Convex Consensus with Asynchronous Fallback

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1.

Why Convex Consensus?

(Almost) everything is convex if you believe!

Consensus (Byzantine Agreement)

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n parties

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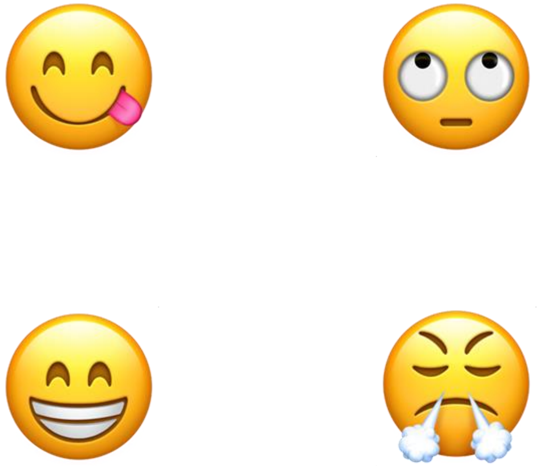

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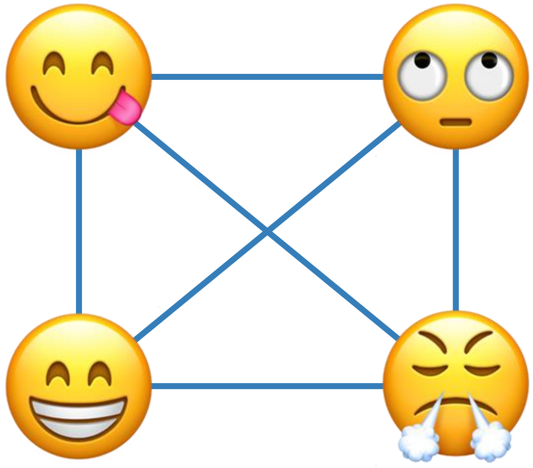

(authenticated bidirectional channels)

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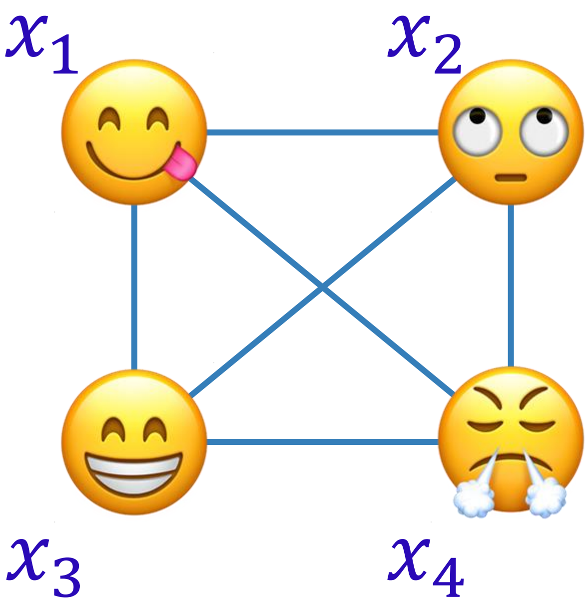

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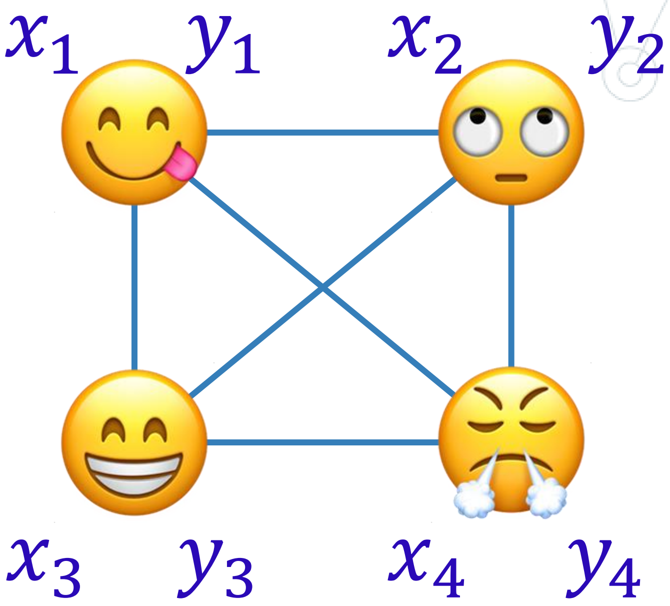

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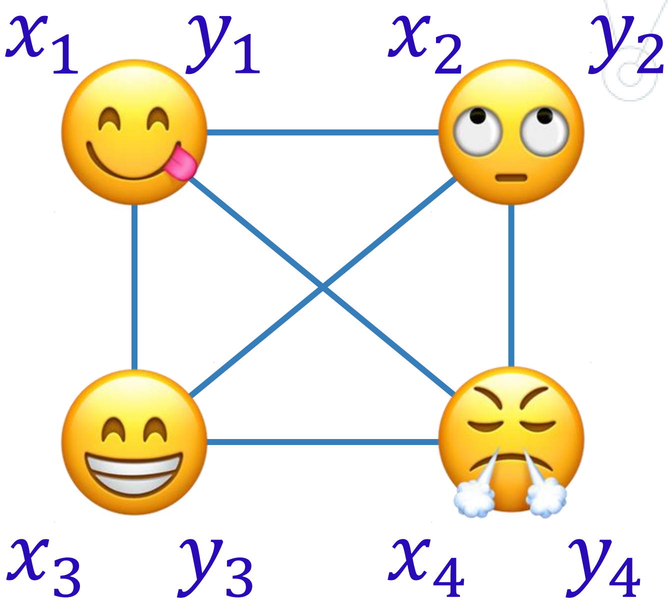

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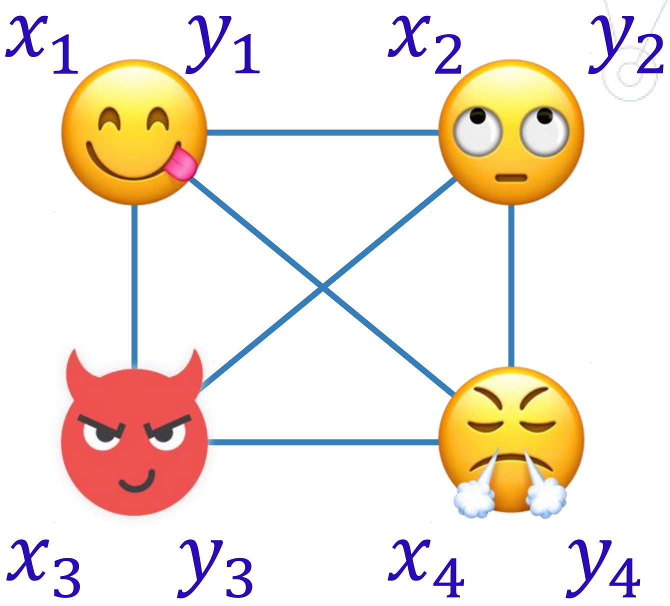

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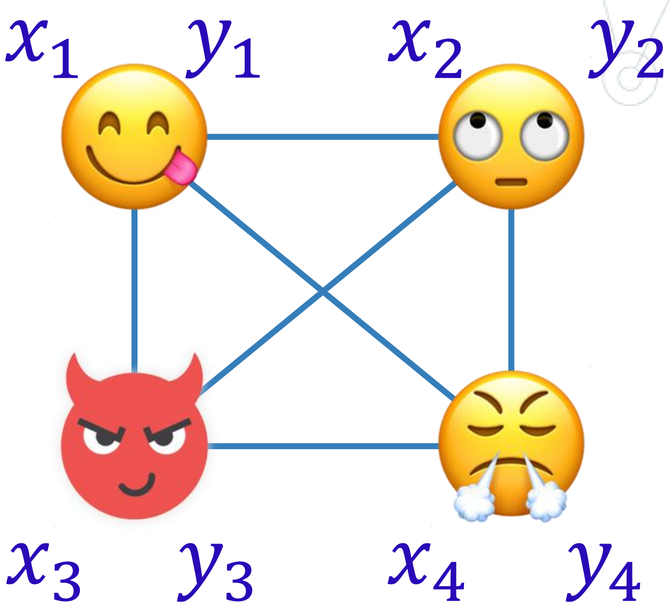

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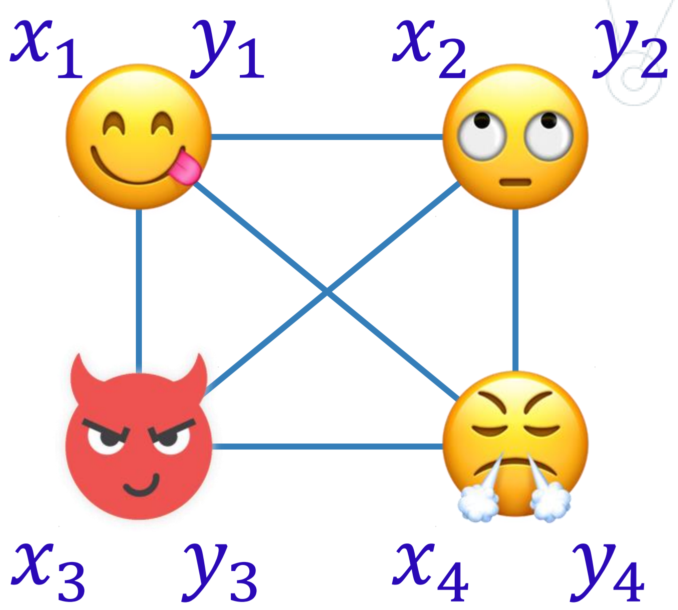

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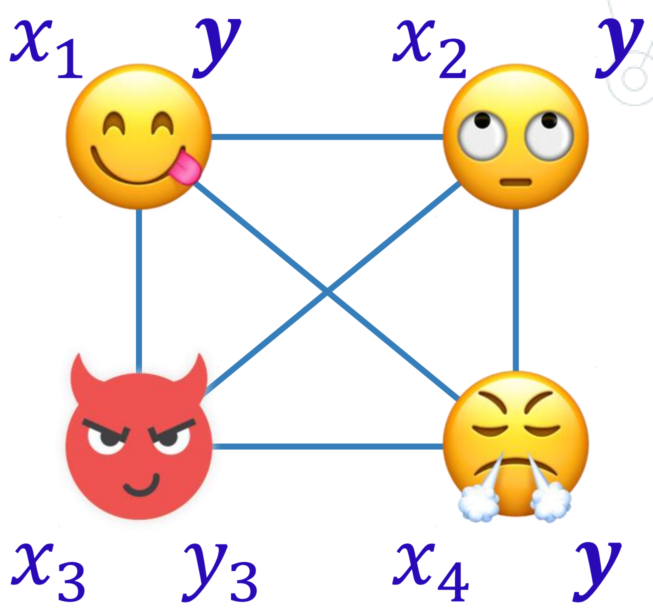

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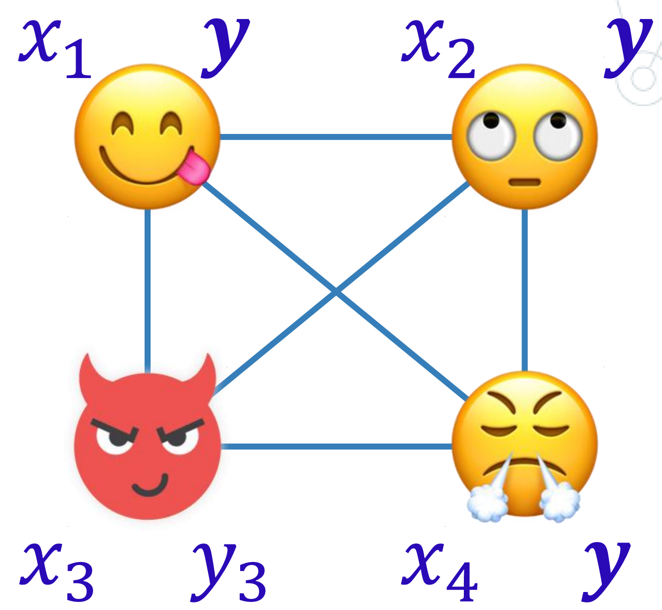

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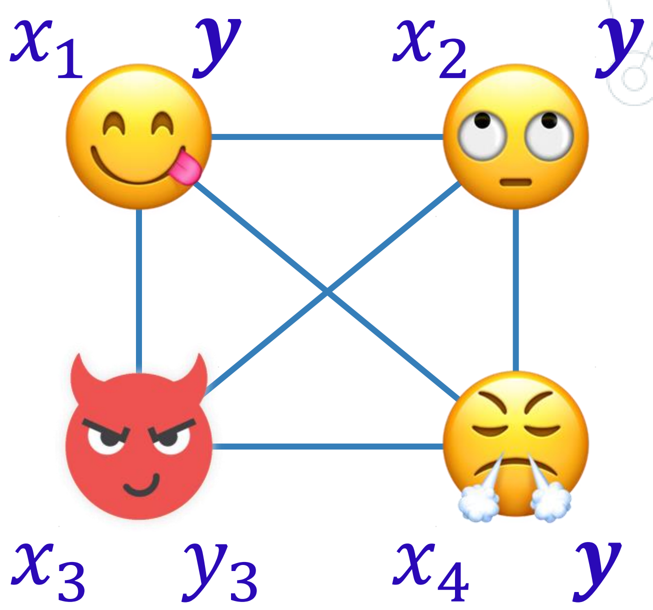

Q: What does *meaningful* mean?

