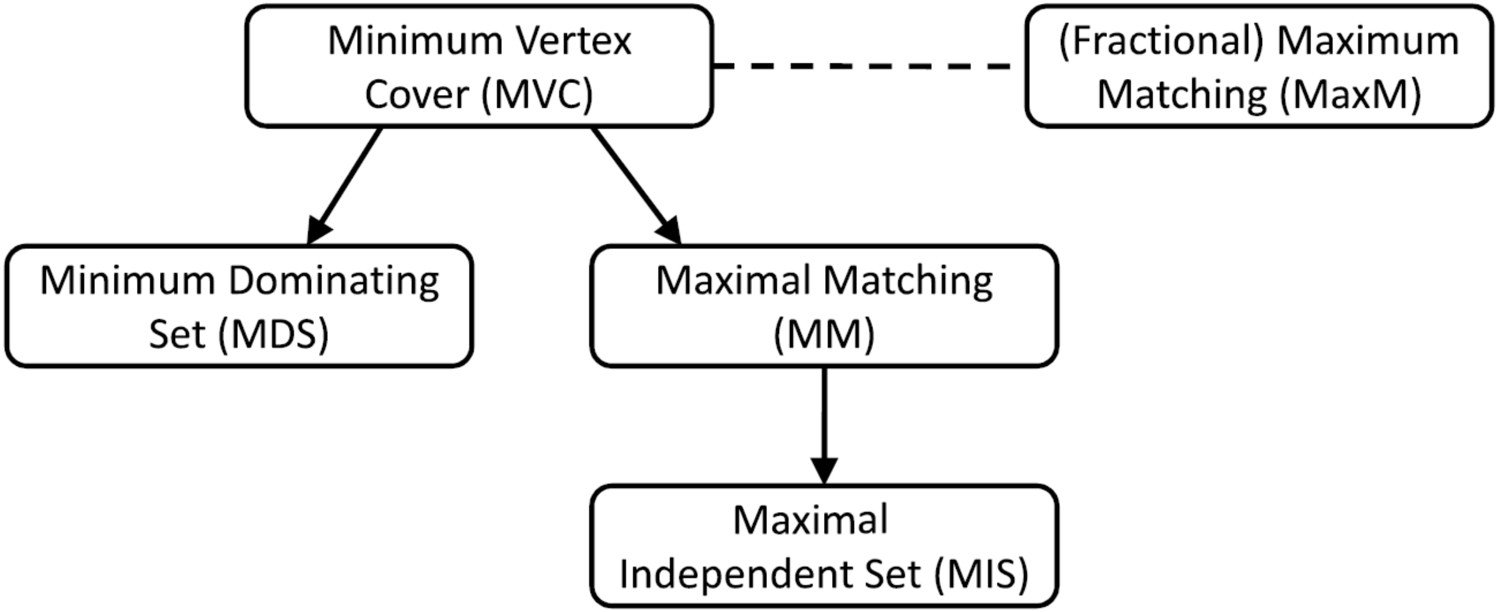
Lower Bound: Reductions

- Many "local looking" problems need non-trivial *t*, in other words, the bounds $\Omega(\log \Delta)$ and $\Omega(\sqrt{\log n})$ hold for a variety of classic problems.



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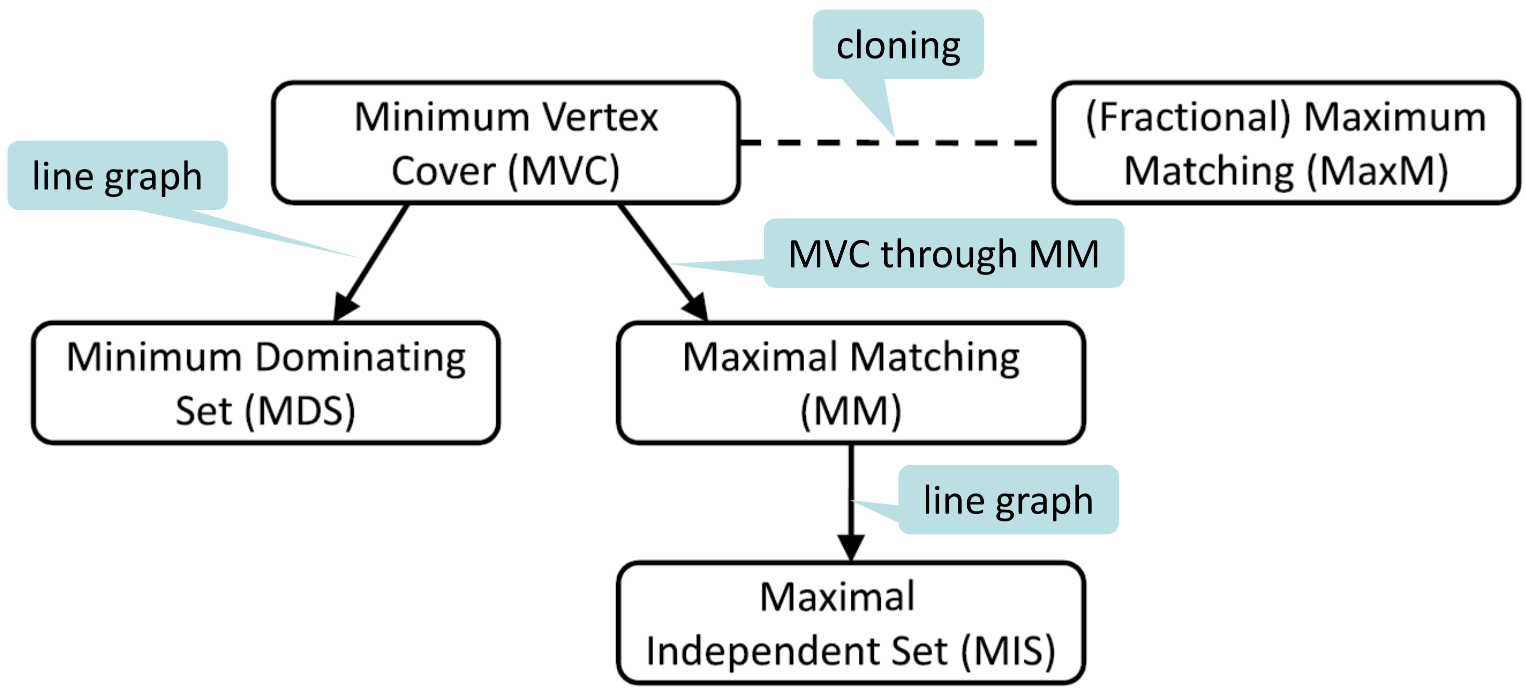

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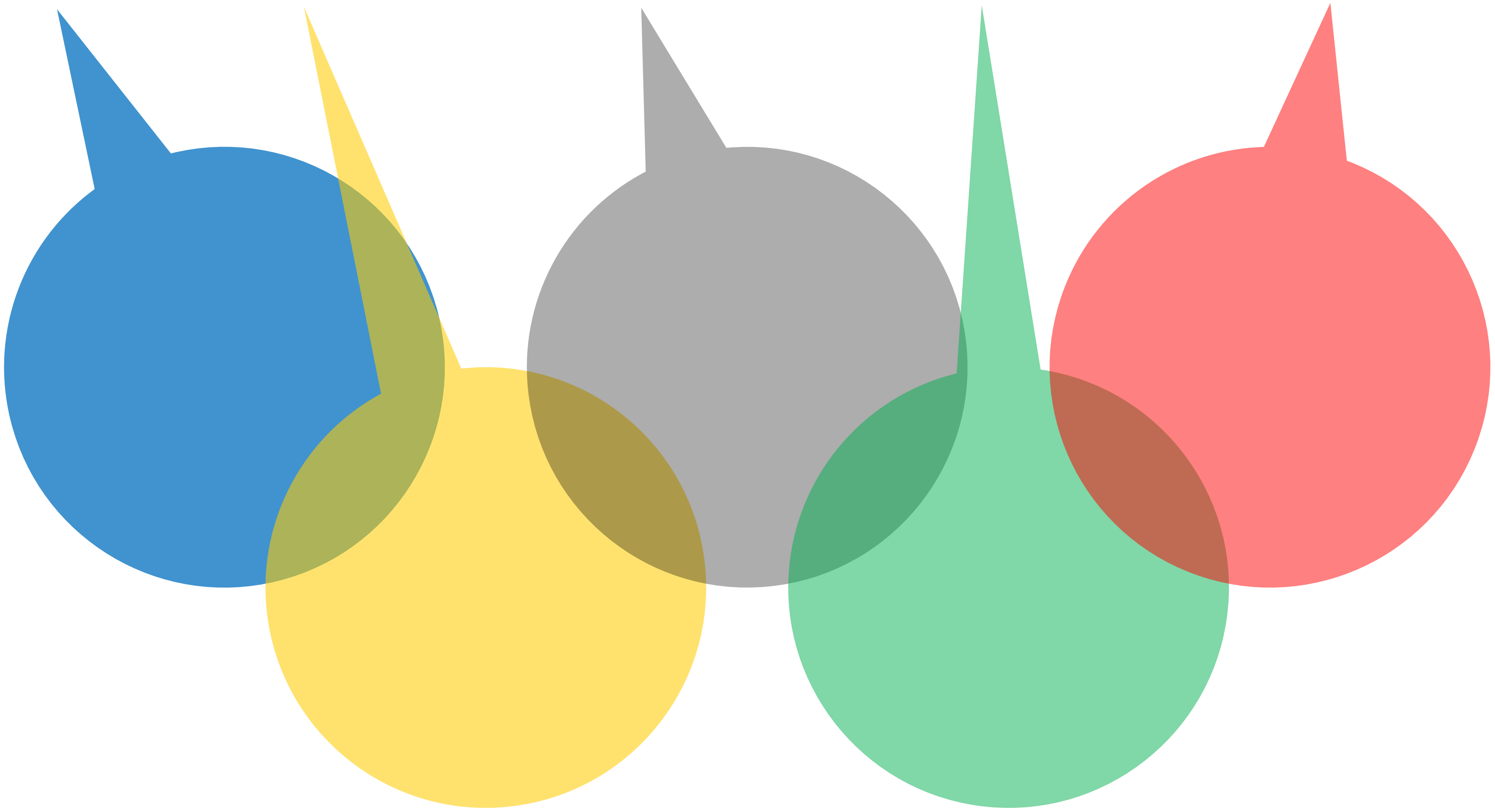
