# Networks Cannot Compute Their Diameter in Sublinear Time

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Roger Wattenhofer

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# Distributed network
# Graph G of n nodes

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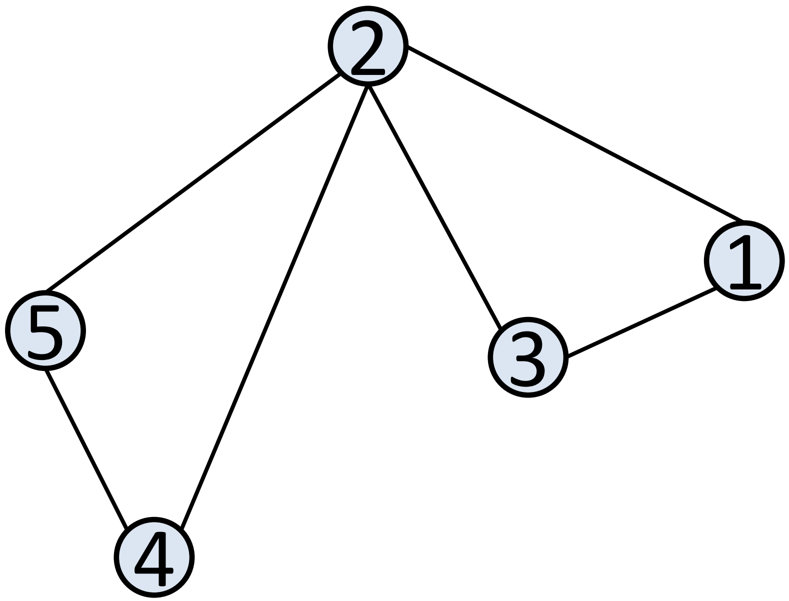

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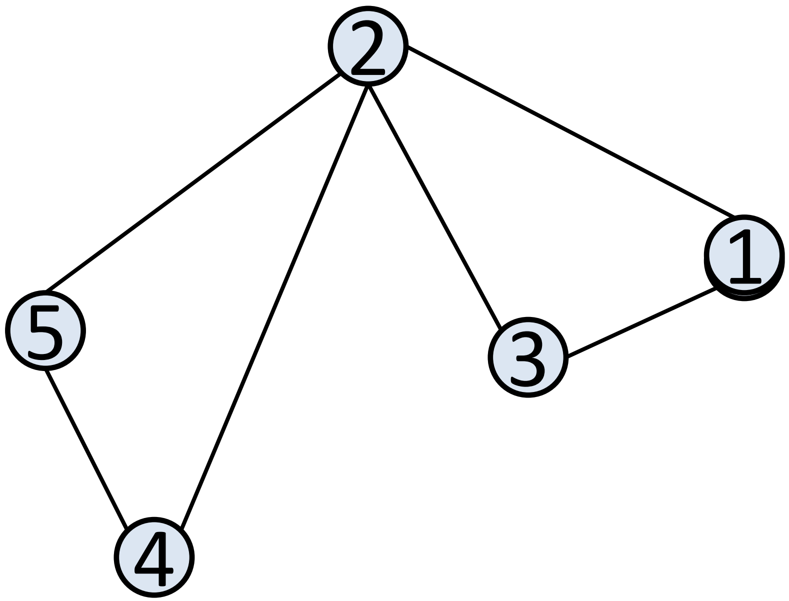


Local information only

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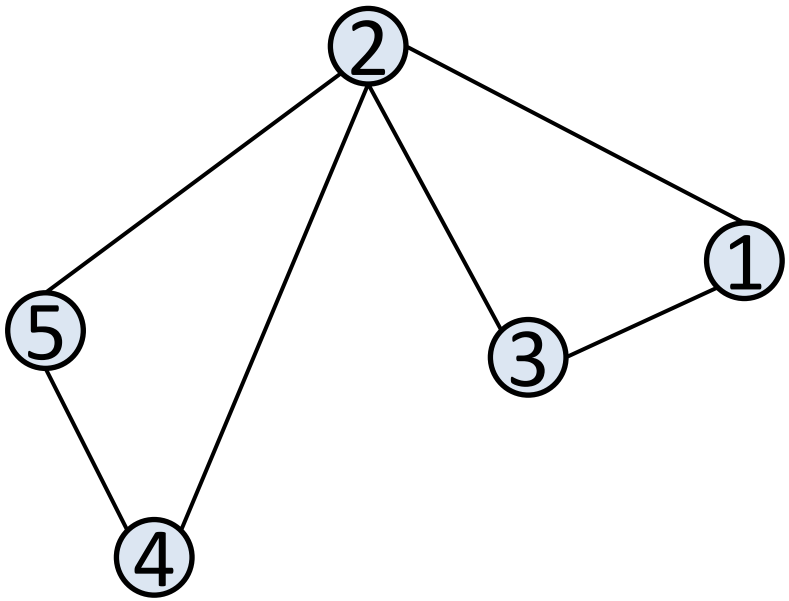