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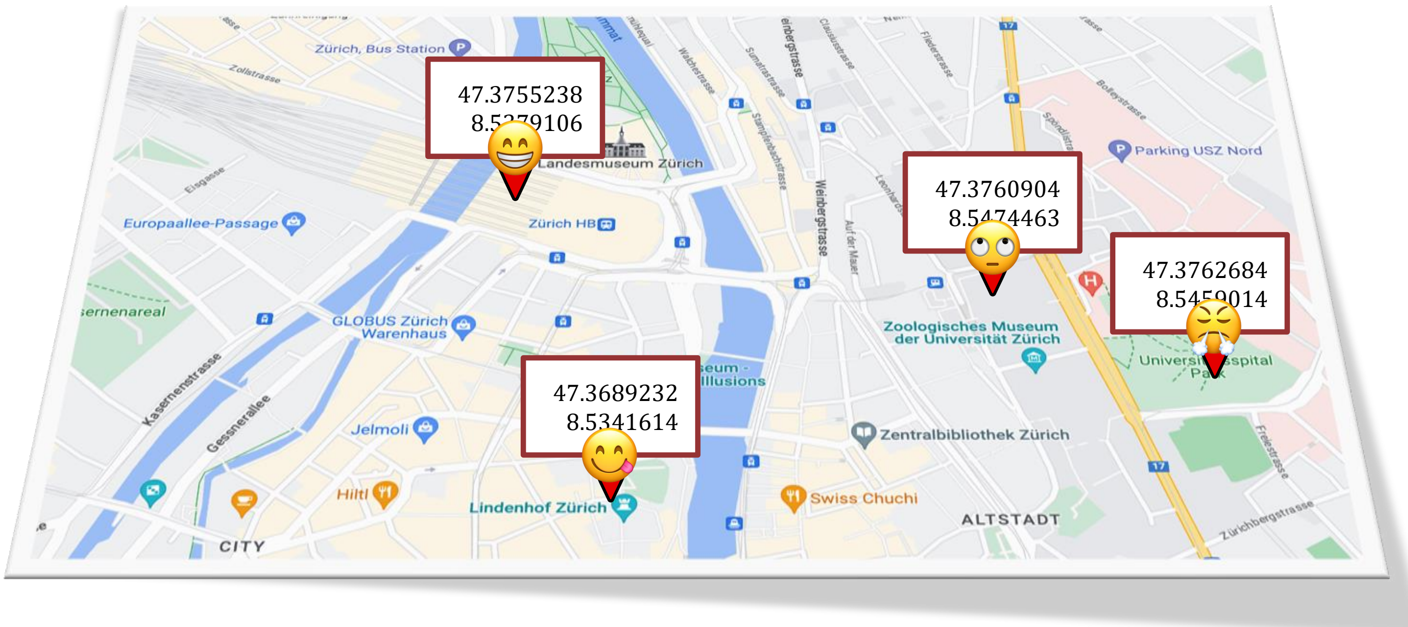

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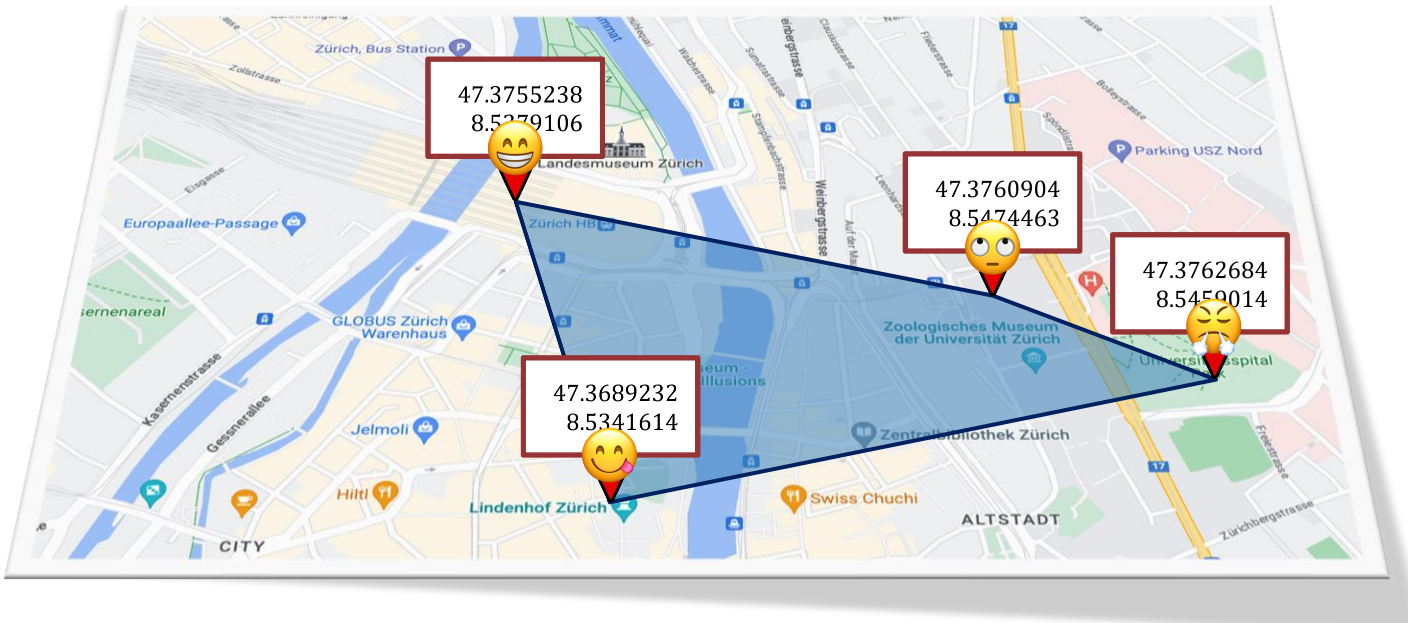

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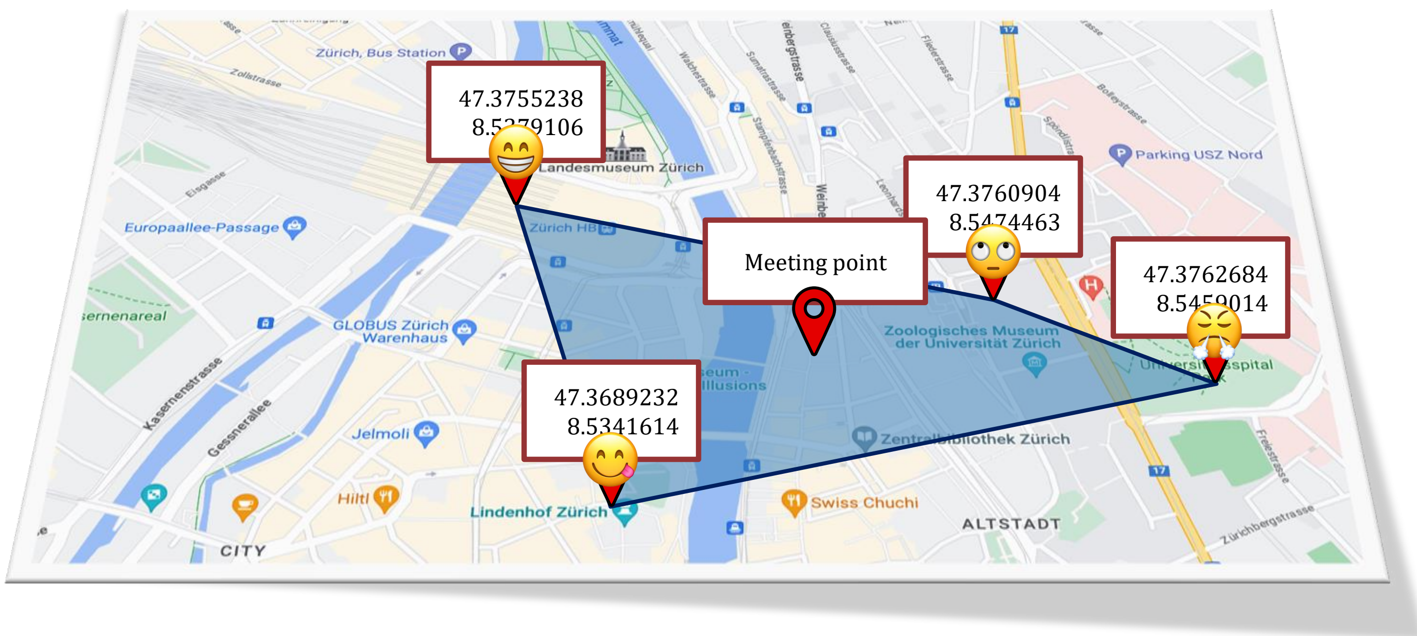

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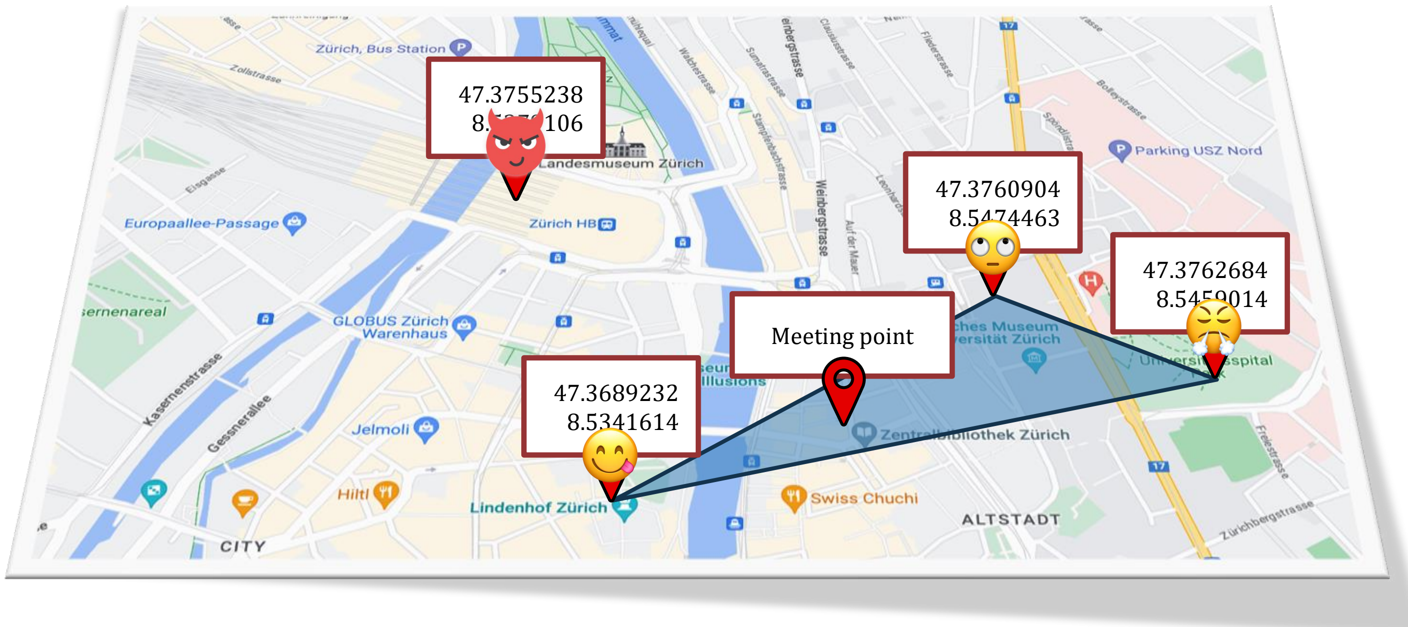

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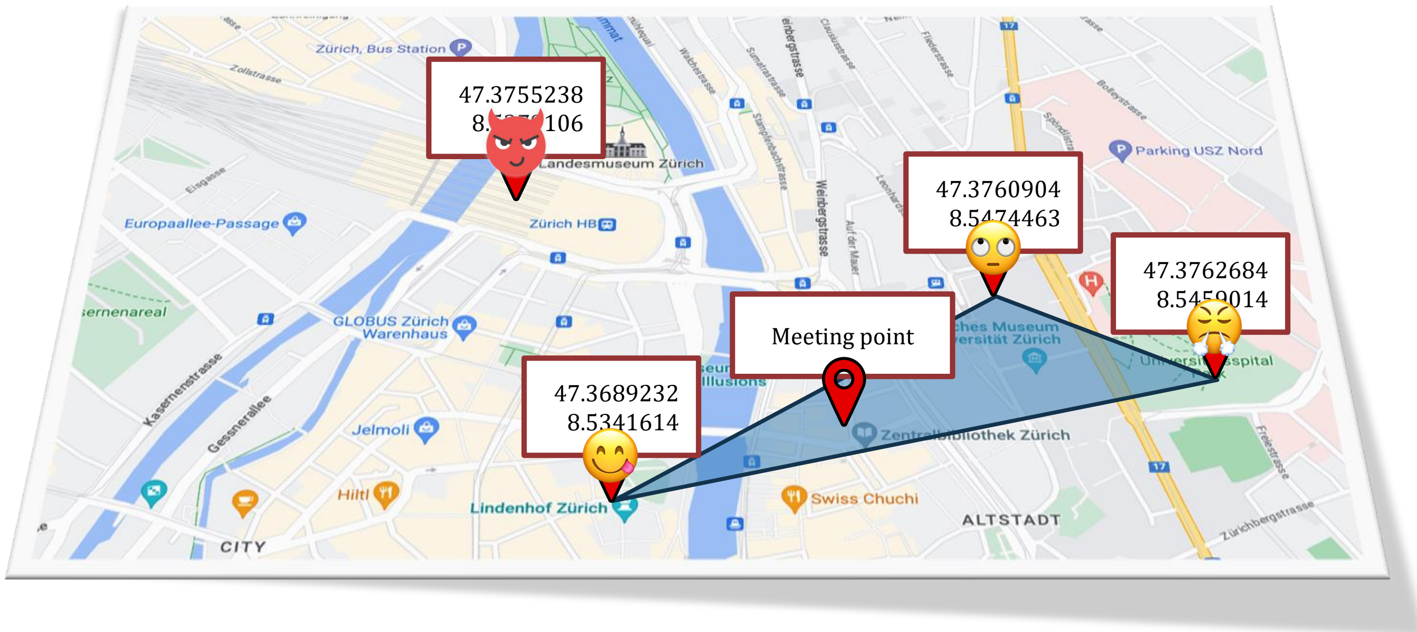