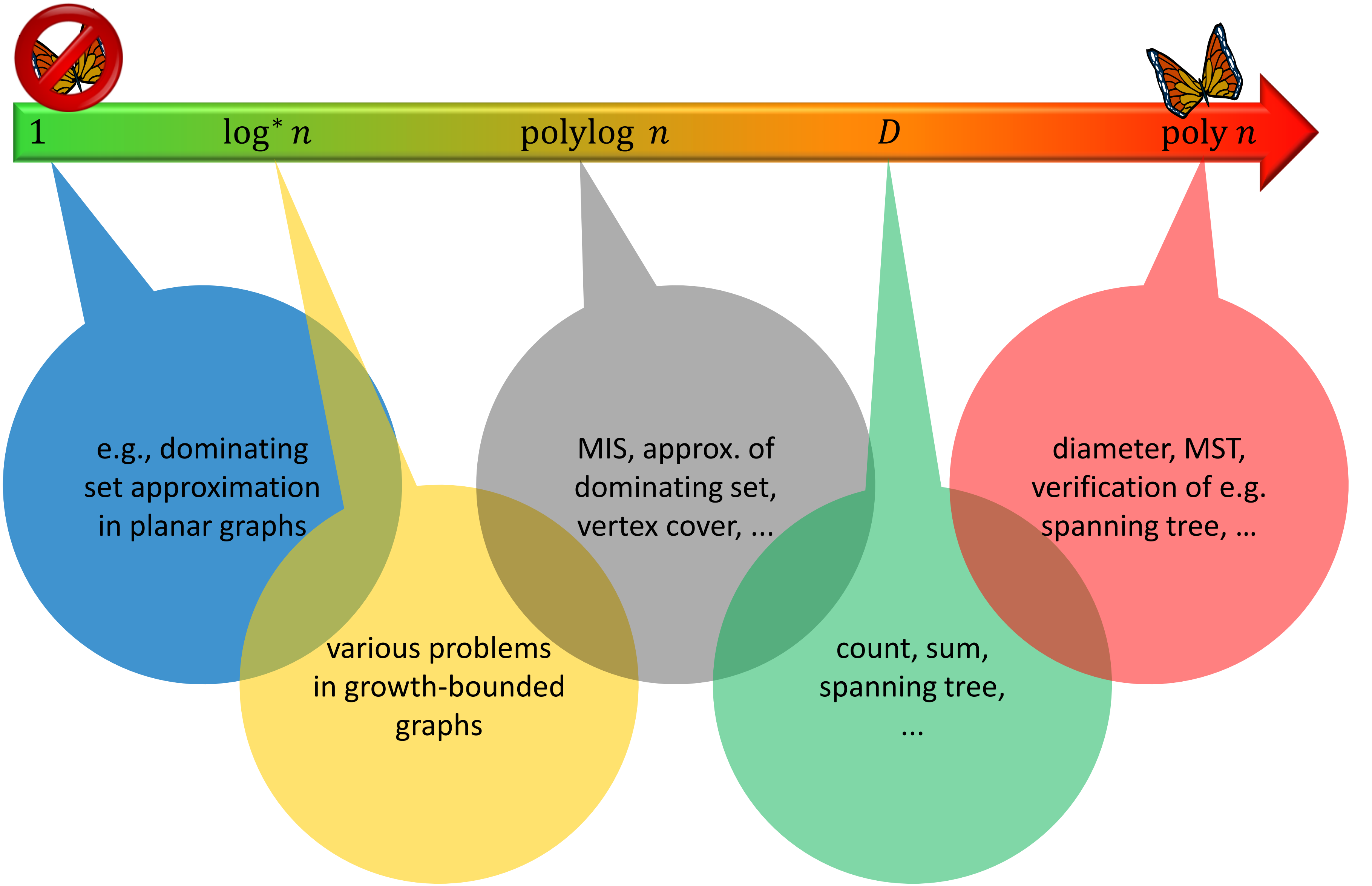
Olympics!



Distributed Complexity Classification



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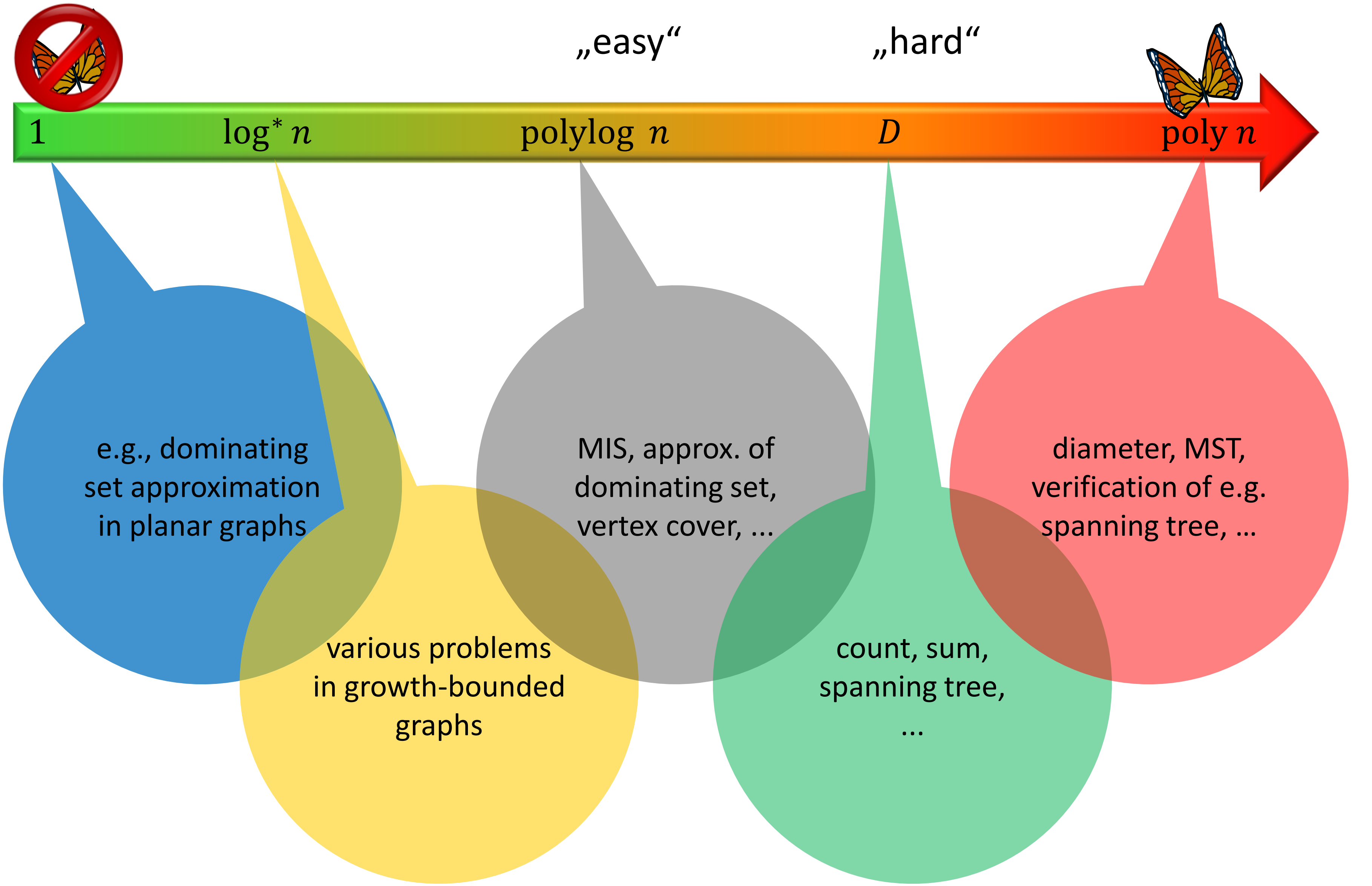
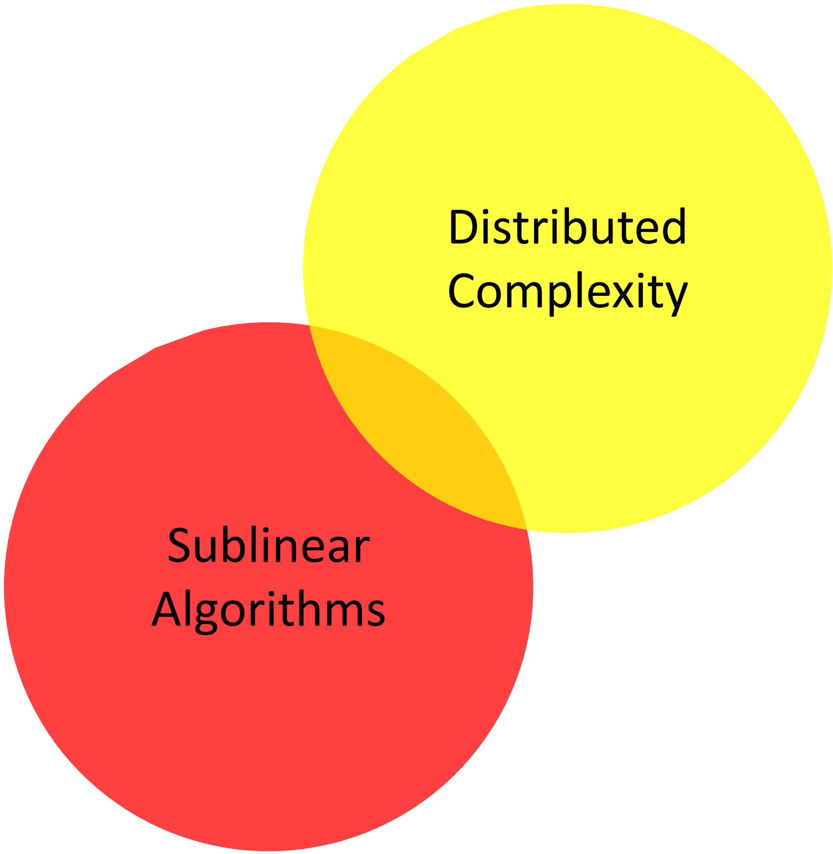


Distributed Complexity



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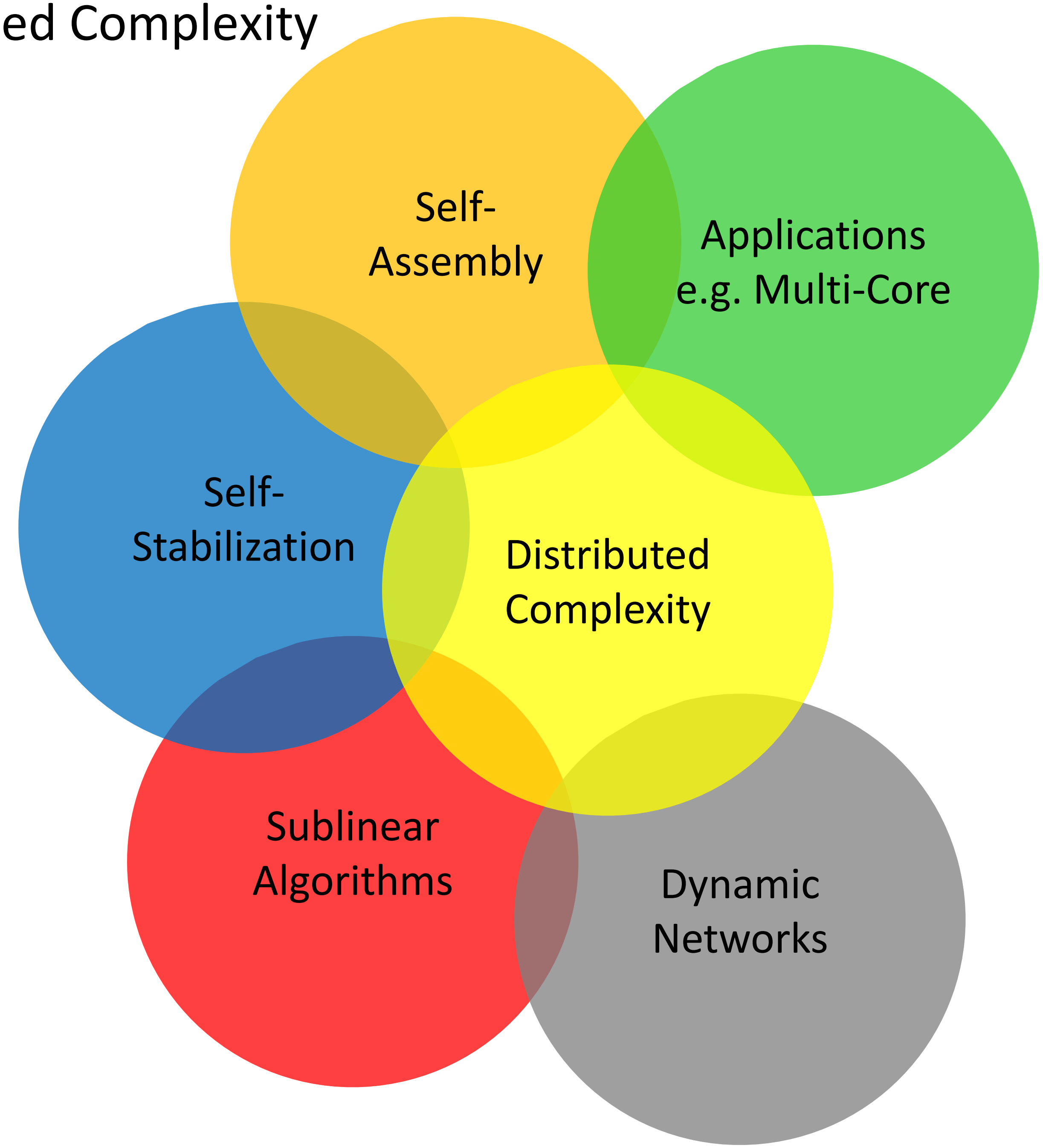

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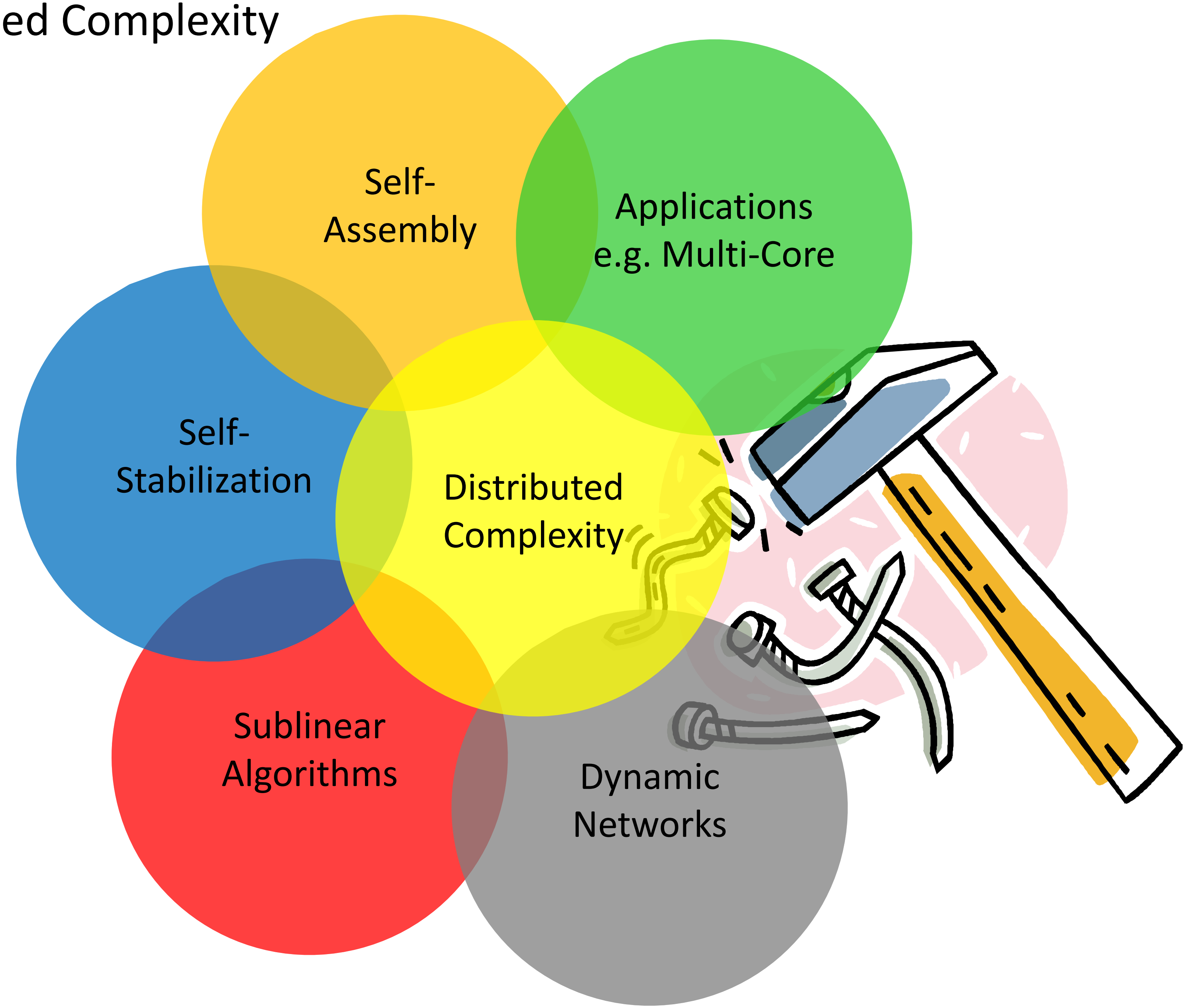