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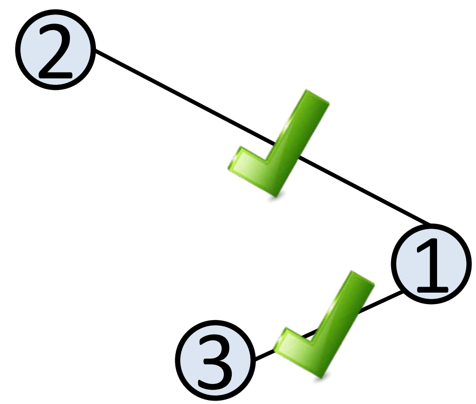


Slide inspired by Danupon Nanongkai

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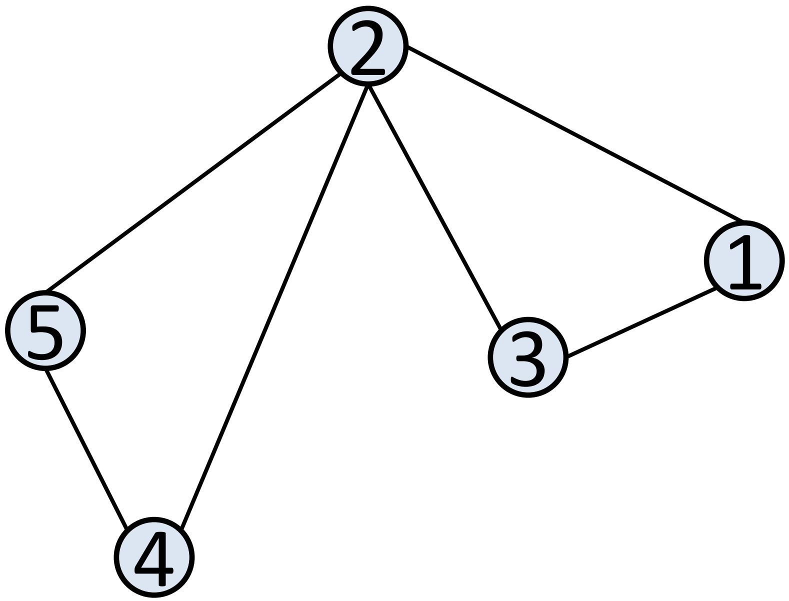


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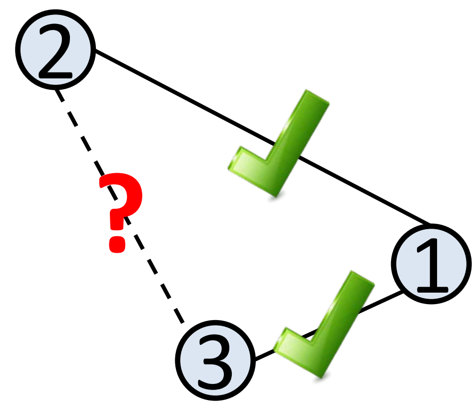

# Distributed network
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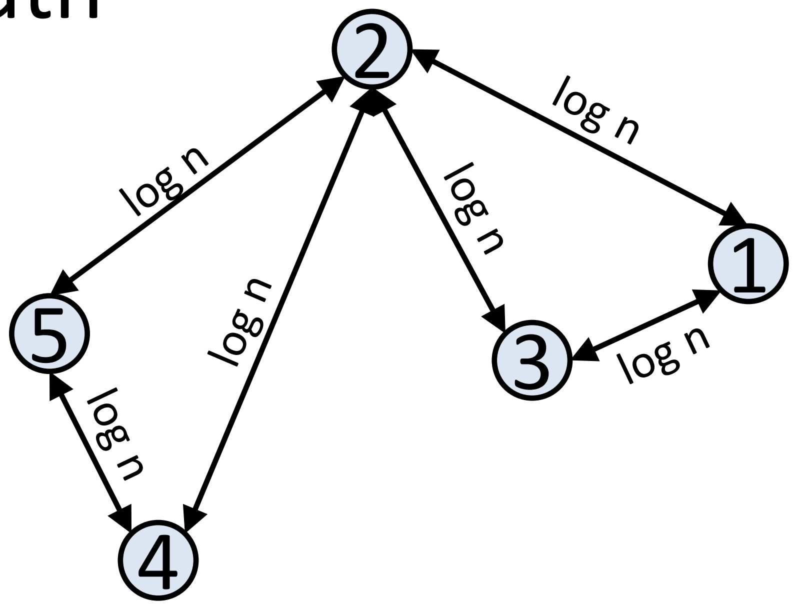
Limited bandwidth