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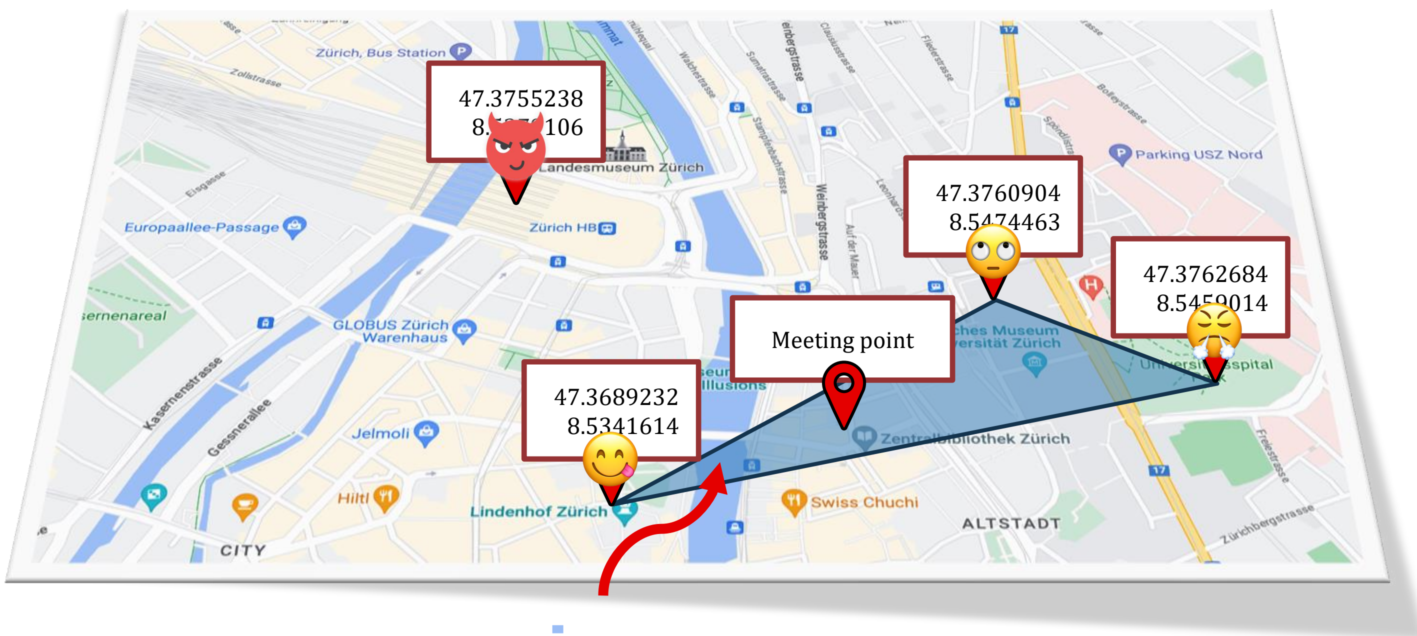

river

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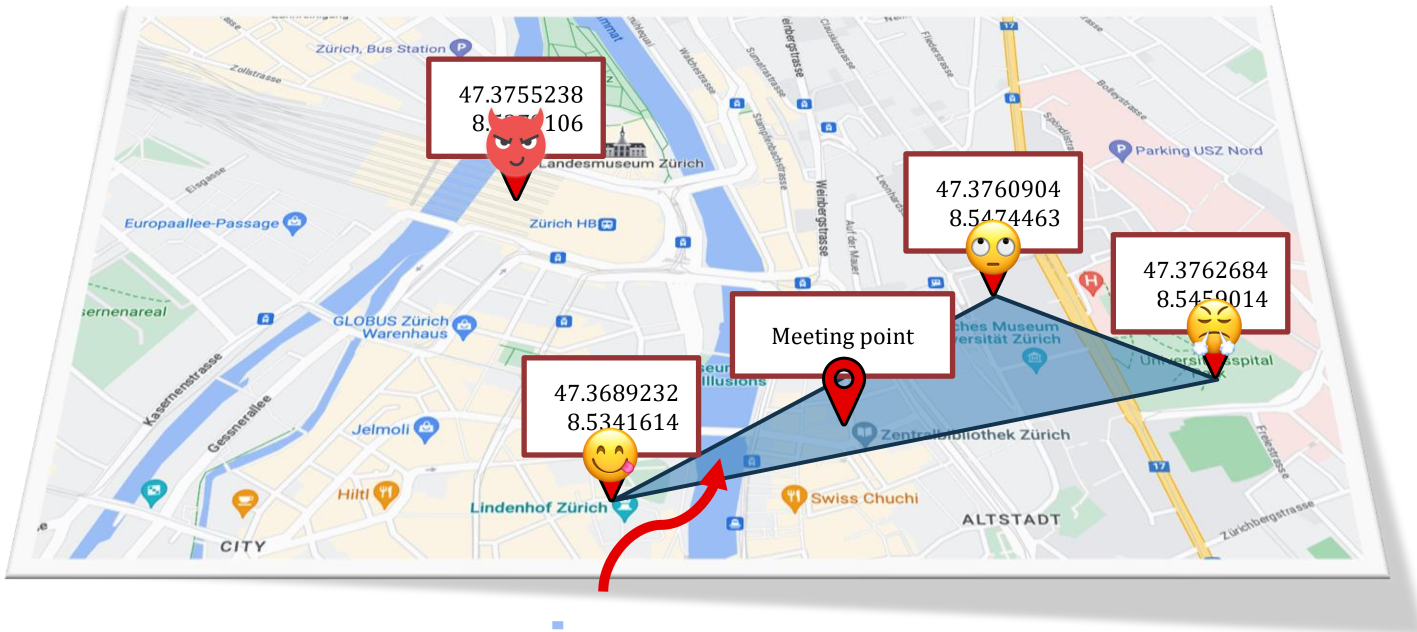

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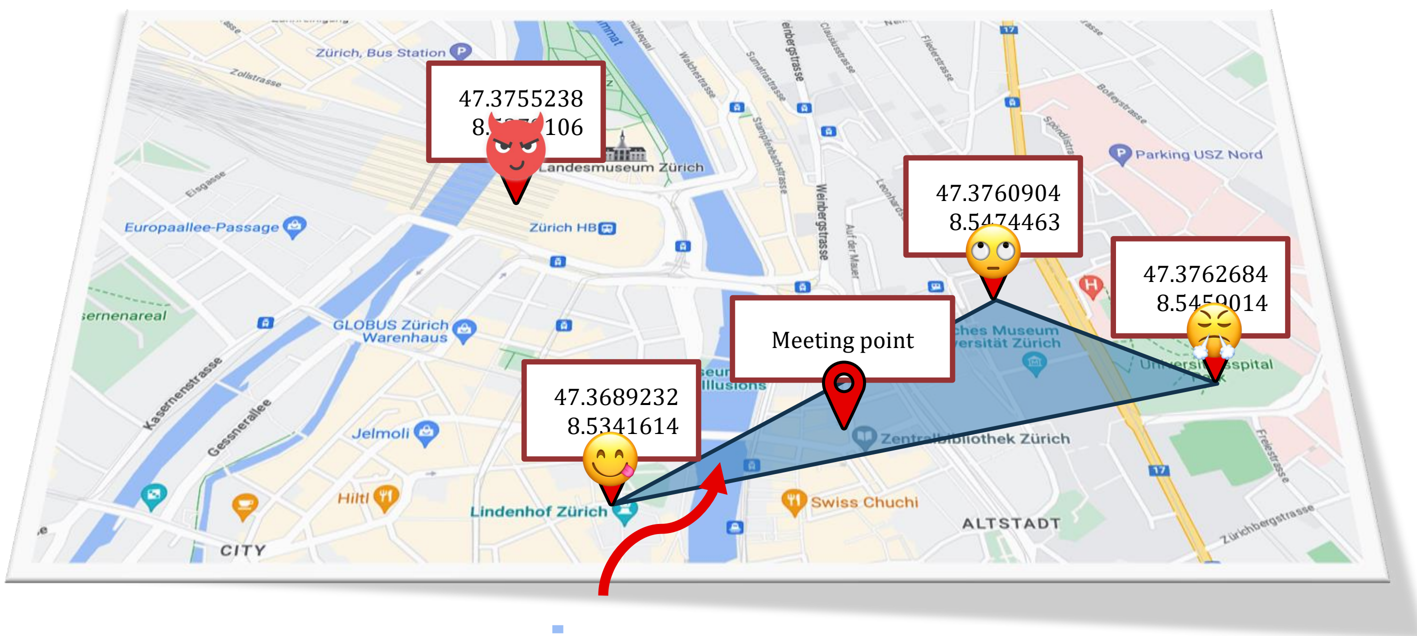

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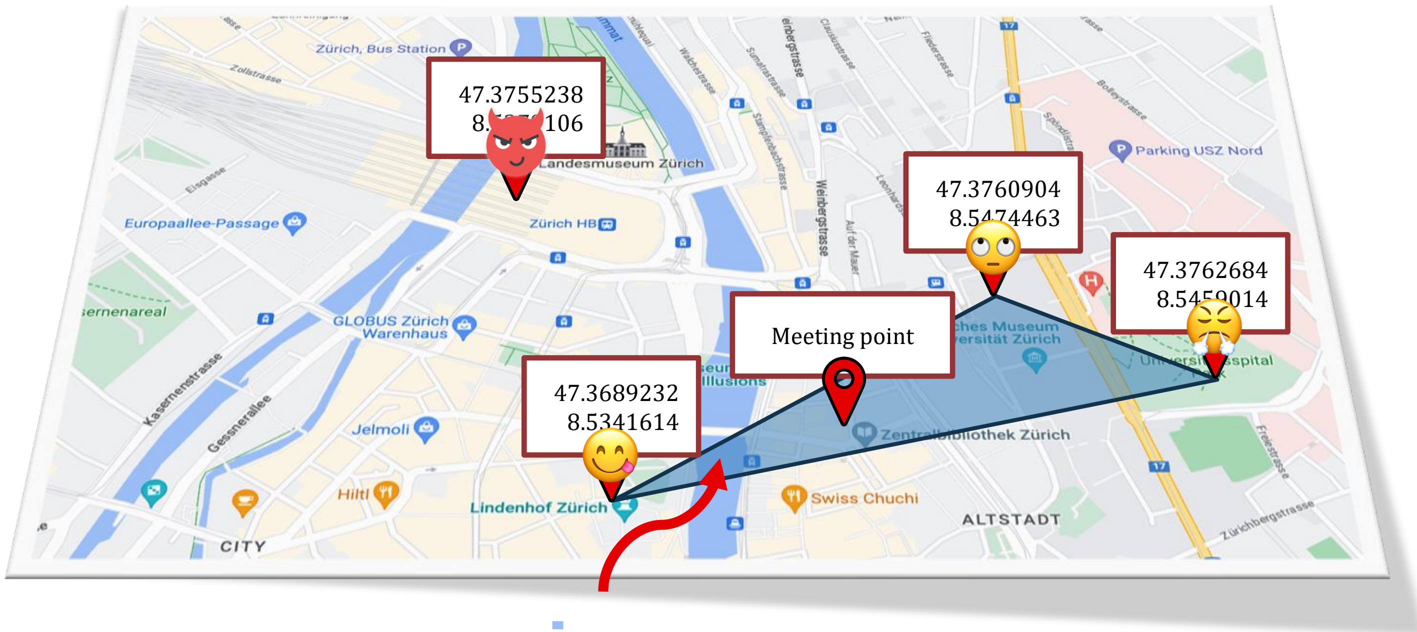

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A geometric invariant

The Helly number

Helly's Theorem

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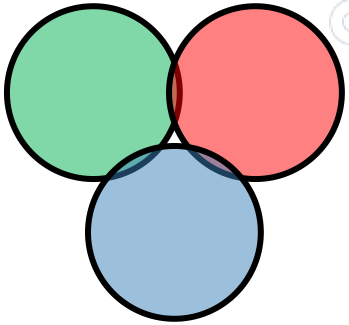

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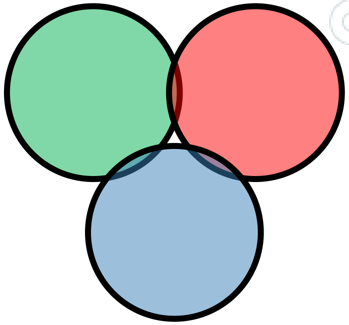

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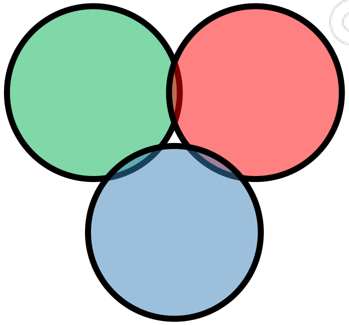

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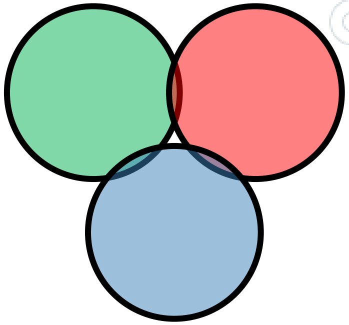

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$\boxed{\text{Convex Consensus (CC)}}$

A: depends on *network model*.

Pb.: How can we even specify t? $\boldsymbol{\omega}$

3.

(A)synchrony

And the best-of-both-worlds

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Network-agnostic model:

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- (**Question here:** what pairs (t_s, t_a) are possible?)

4.

Results

Tight results for all 3 models!