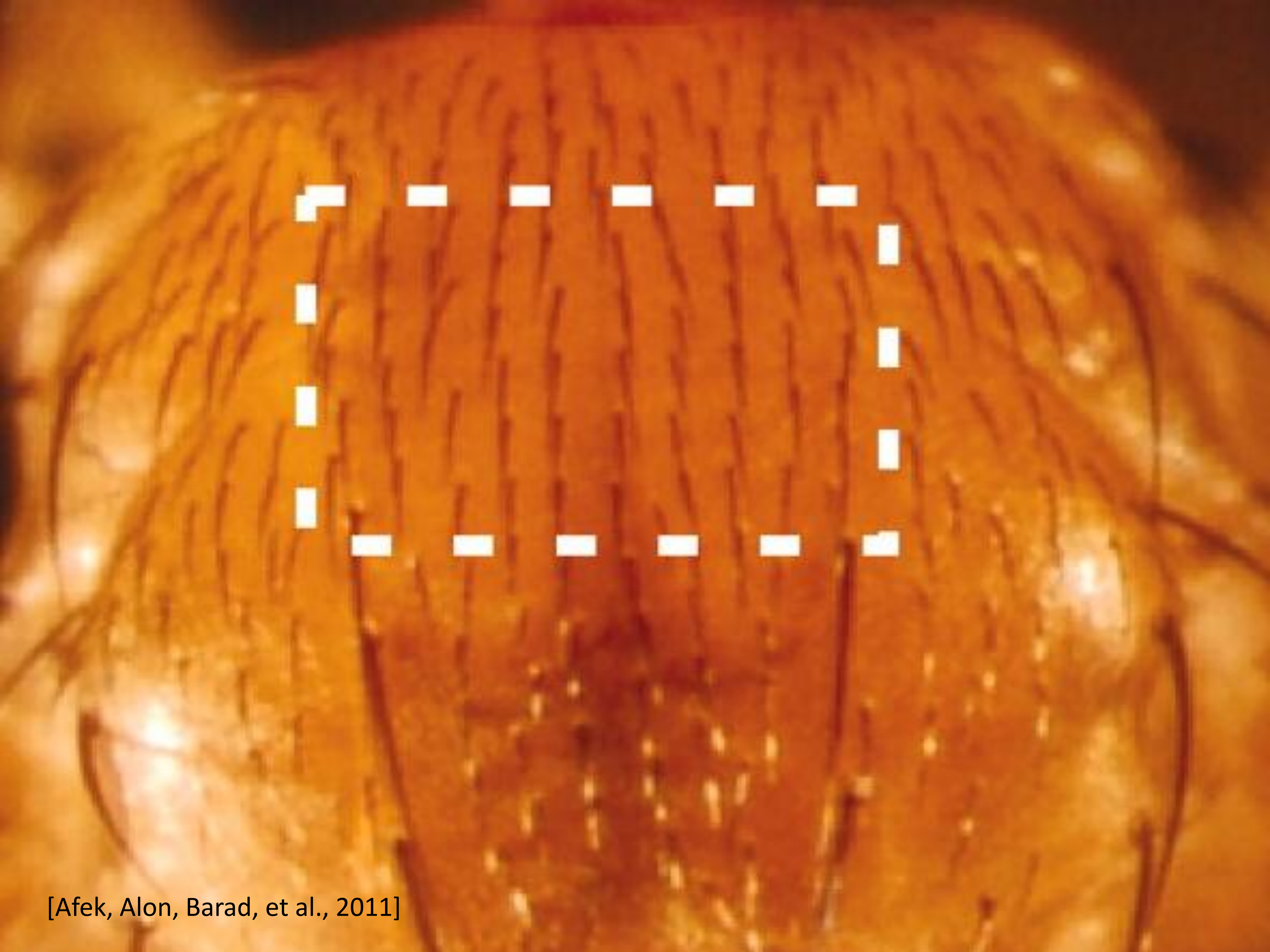
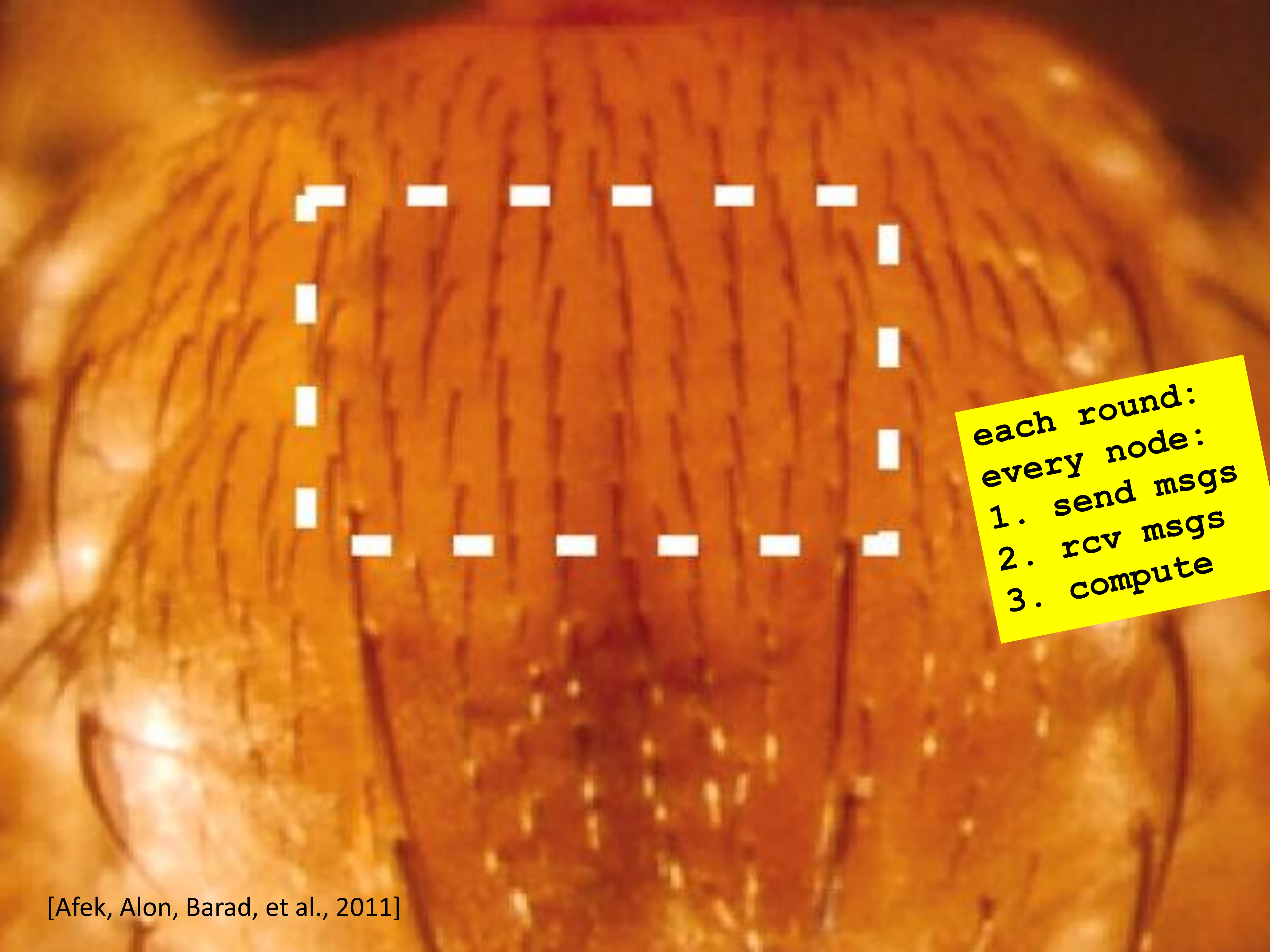




[Afek, Alon, Barad, et al., 2011]



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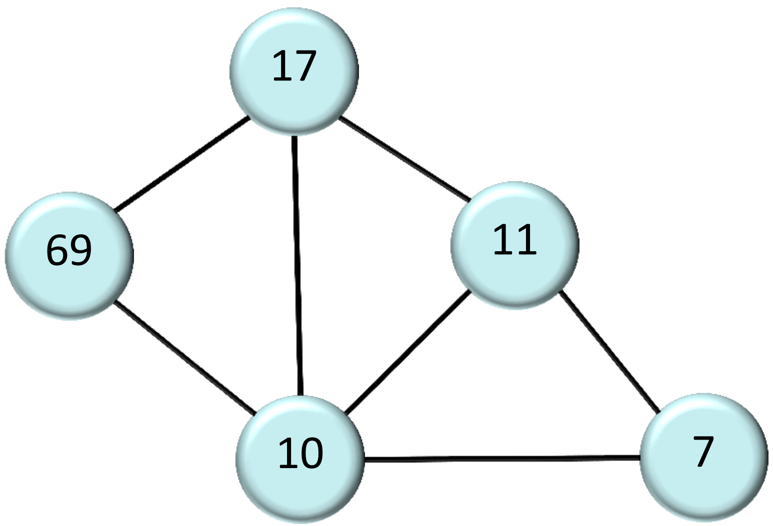
Maximal Independent Set (MIS)

- Given a network with *n* nodes, nodes have unique IDs.
- Find a Maximal Independent Set (MIS)
  - a non-extendable set of pair-wise non-adjacent nodes