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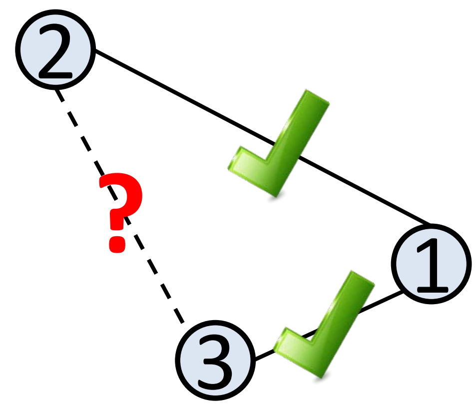

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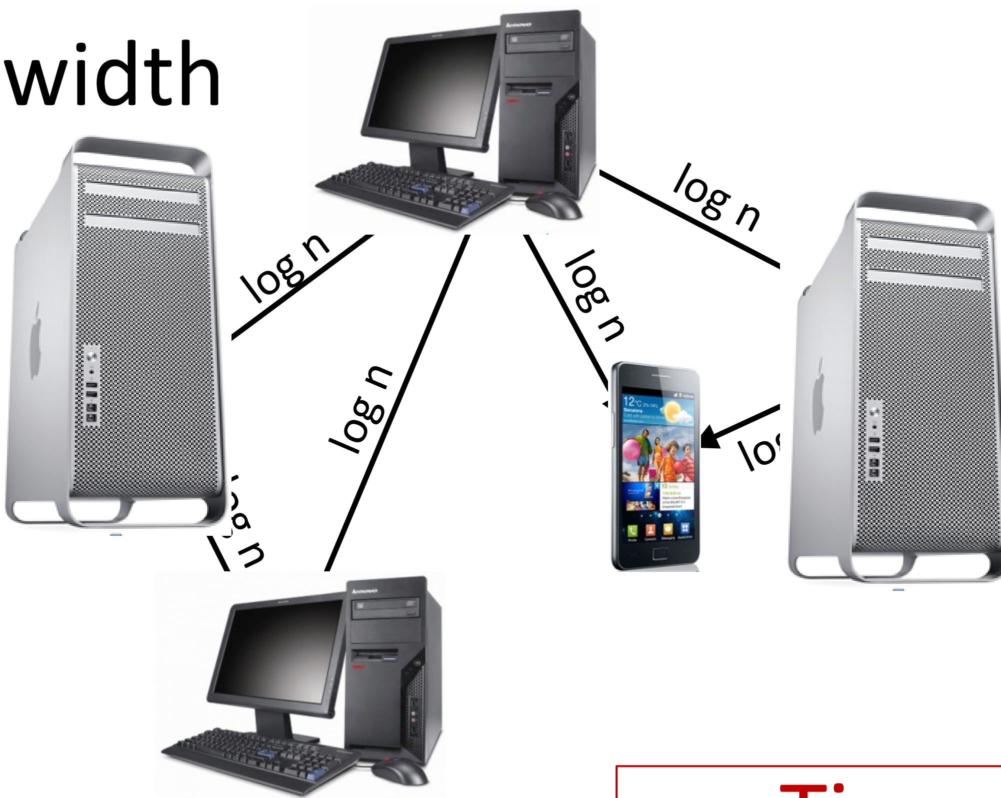
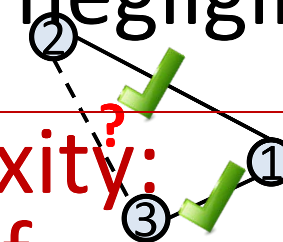