CC Solvable Iff

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Synchronous model:

CC Solvable Iff

Synchronous model: $\color{red}{t} < \frac{n}{\omega}$

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Synchronous model: $t < \frac{n}{\omega}$

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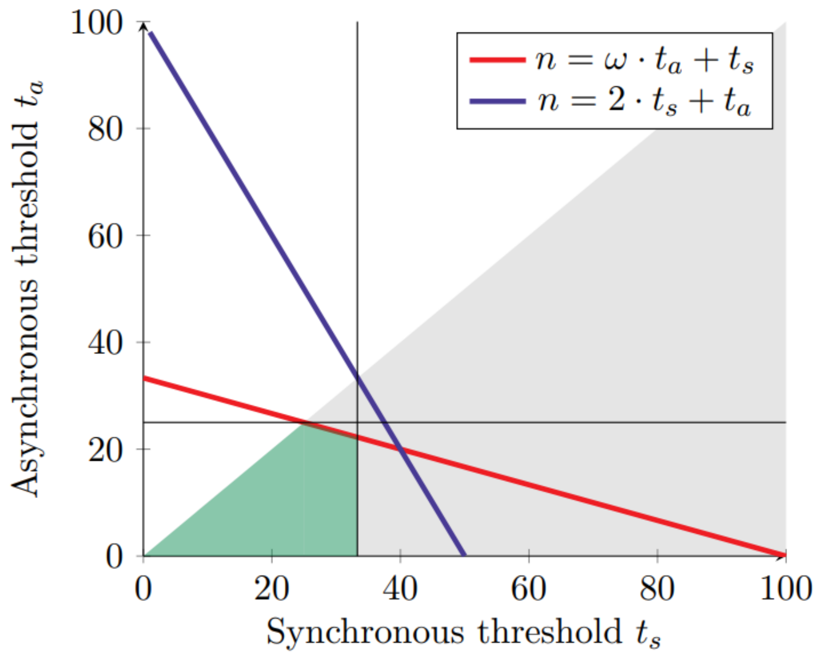

(b) $\omega = 3$

CC Solvable Iff

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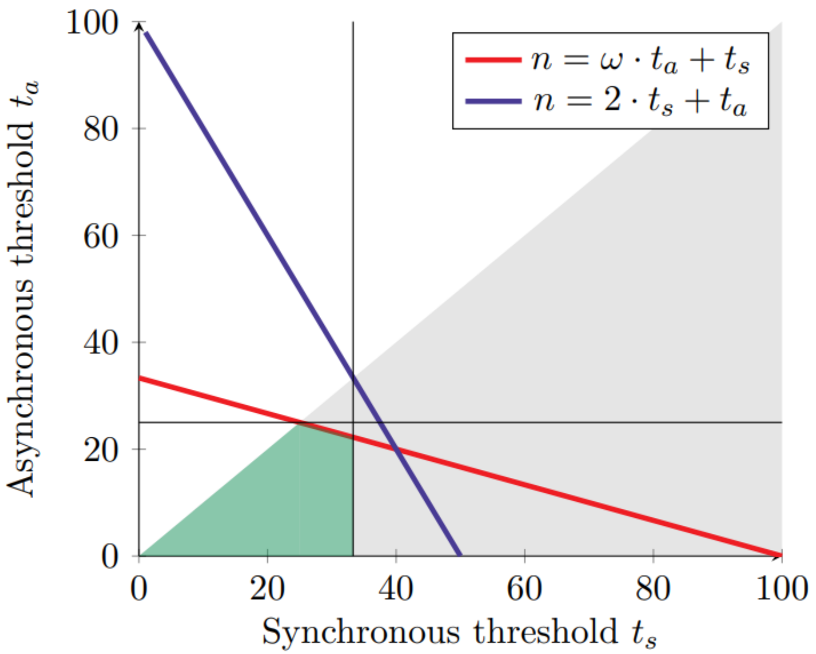
Imp1 Imp2 Imp3



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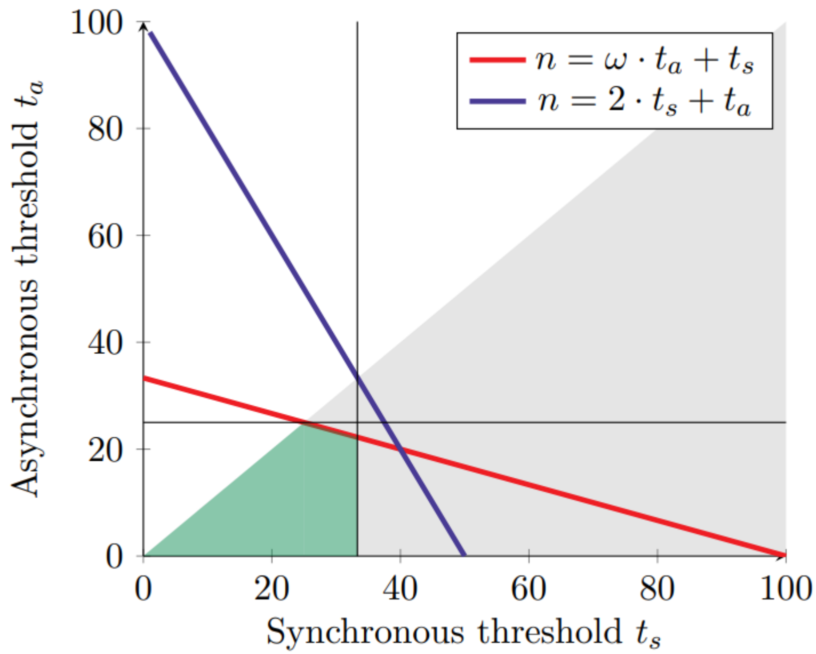

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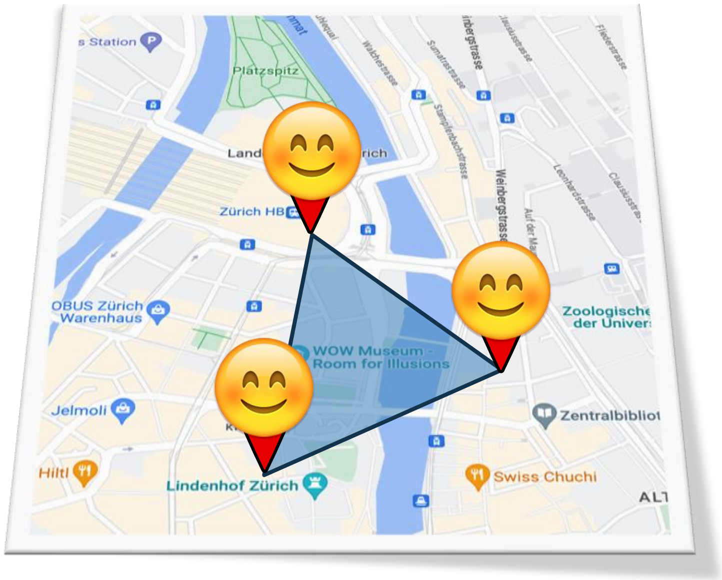

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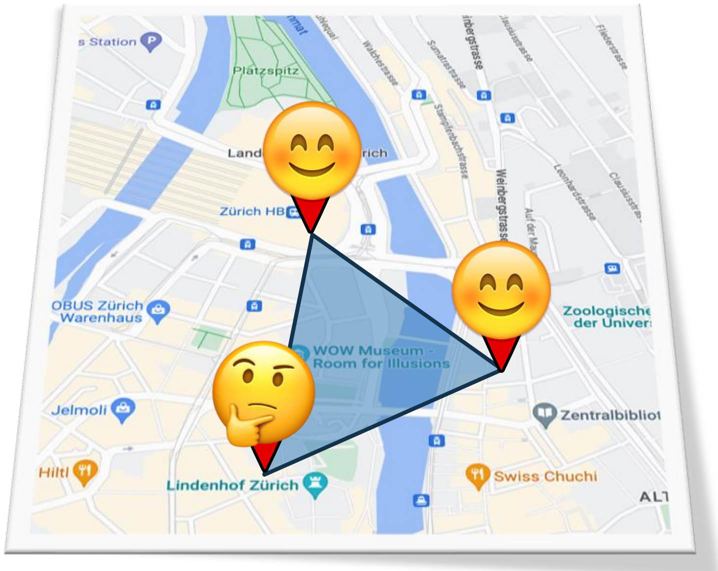

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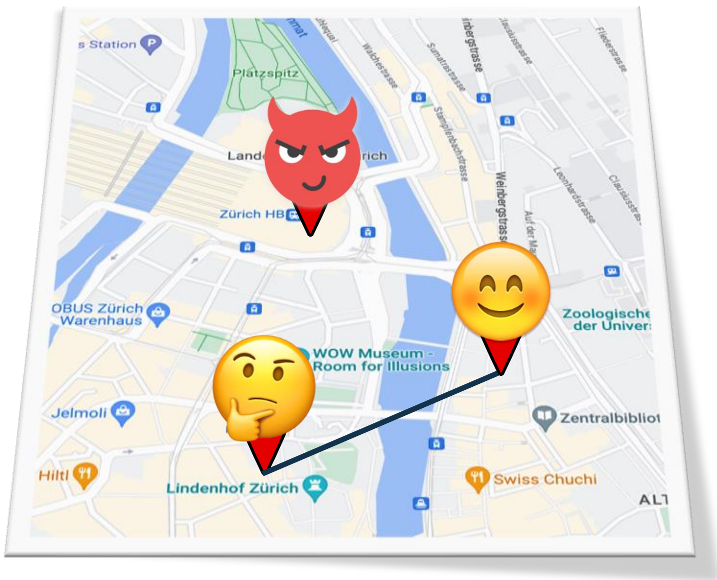

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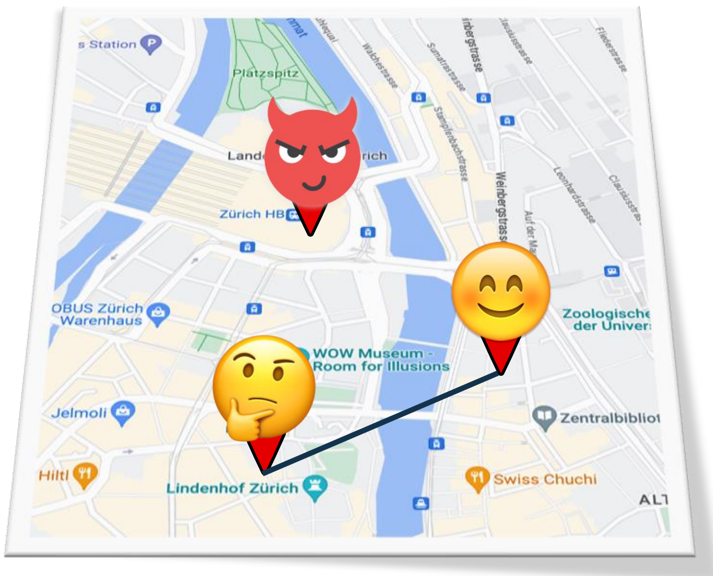

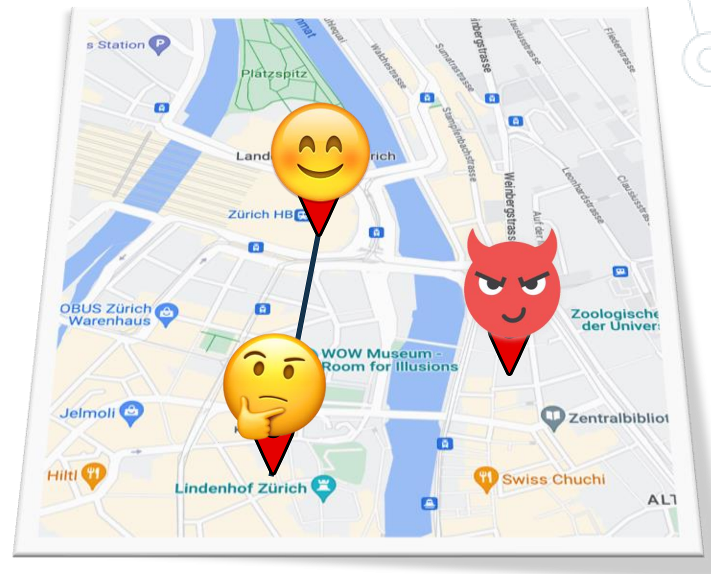

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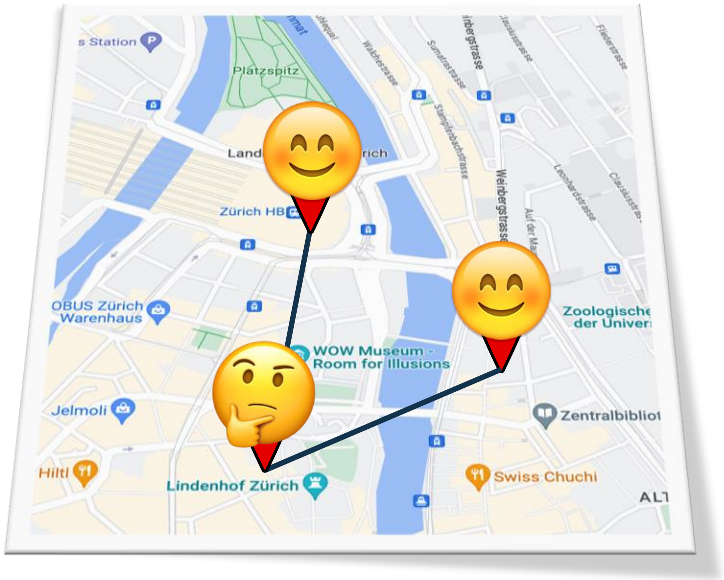

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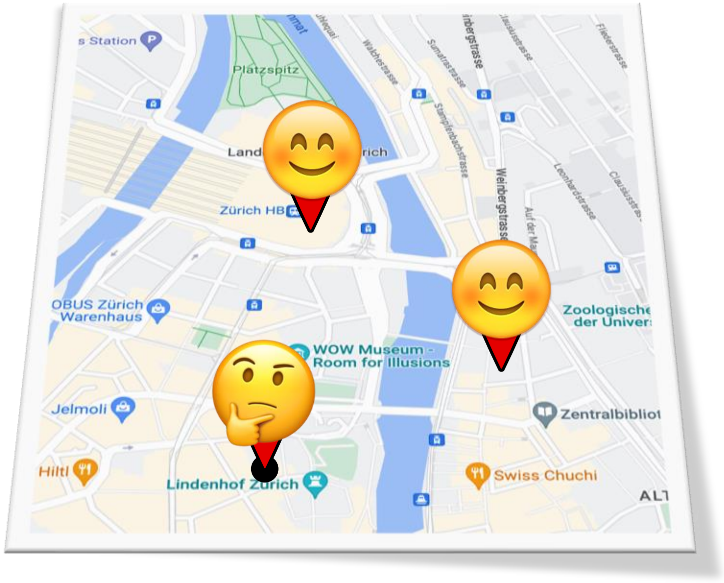