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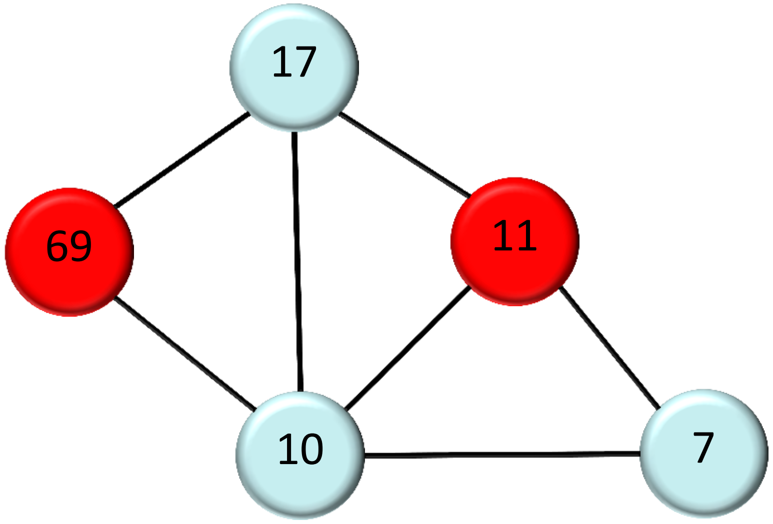

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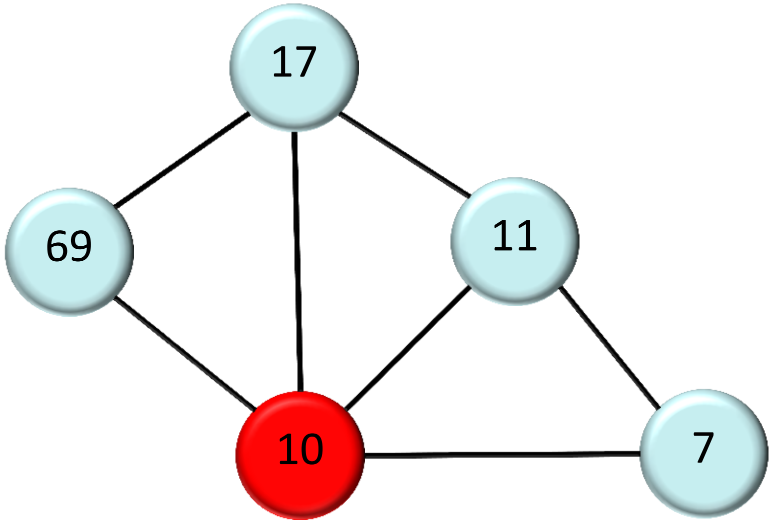

```
given: id, degree
synchronized while (true) {
    p = 1 /(2*degree);
    if (random value between 0 and 1 < p) {
        transmit "(degree, id)";
        …
```

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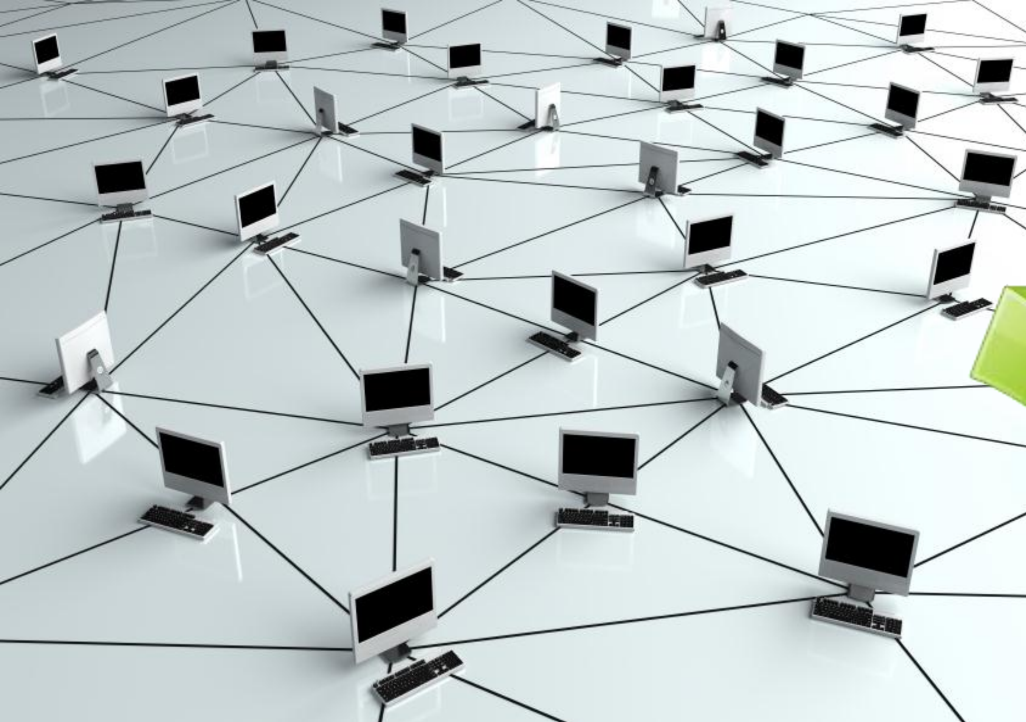
```

?!