Synchronized

Internal computations negligible

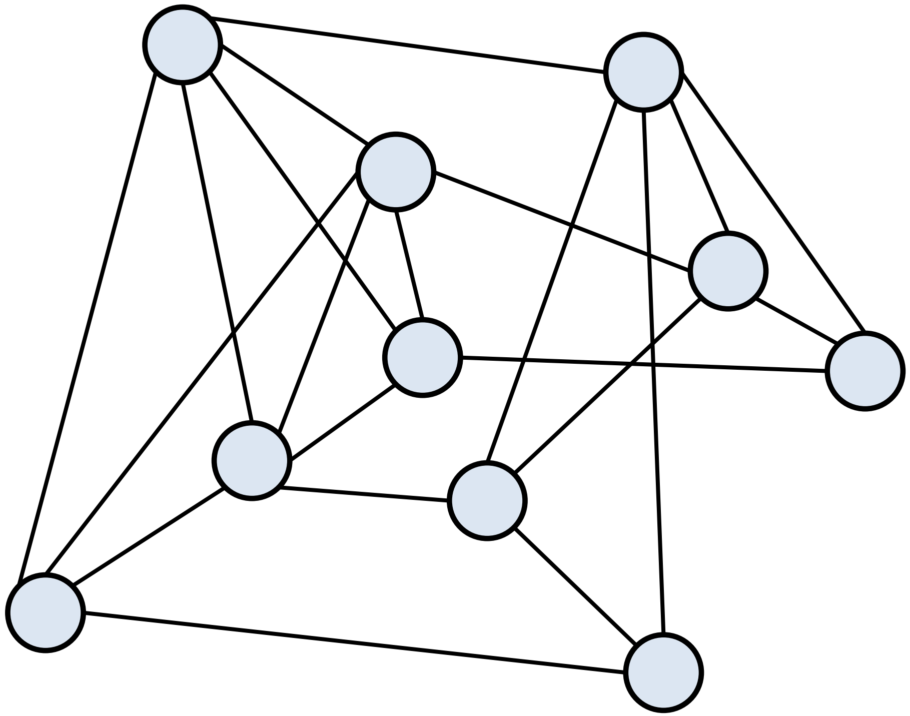
Local information only




Time complexity: number of communication rounds

# Distributed algorithms: a simple example

# Count the nodes!



# Count the nodes!

1. Compute