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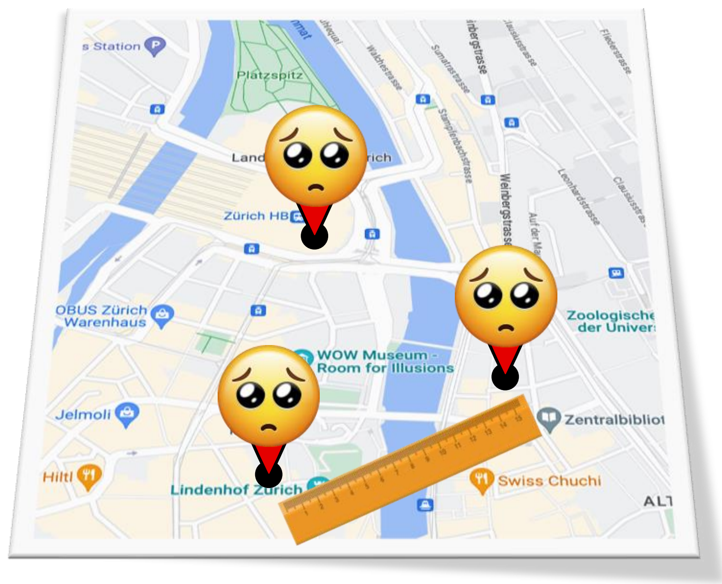

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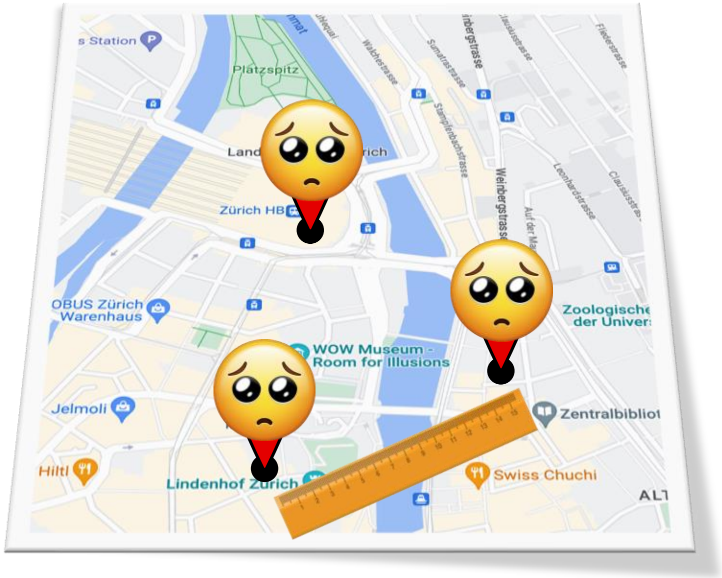

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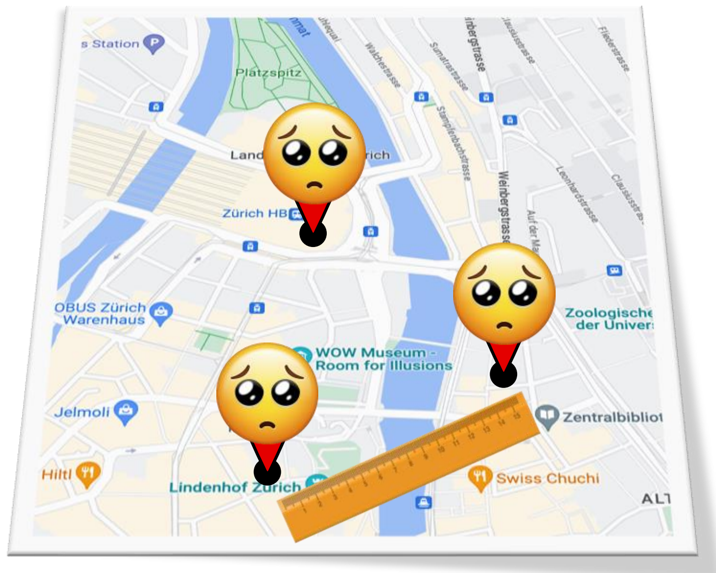

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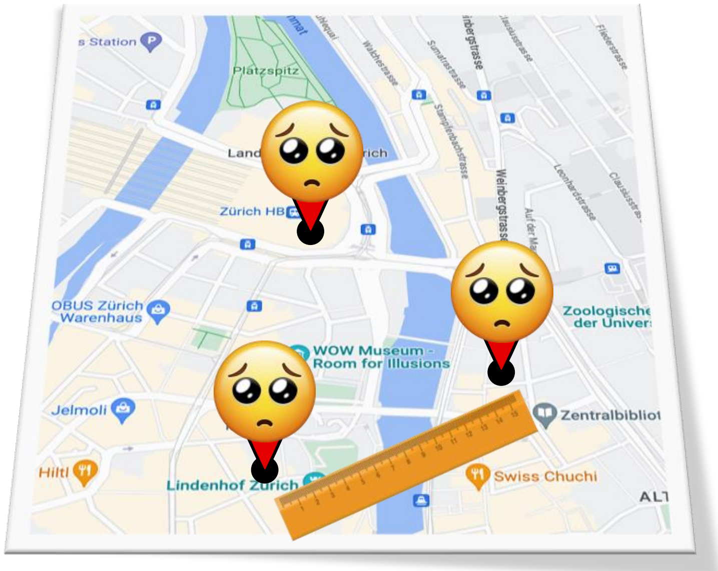

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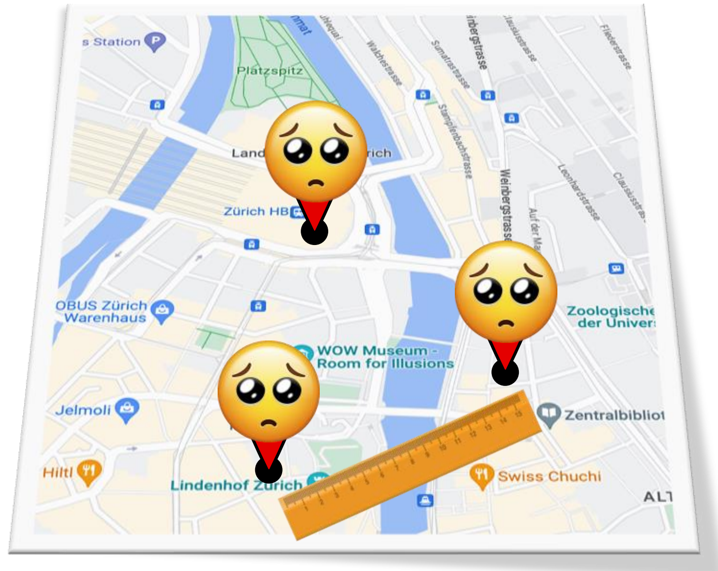

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- *Bad* instance guaranteed by def. of ω.
- Previously known: $\mathbb{R}^D$ and *convex geometries*.

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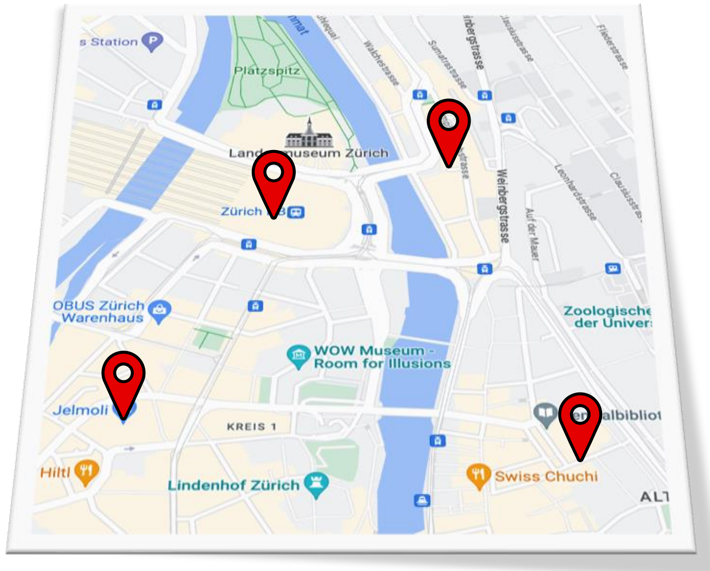
Safe Area

Safe Area

e.g., 4 values in the common view, ≤ 1 corrupted, what to do?



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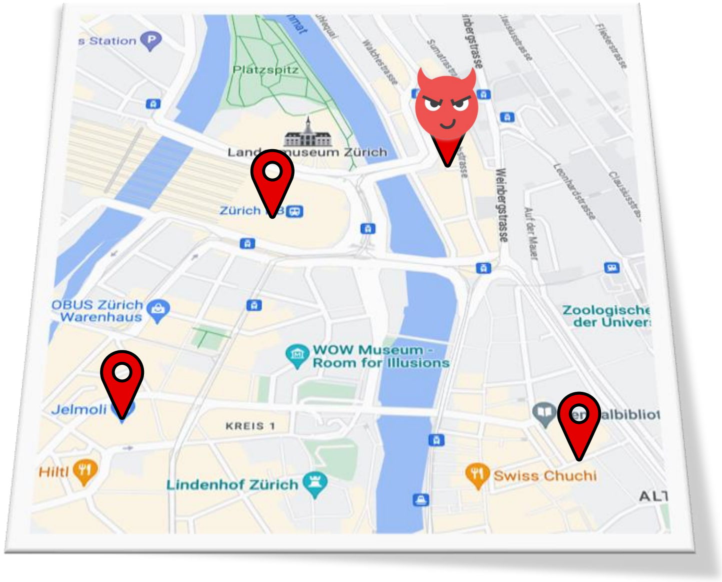

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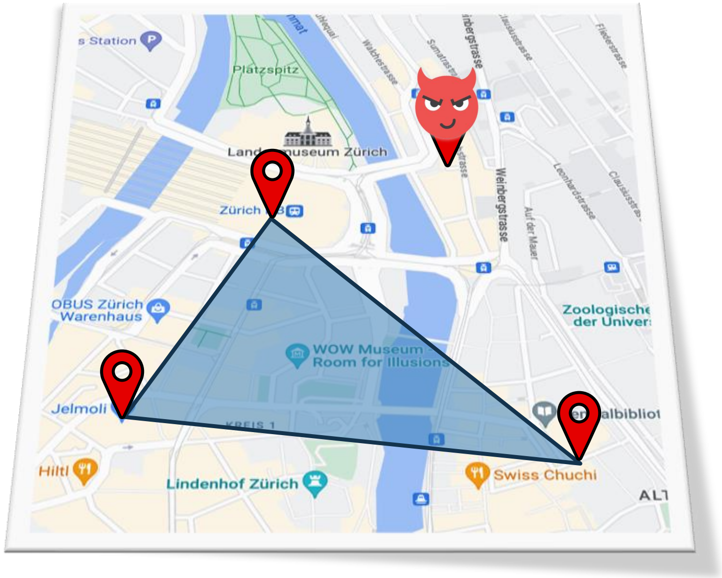

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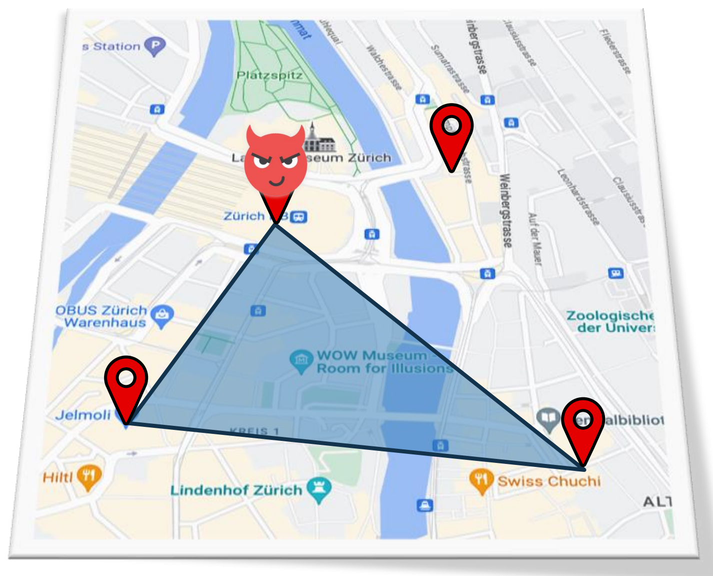

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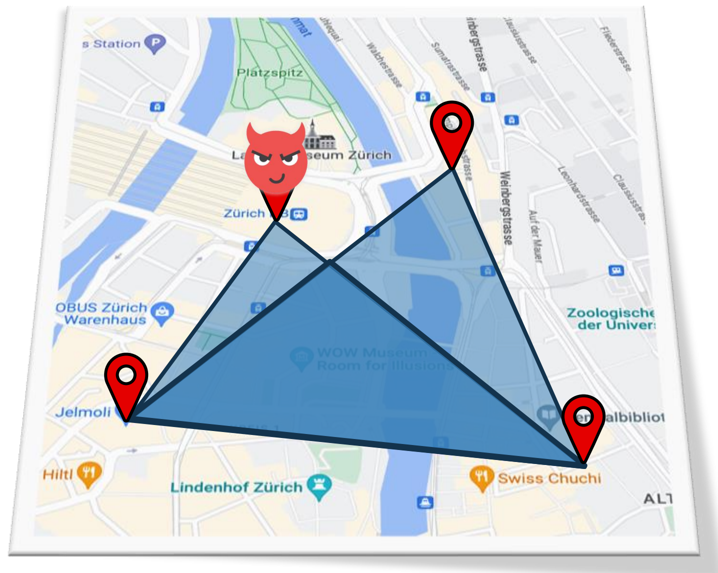