$\ll$

Distributed Computing Without Computing!



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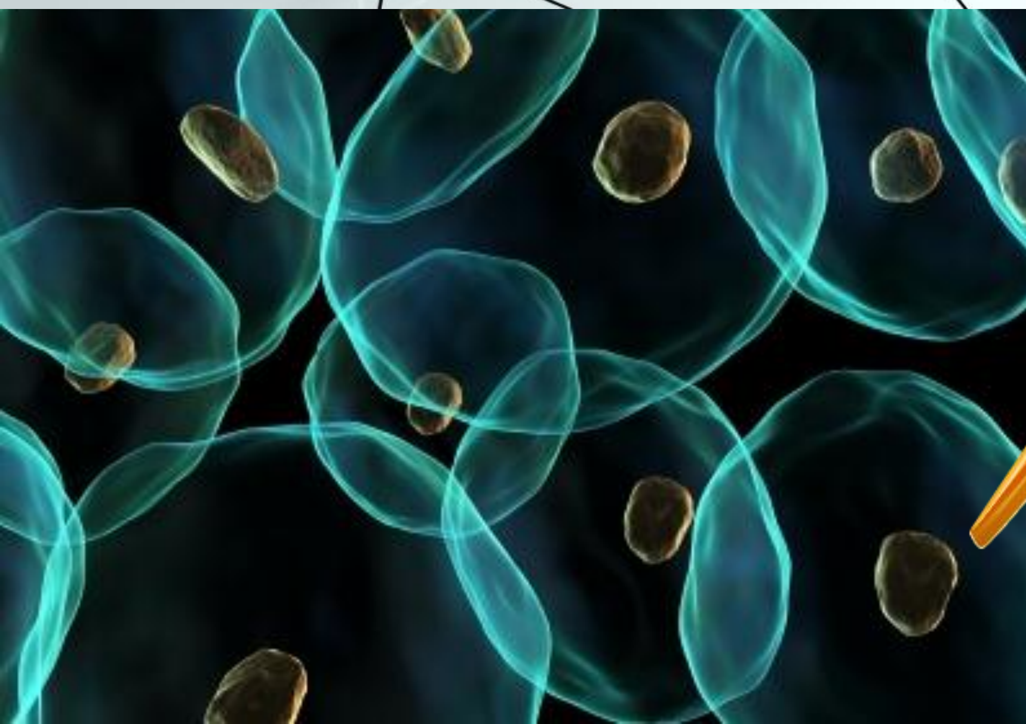
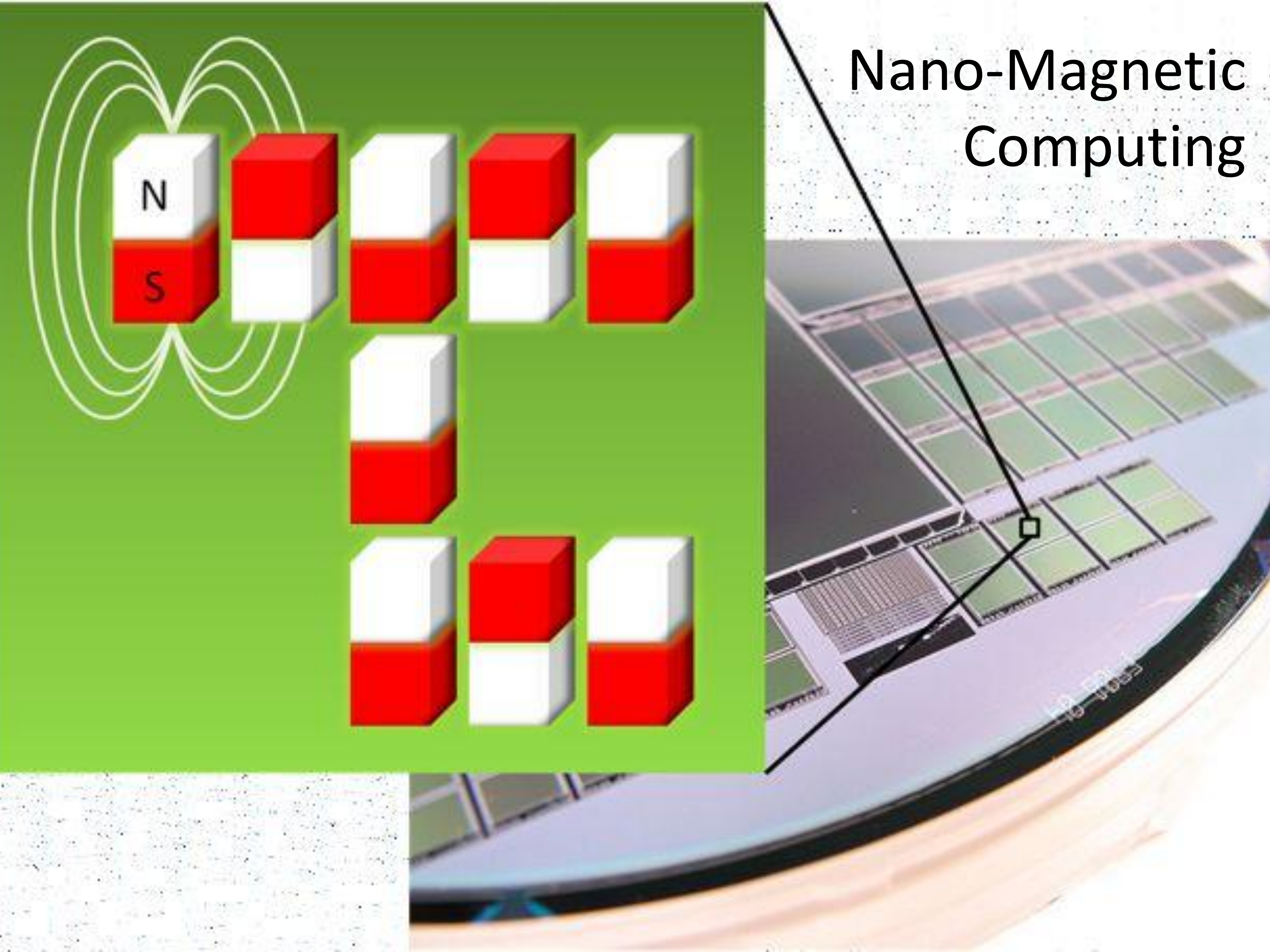


Nano-Magnetic Computing




Stone Age
Distributed Computing

nFSM: networked Finite State Machine

- Every node is the same finite state machine, e.g. no IDs
- Apart from their state, nodes cannot store anything
- Nodes know nothing about the network, including e.g. their degree
- Nodes cannot explicitly send messages to selected neighbors,
  i.e. nodes can only implicitly communicate by changing their state
- Operation is asynchronous
- Randomized next state okay, as long as constant number
- Nodes cannot compute, e.g. cannot count

One, Two, Many Principle

Piraha

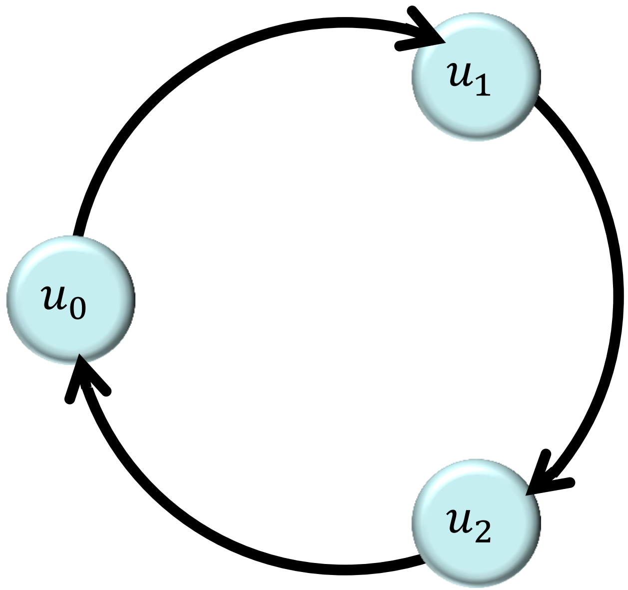
Walpiri

One, Two, Many Principle

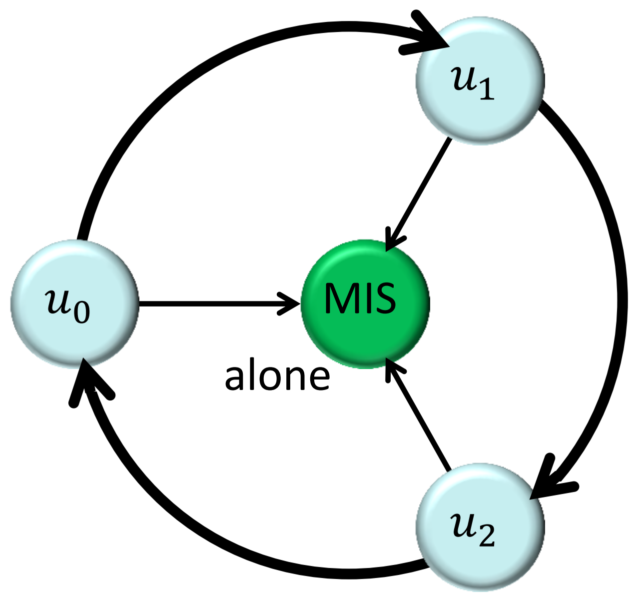
- Not okay
  - while (k < log n) {
  - At least half of neighbors in state s?