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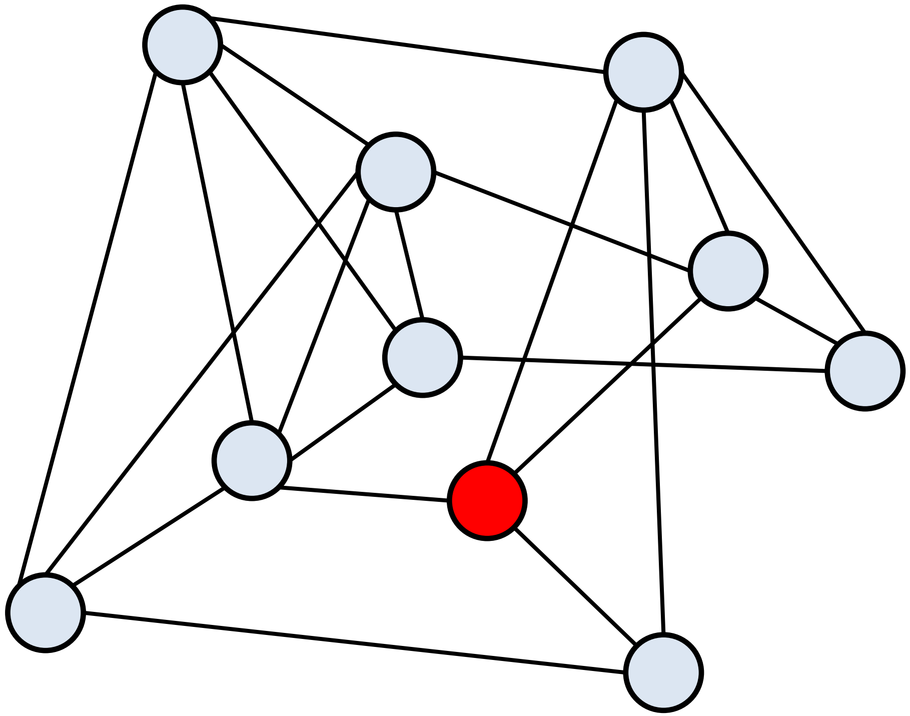

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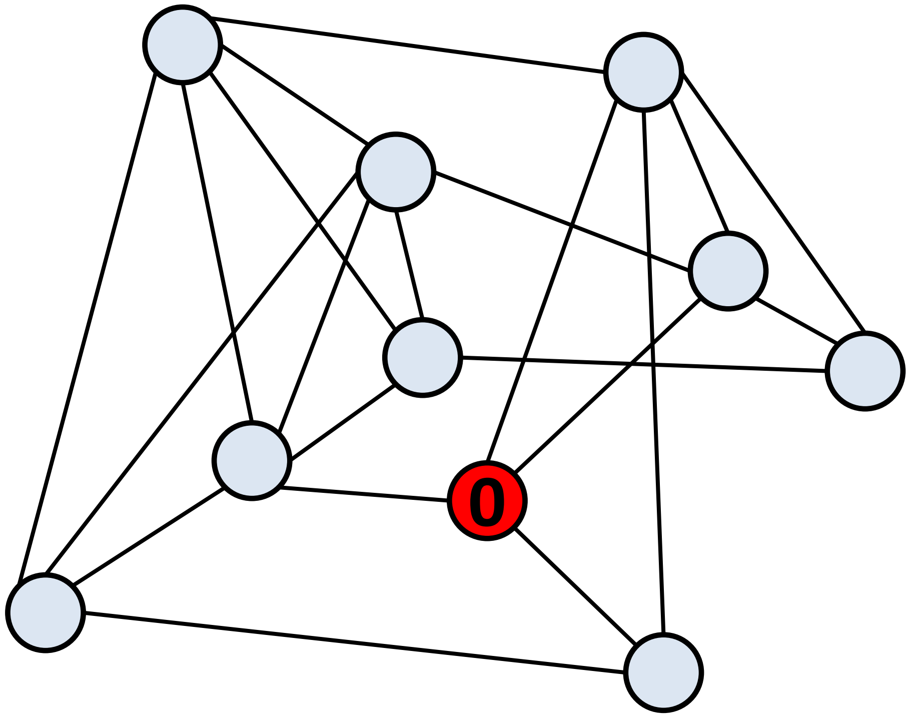

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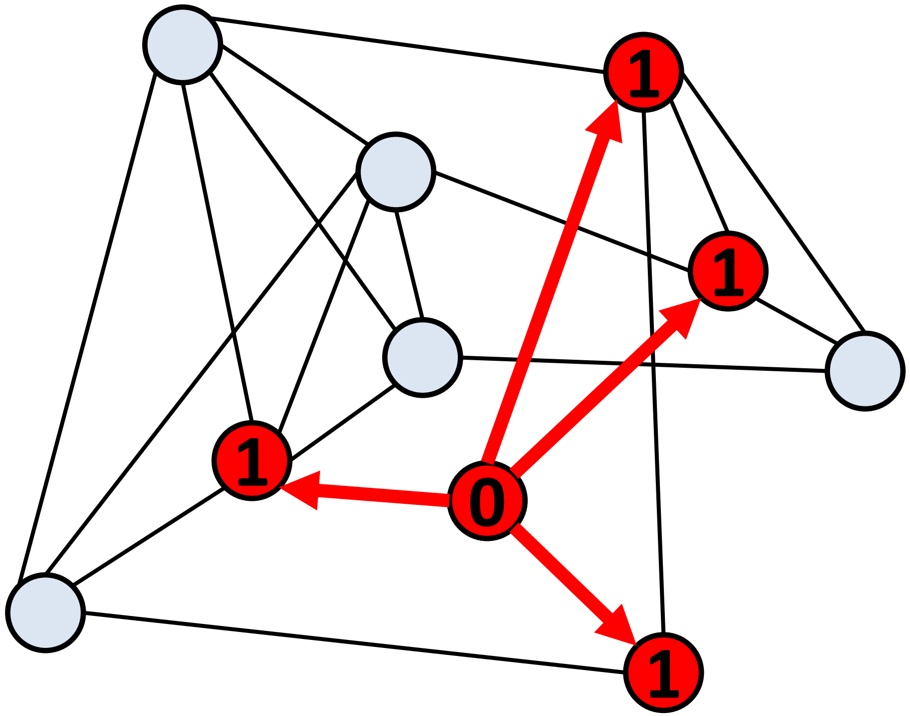