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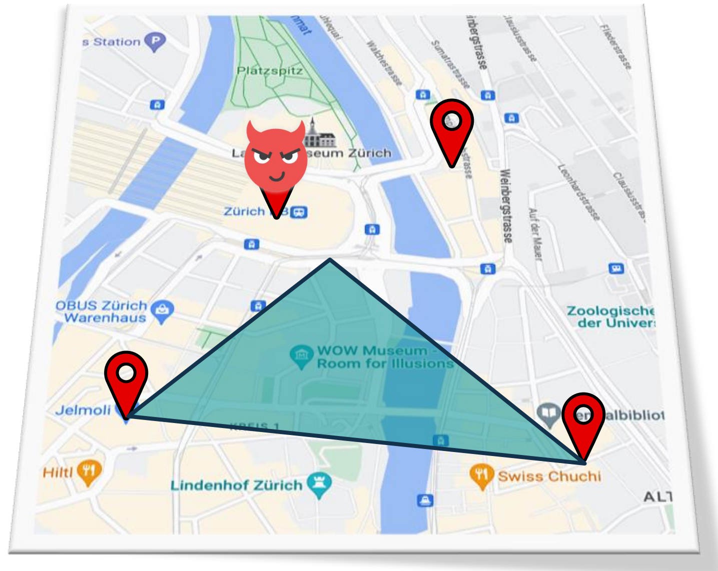

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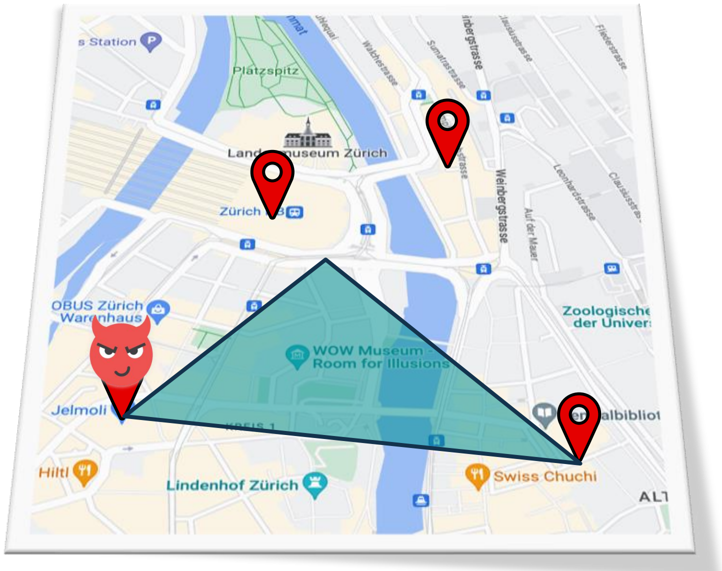

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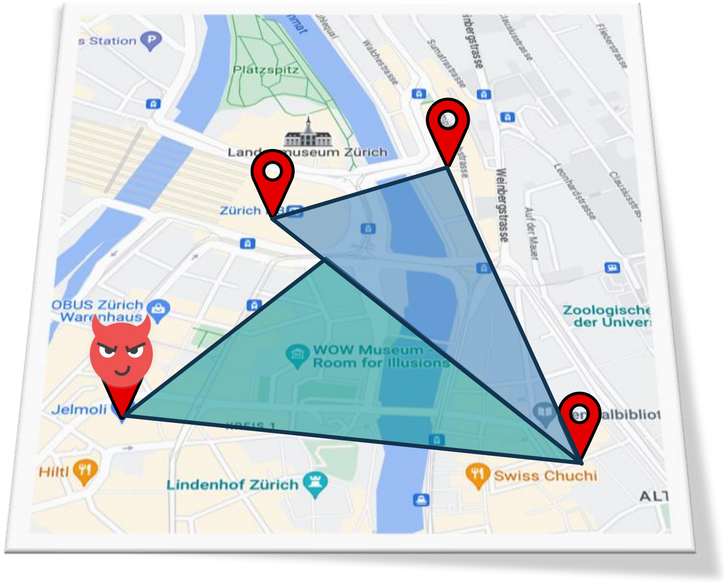

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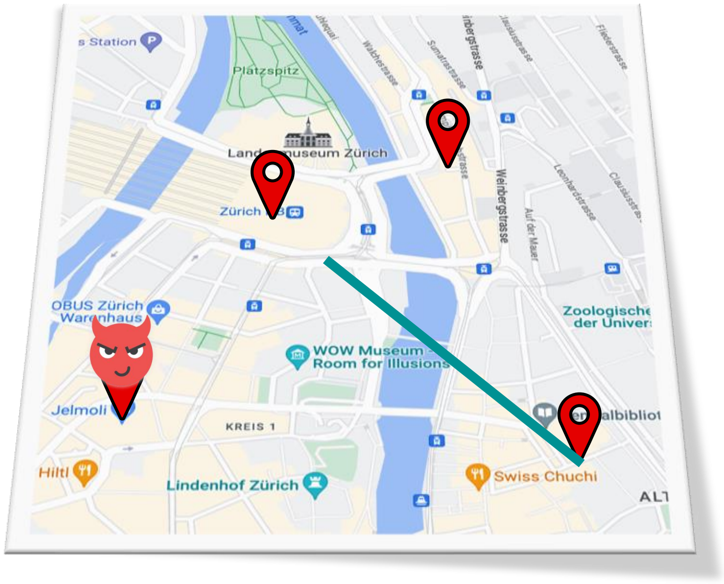

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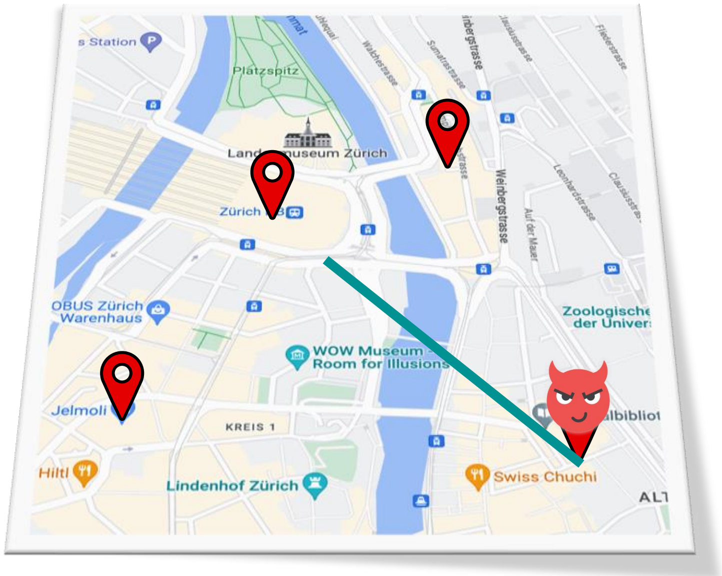

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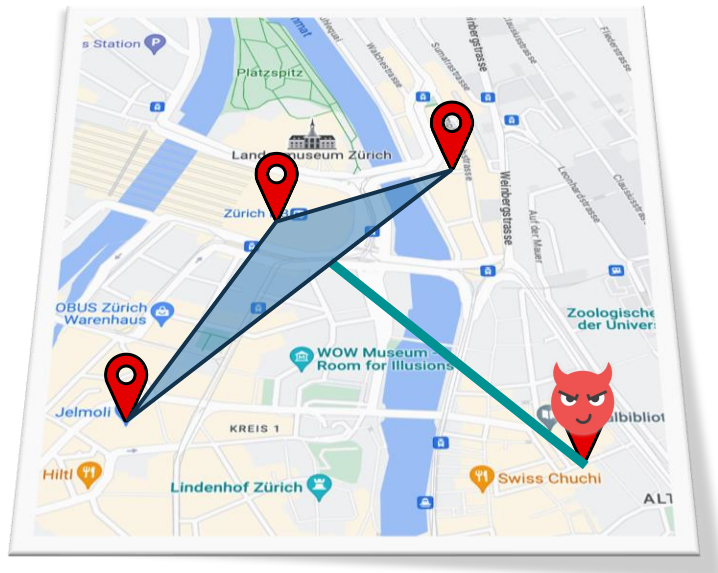

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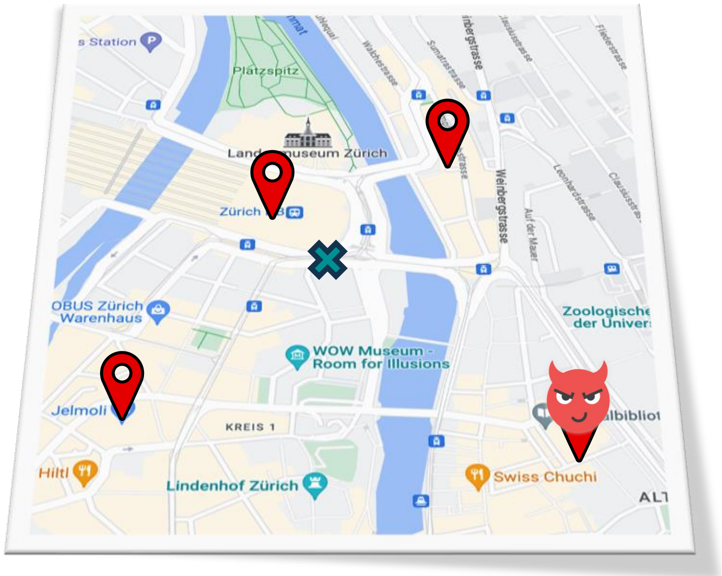

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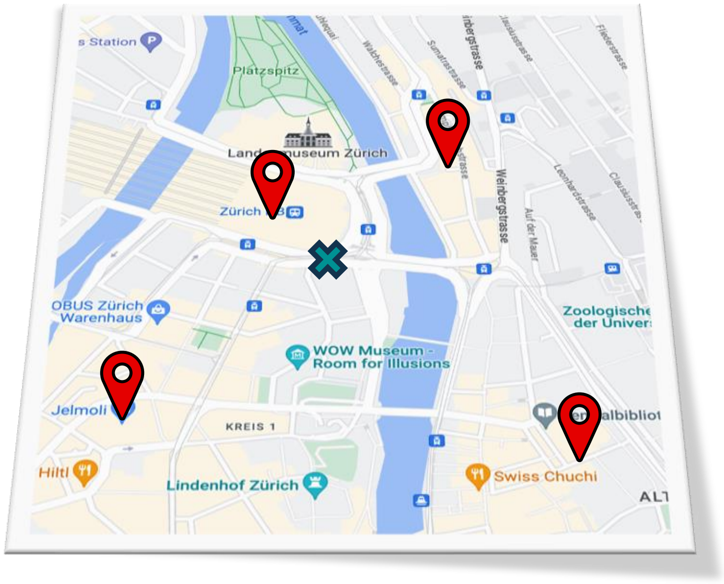

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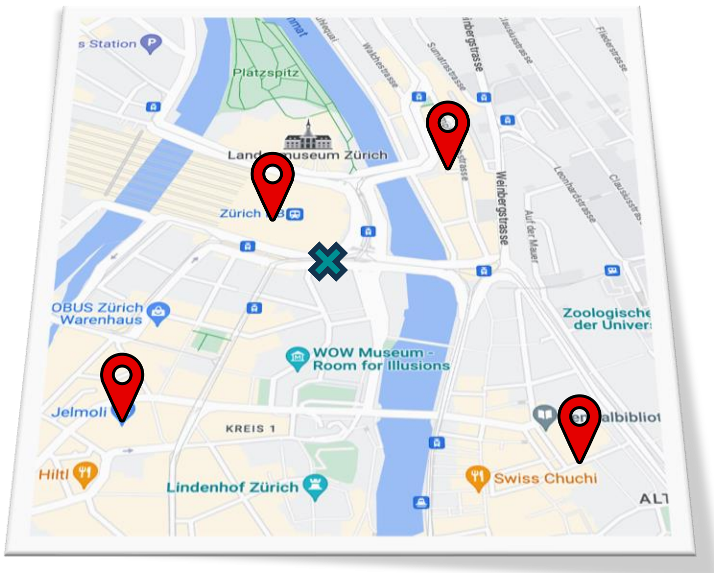

Generally: a values in the common view, $\leq$ b corrupted

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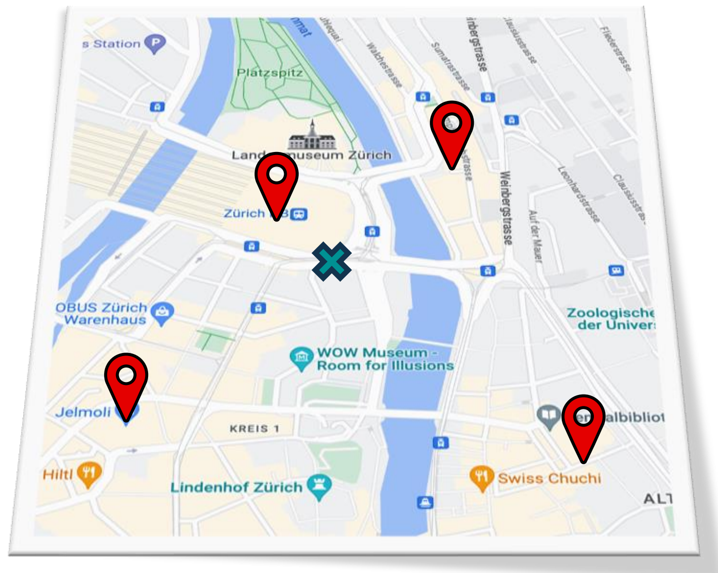

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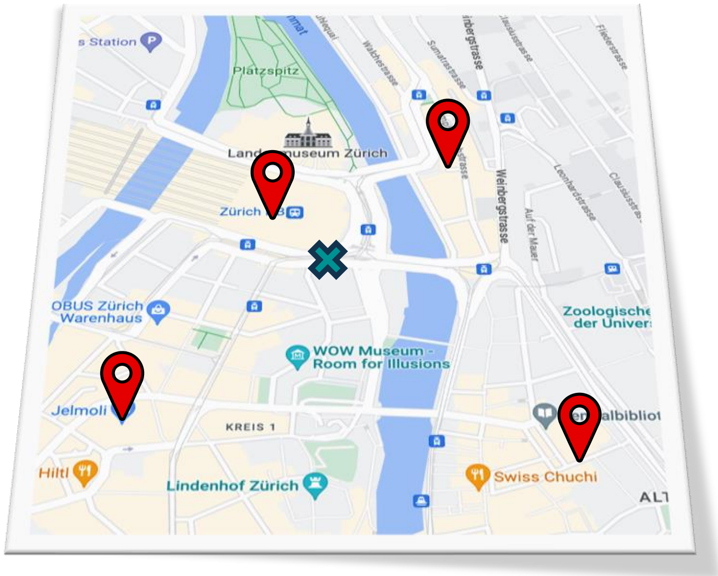

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Safe area: intersection of convex hulls of subsets of size $a - b$

Any point in safe area is valid, select one *deterministically*.