  - More neighbors in state s than in state t?

- Okay
  - No neighbor in state s?
  - Some neighbor in state s?
  - At most two neighbors in state s?

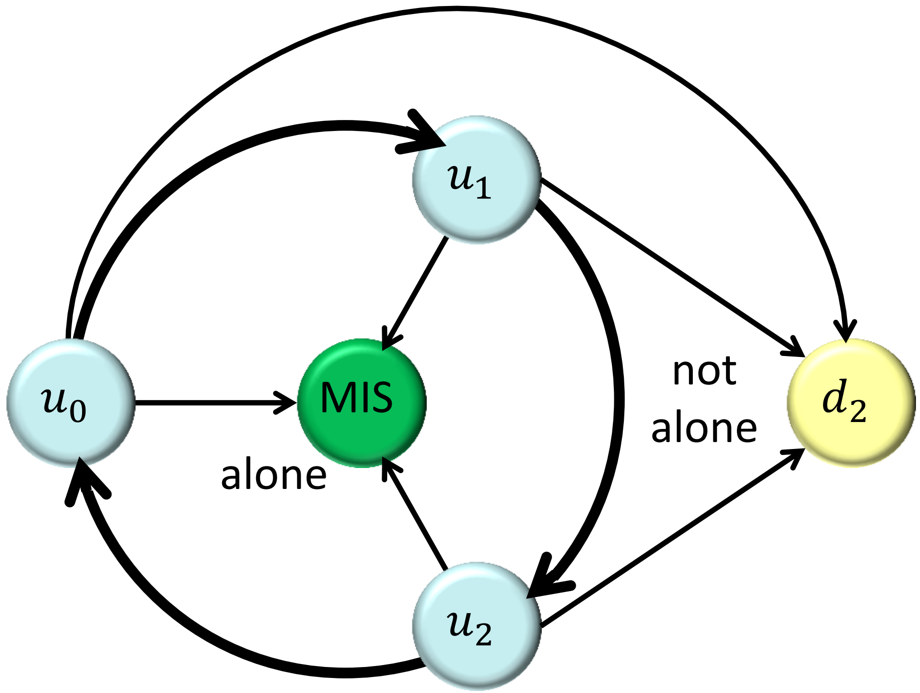
PRIMITIVE CULTURES DEVELOP SESAME STREET.



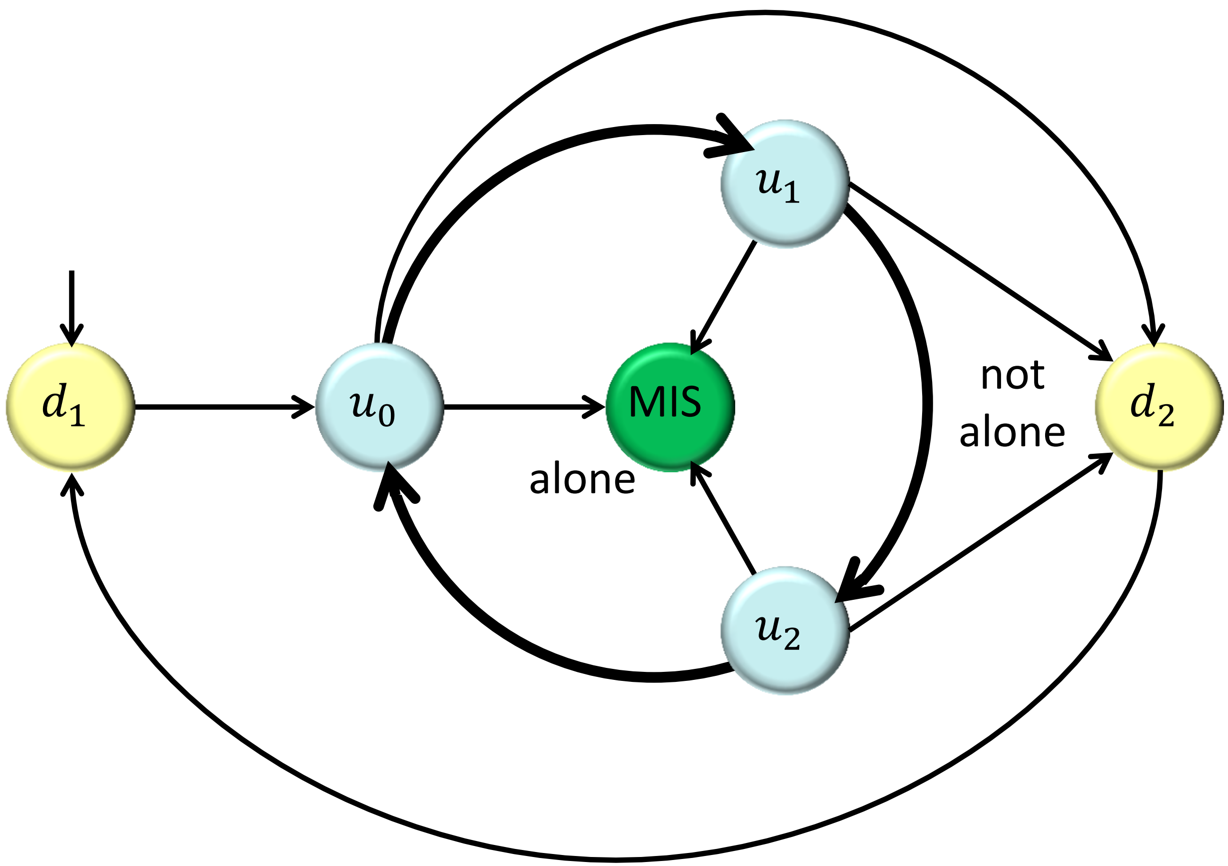
u_1
u_0
u_2



u_1
u_0
MIS
alone
u_2



u_1
u_0
MIS
alone
not
alone
d_2
u_2



d_1
u_0
MIS
alone
u_1
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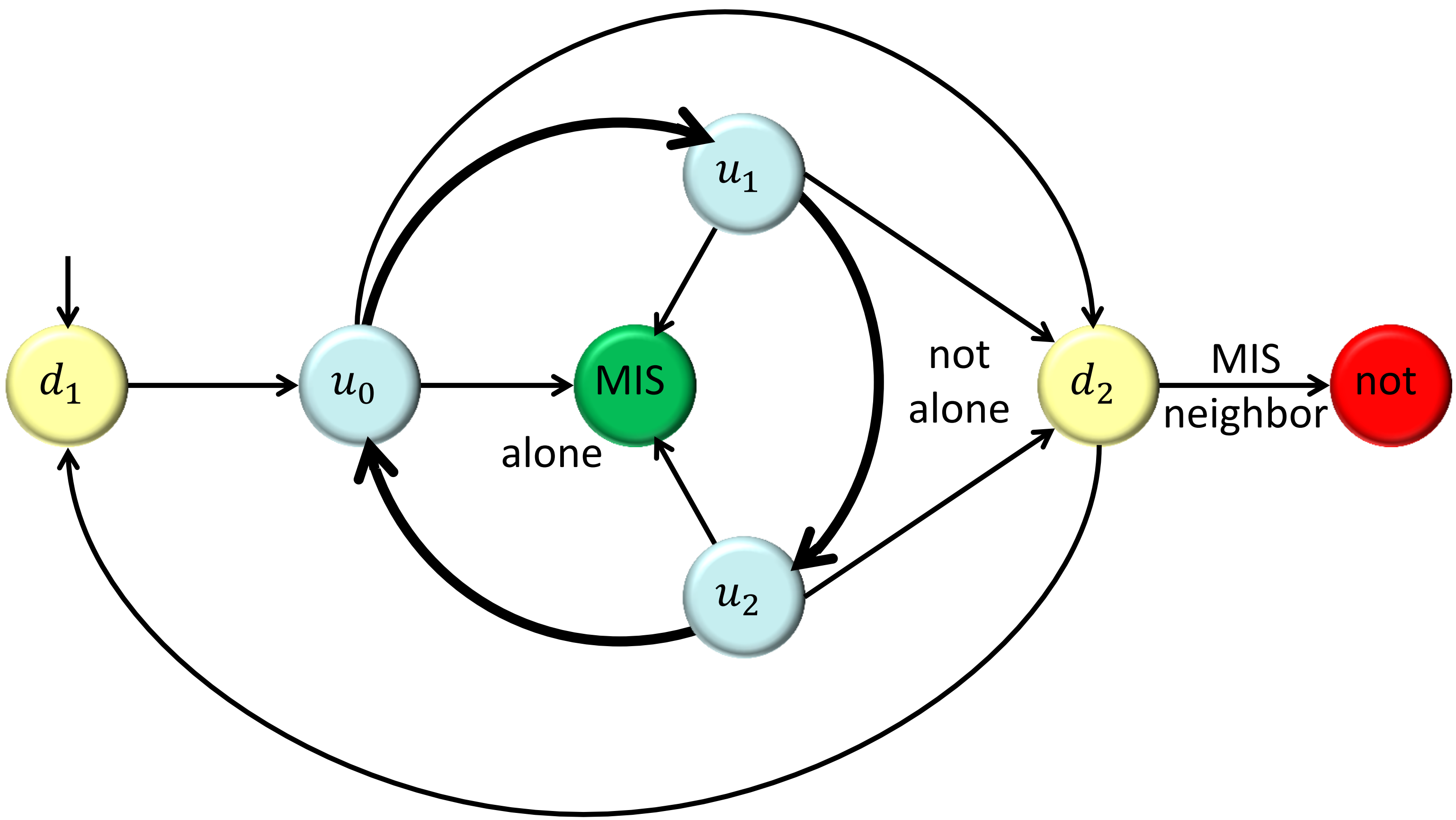
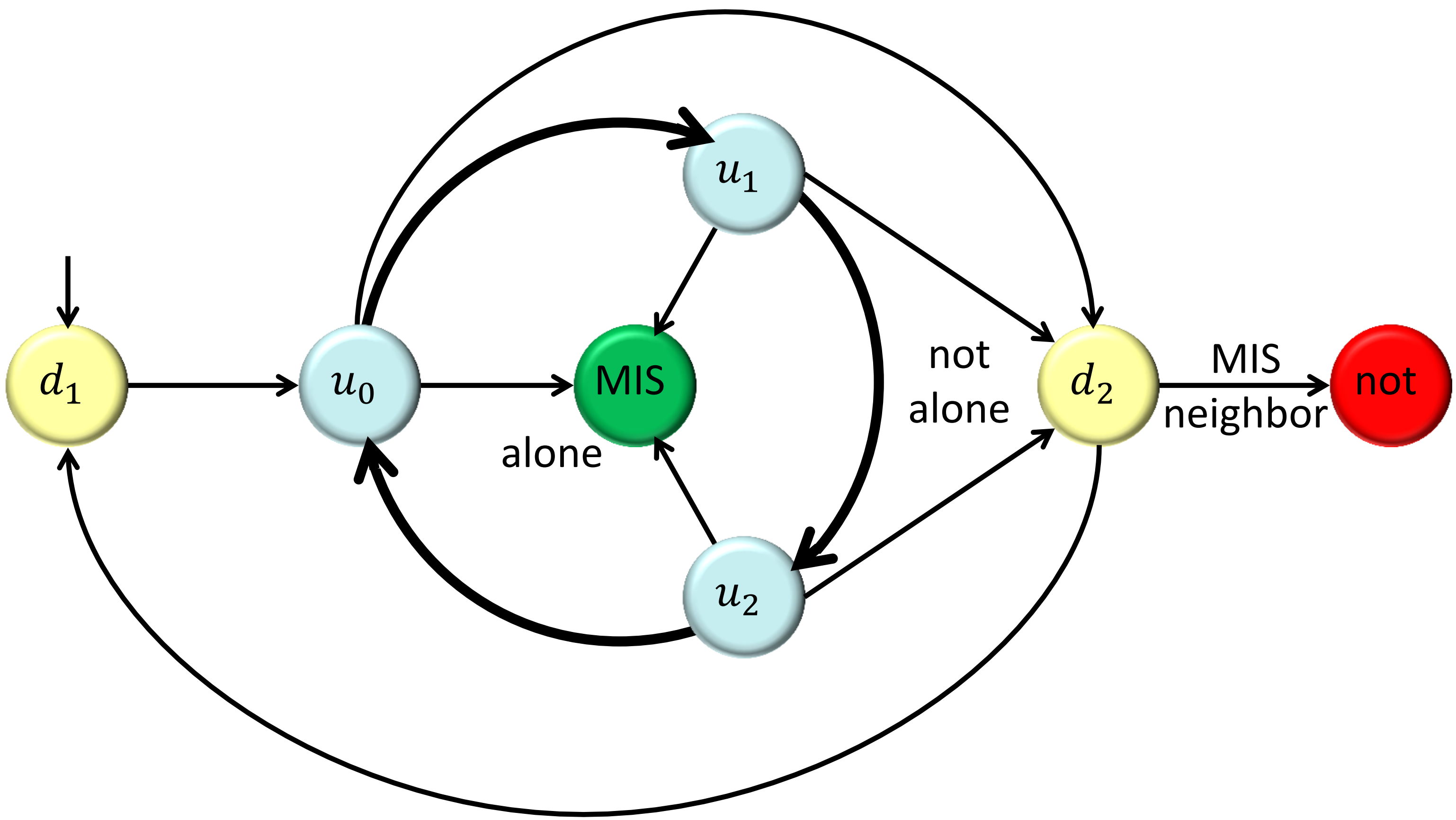
not
alone
d_2



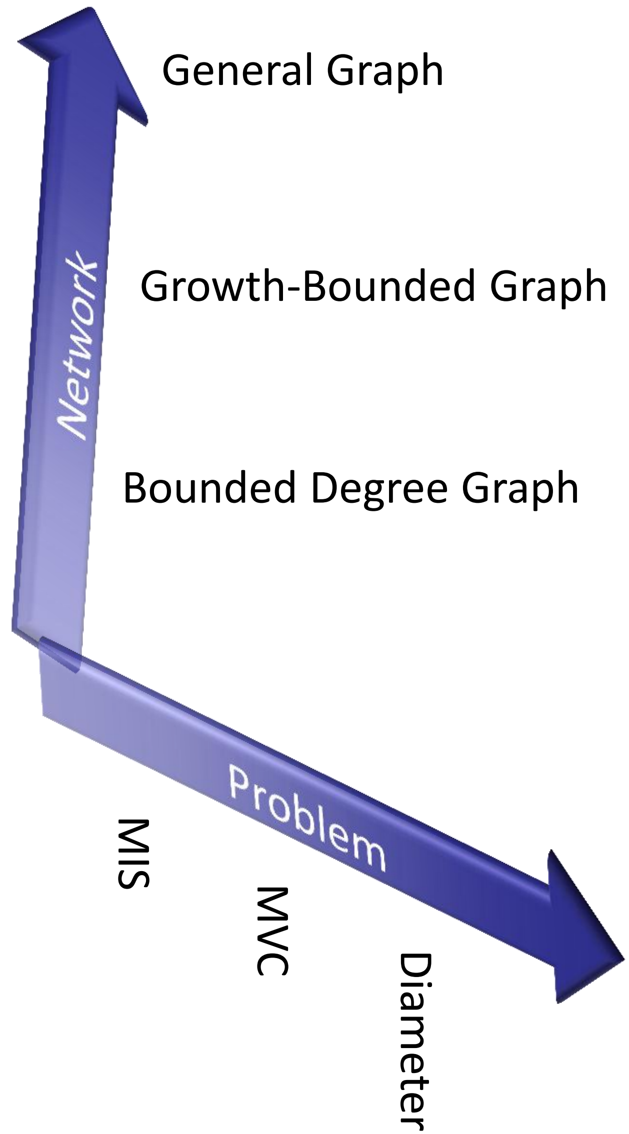
d_1
u_0
MIS
alone
u_1
u_2
not
alone