Safe Area (cont’d)

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intersect; Pigeonhole Principle; $\max(\omega t_s, \omega t_a + t_s) < n$

5.

Outlook

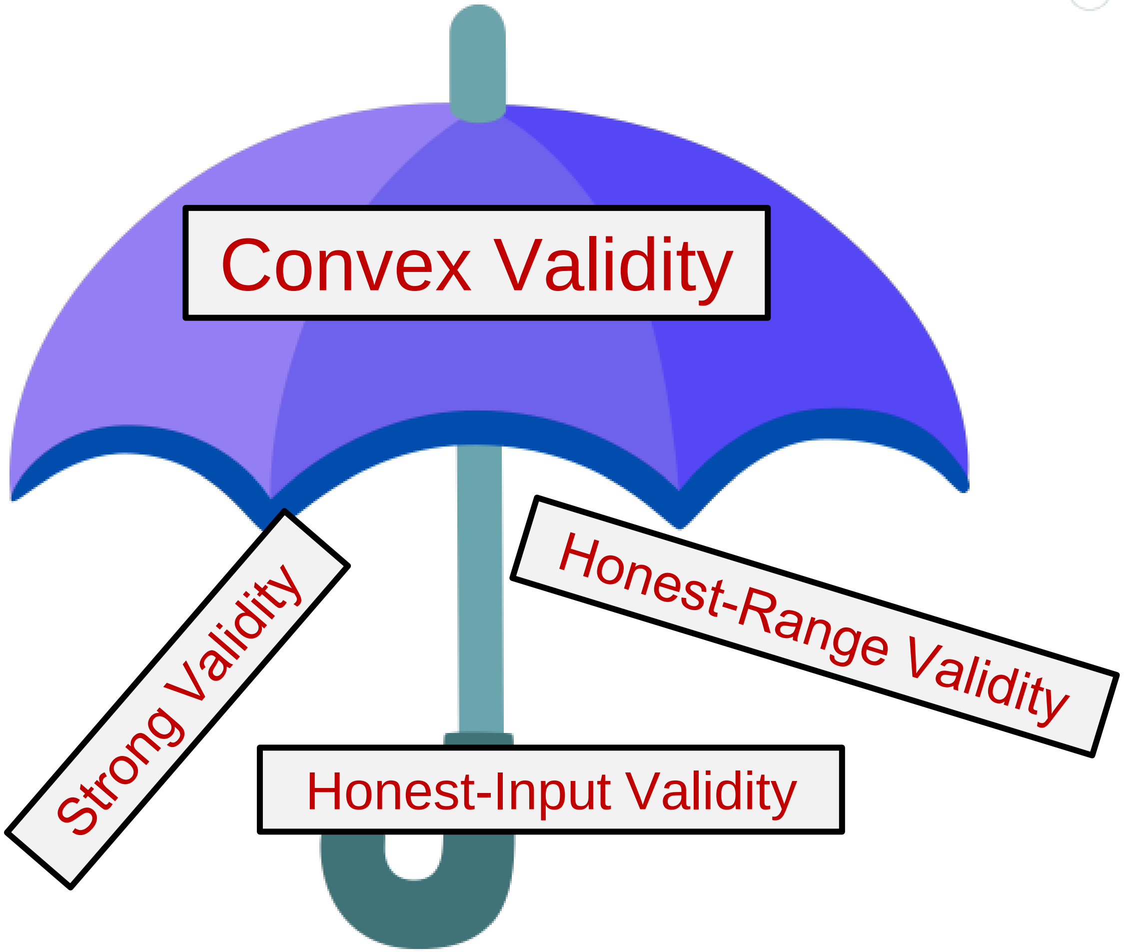
And shameless self-advertising

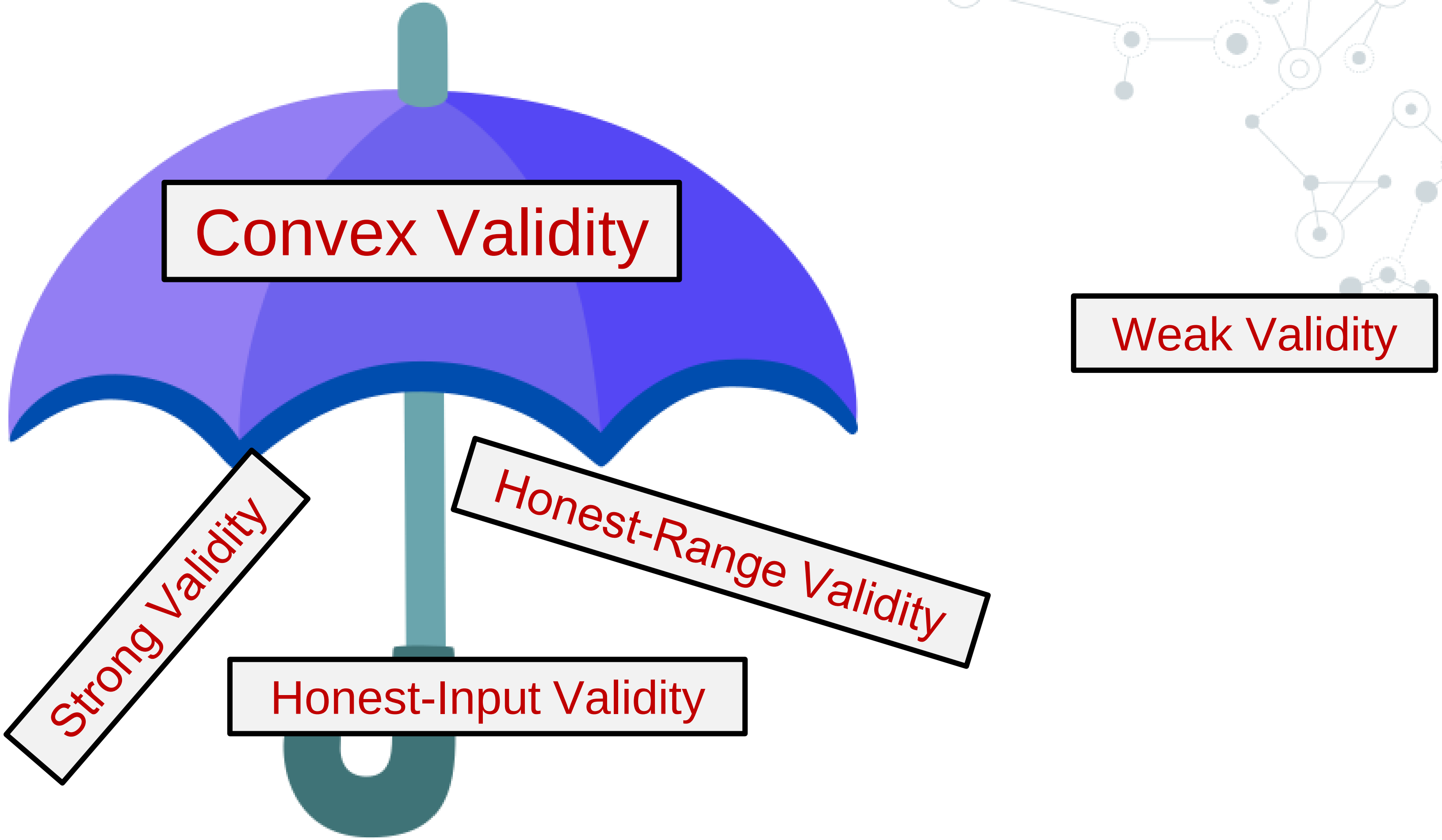
Convex Validity

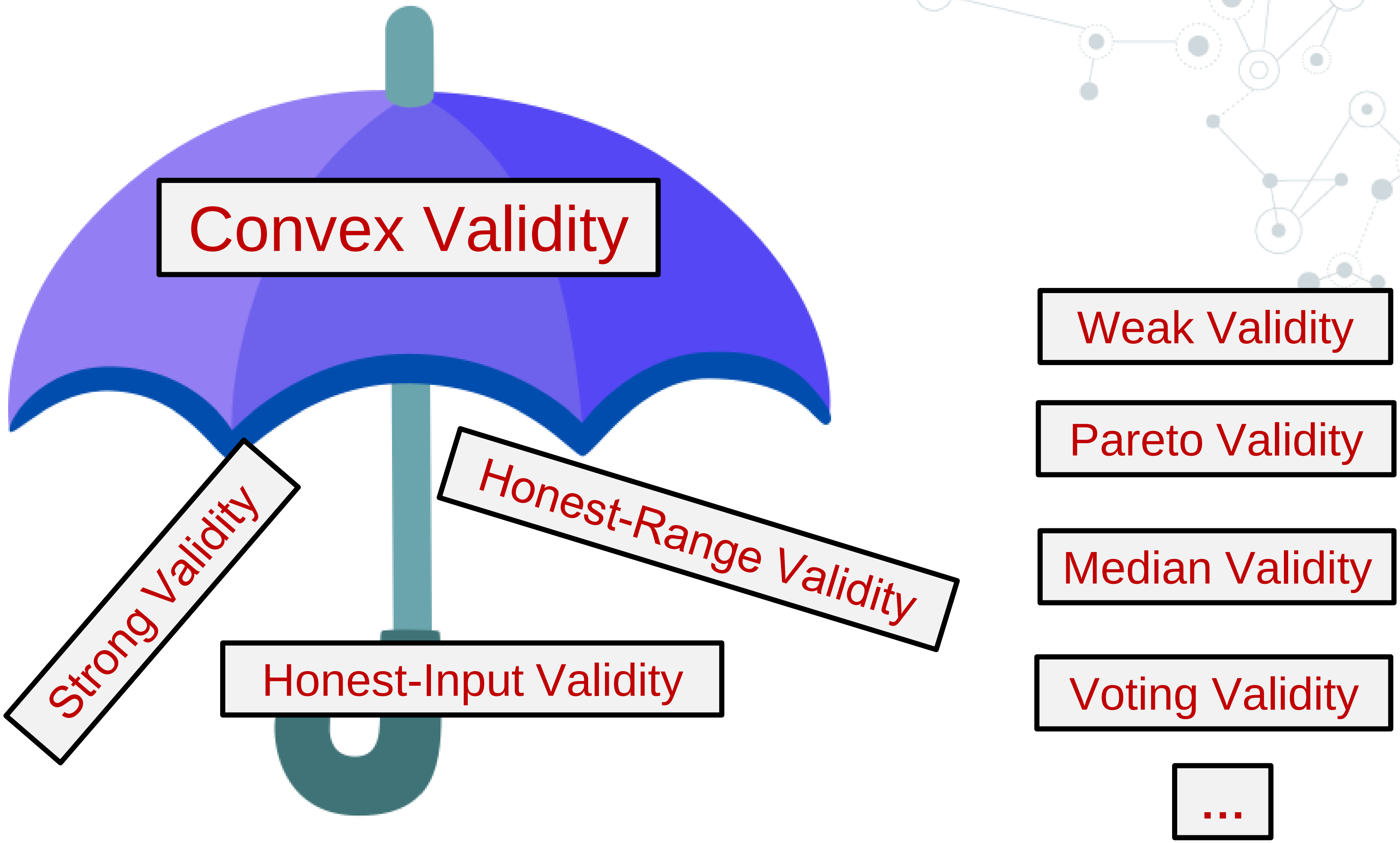
Strong Validity

Honest-Range Validity

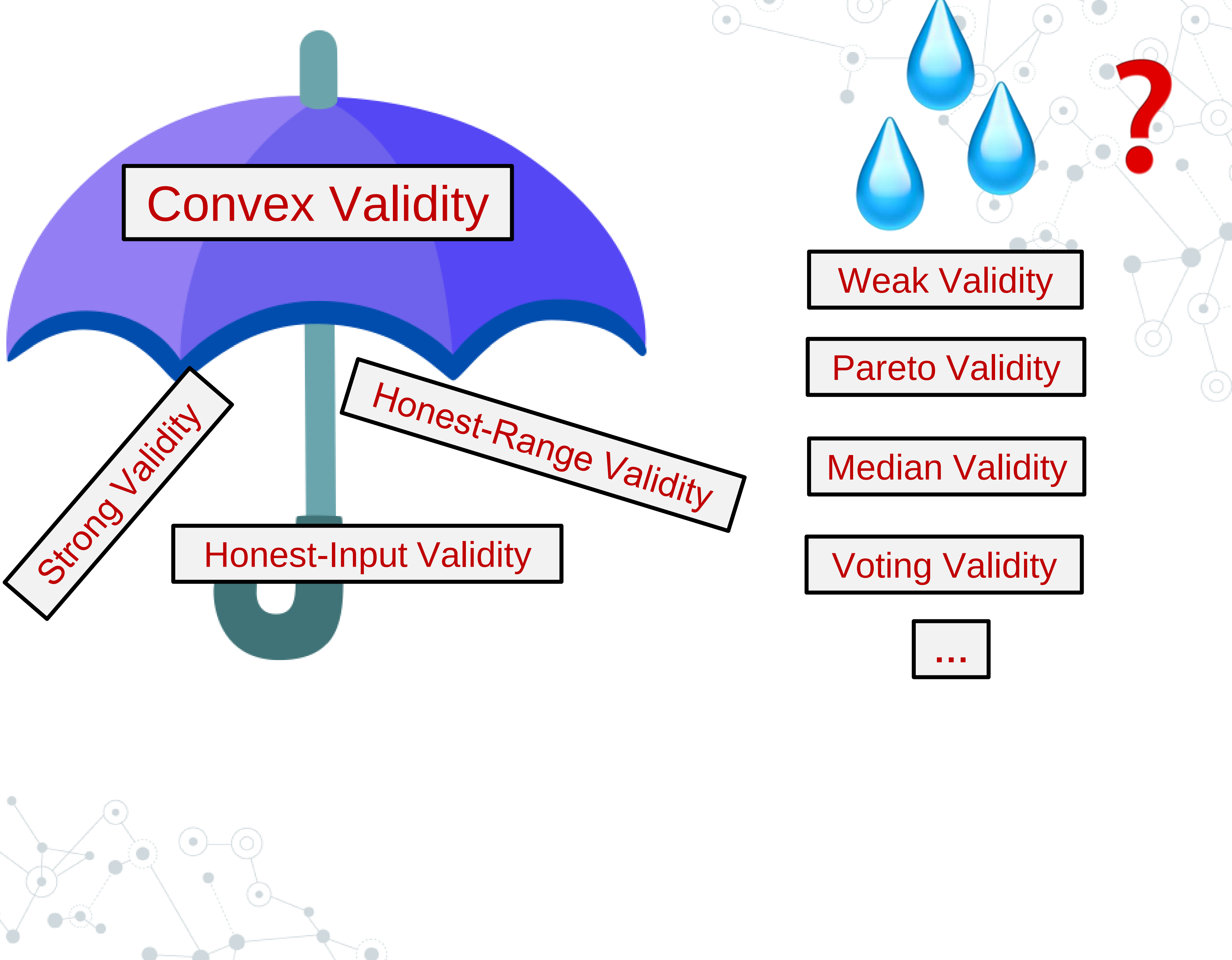
Honest-Input Validity


Convex Validity
Strong Validity
Honest-Range Validity
Honest-Input Validity

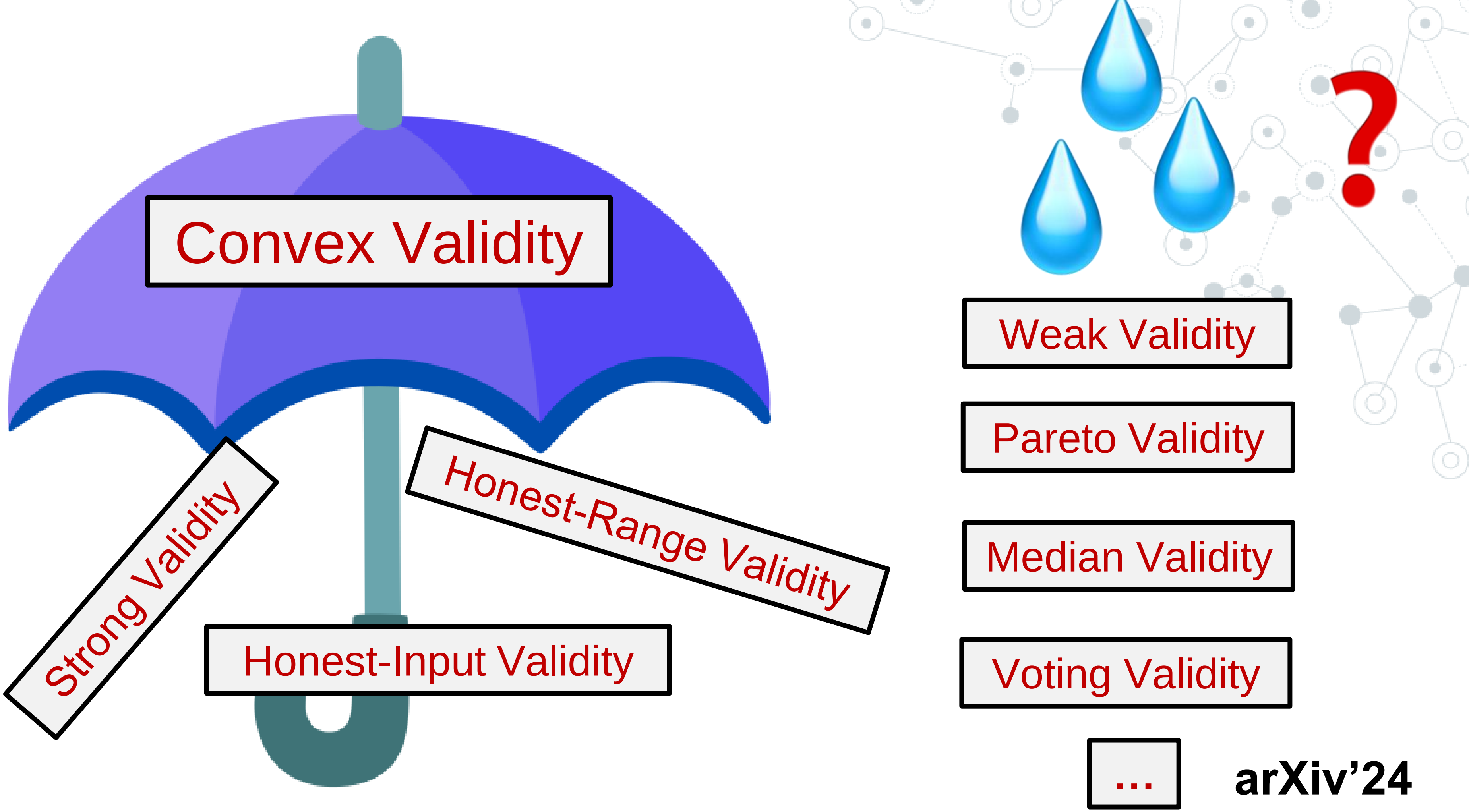

Convex Validity
Strong Validity
Honest-Range Validity
Honest-Input Validity
Weak Validity