d_2
MIS
neighbor
not

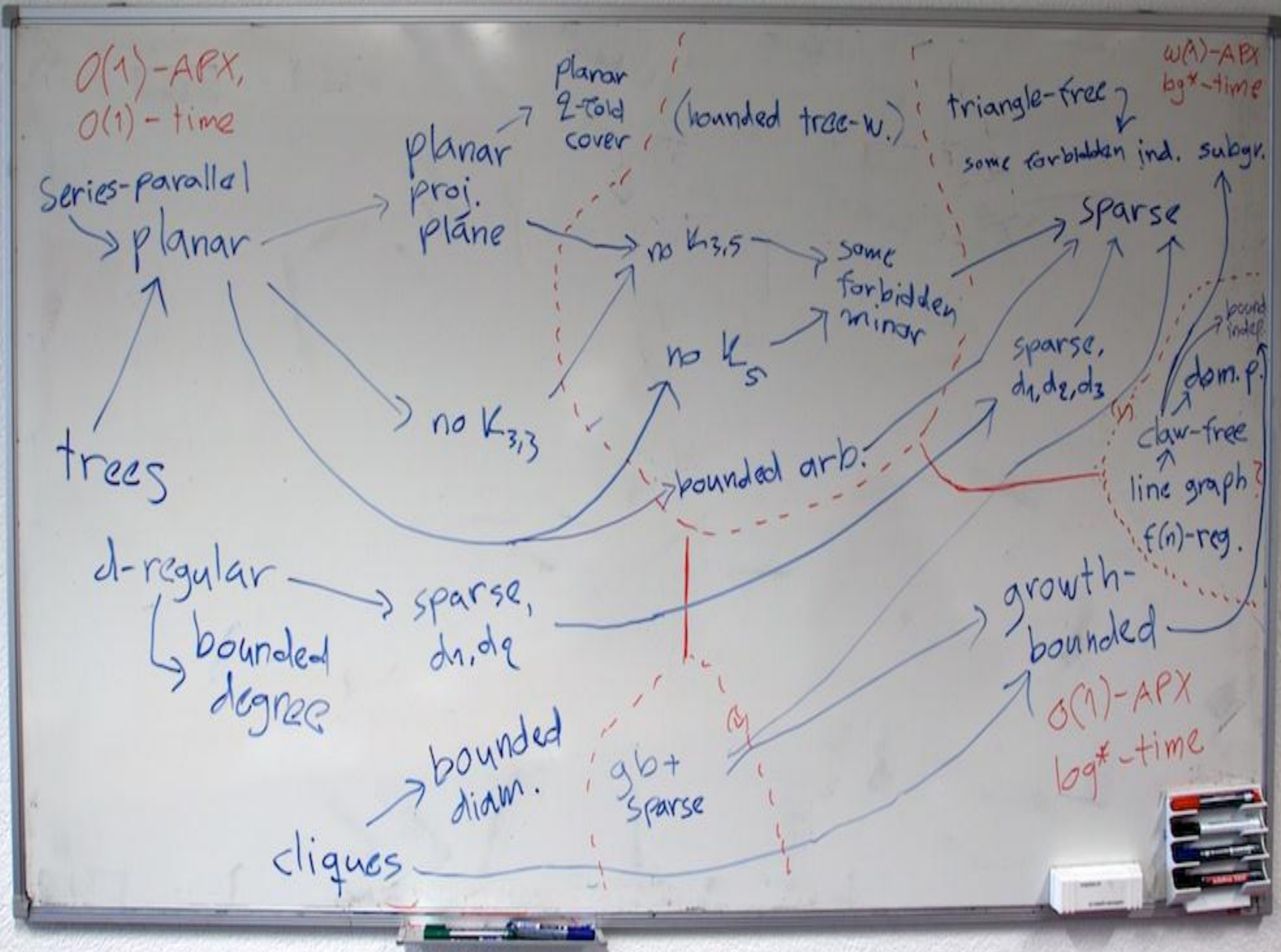
nFSM solves MIS whp in time $O(\log^2 n)$



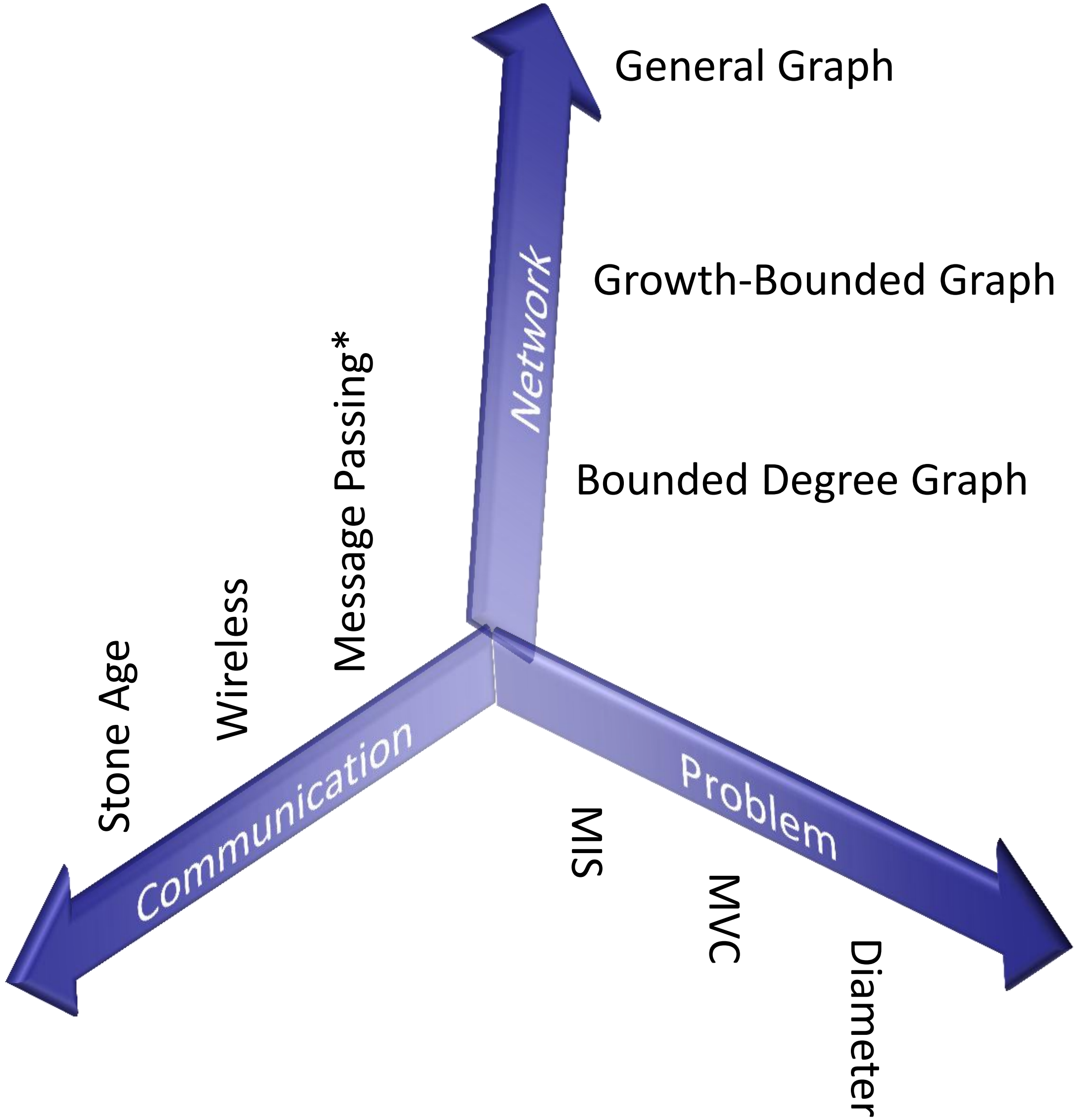
[Emek, Smula, W, in submission, also in arXiv]

Overview

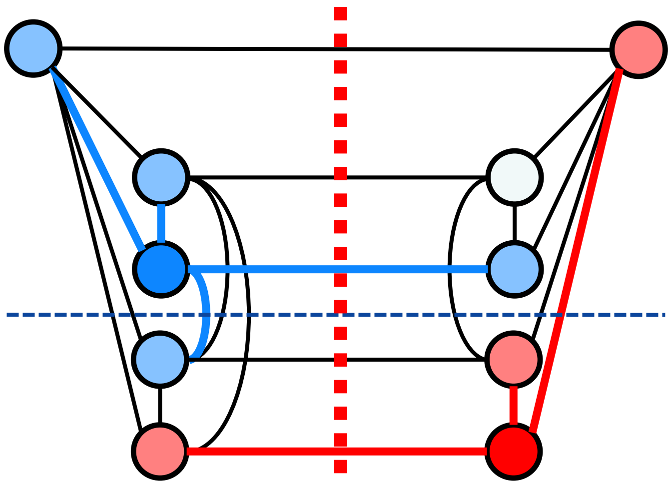

General Graph
Growth-Bounded Graph
Bounded Degree Graph
Network
Problem
MIS
MVC
Diameter

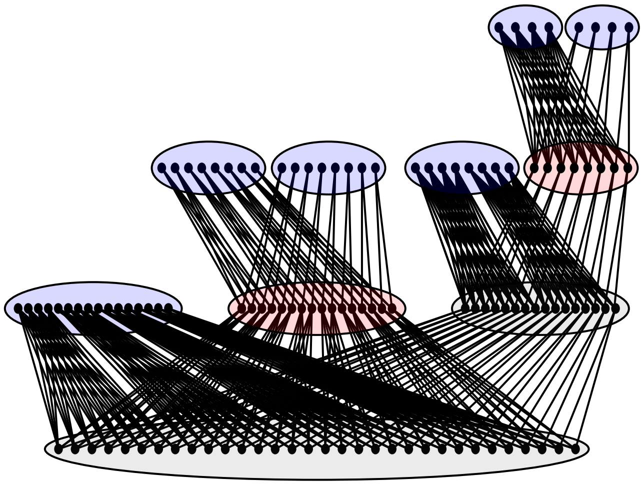

O(1)-APX,
O(1)-time
planar
2-fold
cover
(bounded tree-w.)
triangle-free
ω(1)-APX
log*-time
some forbidden ind. subgr.
planar
proj.
plane
Series-parallel
planar
sparse
no $K_{3,5}$
some
forbidden
minor
no K_5
sparse,
d_1, d_2, d_3