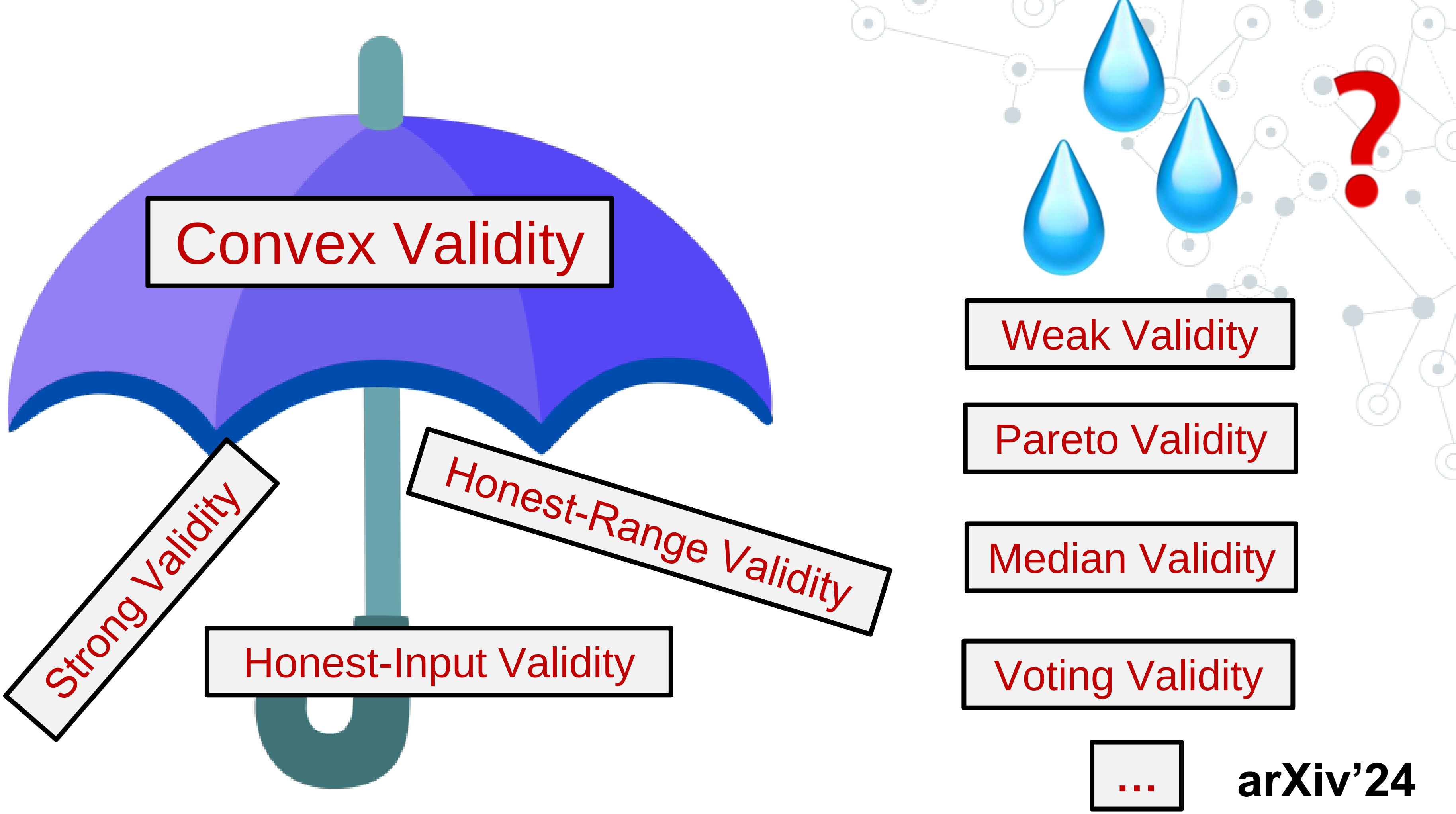
Convex Validity
Strong Validity
Honest-Range Validity
Honest-Input Validity
Weak Validity
Pareto Validity
Median Validity
Voting Validity
...



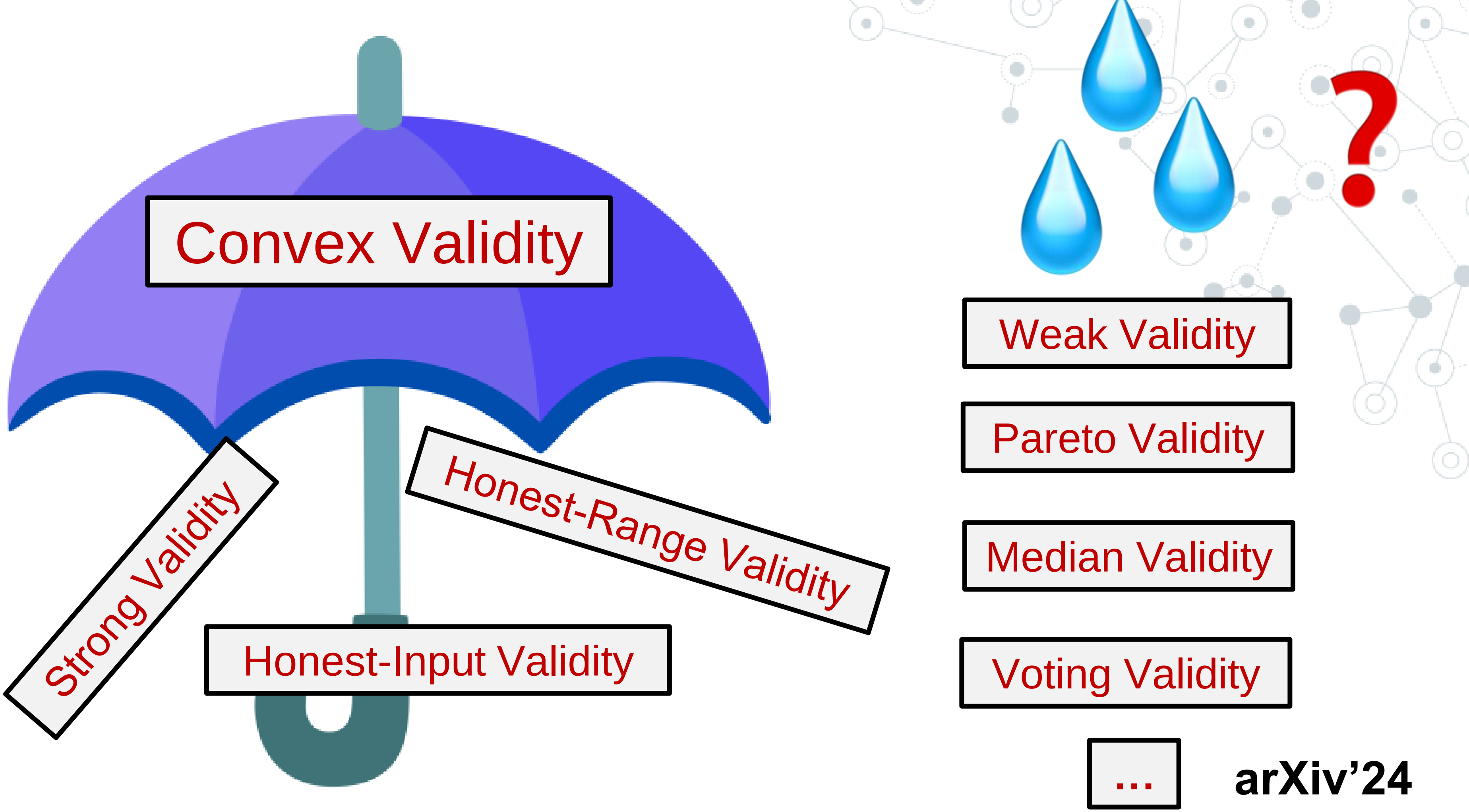
Convex Validity
Strong Validity
Honest-Range Validity
Honest-Input Validity
Weak Validity
Pareto Validity
Median Validity
Voting Validity
...



Convex Validity
Strong Validity
Honest-Range Validity
Honest-Input Validity
Weak Validity
Pareto Validity
Median Validity
Voting Validity
...
arXiv'24



Approximate Agreement?



Approximate Agreement?

Other adversaries?

Hope you enjoyed!

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