bound
indep.
dom. p.
no $K_{3,3}$
claw-free
trees
bounded arb.
line graph?
f(n)-reg.
d-regular
sparse,
d_1, d_2
growth-
bounded
bounded
degree
O(1)-APX
log*-time
bounded
diam.
gb+
sparse
cliques

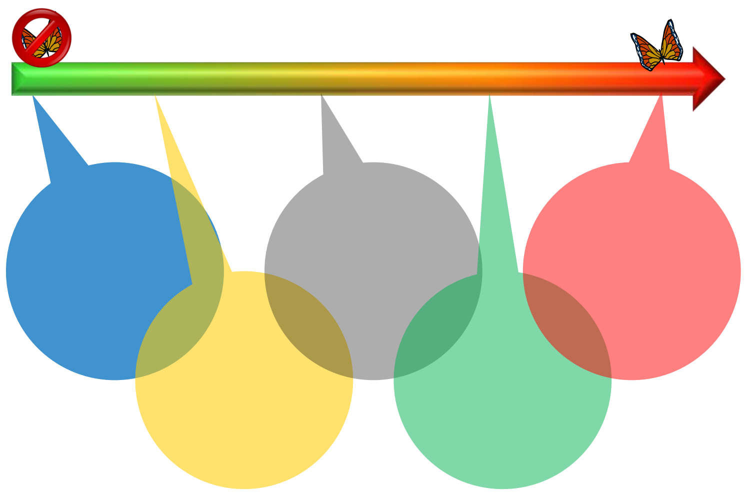
Overview


General Graph
Growth-Bounded Graph
Bounded Degree Graph
Network
Message Passing*
Wireless
Stone Age
Communication
Problem
MIS
MVC
Diameter

Summary







Thank You!

Questions & Comments?

Thanks to my co-authors
Yuval Emek
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Thomas Moscibroda
Jasmin Smula

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