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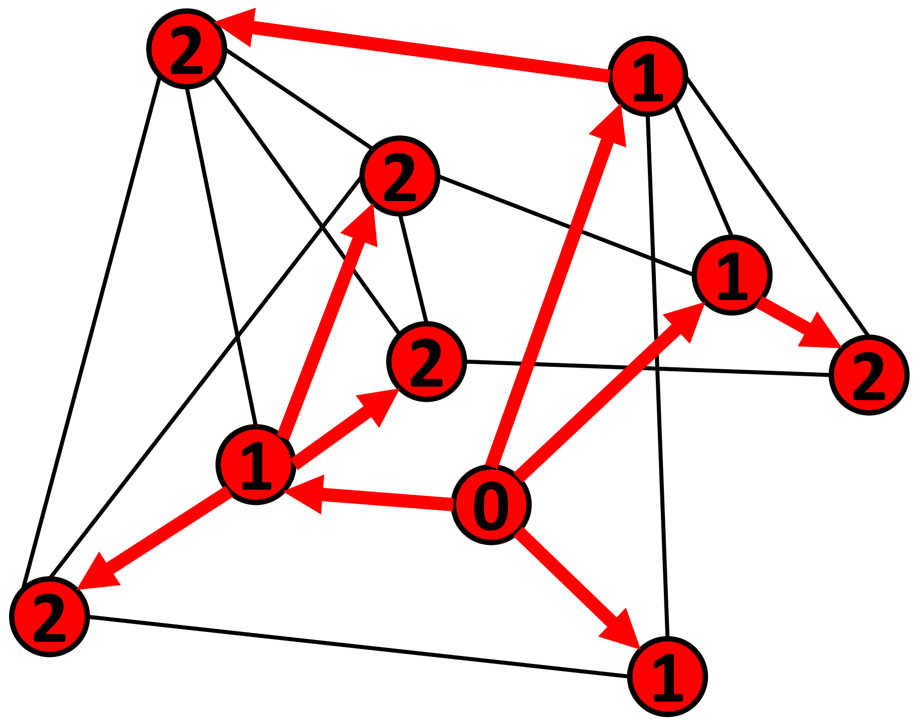
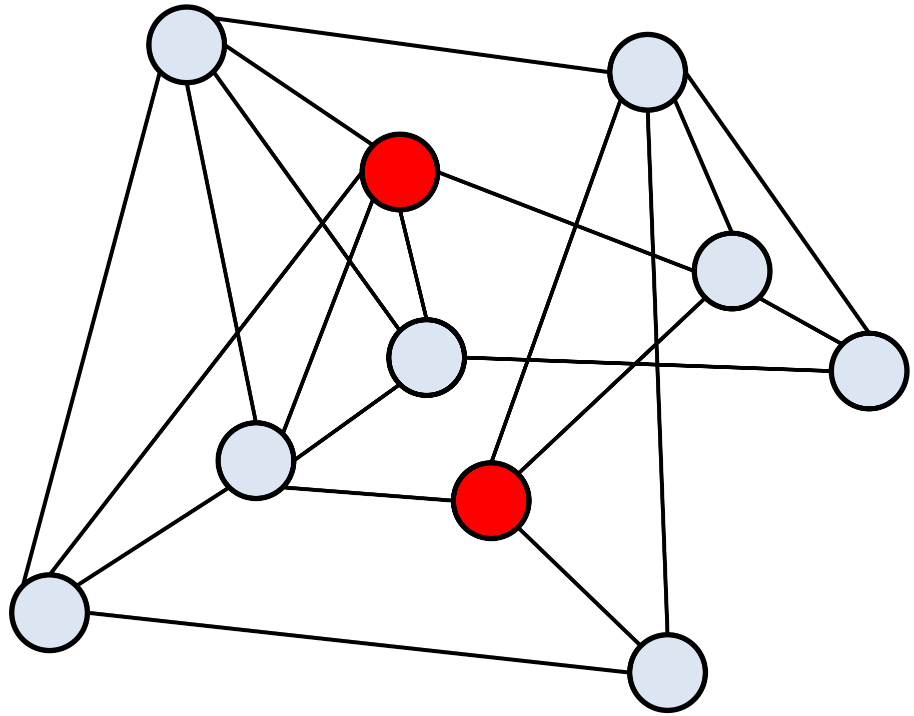



2. Count nodes in subtrees

Runtime: Diameter

# Diameter of a network



- Distance between two nodes = Number of hops of shortest path

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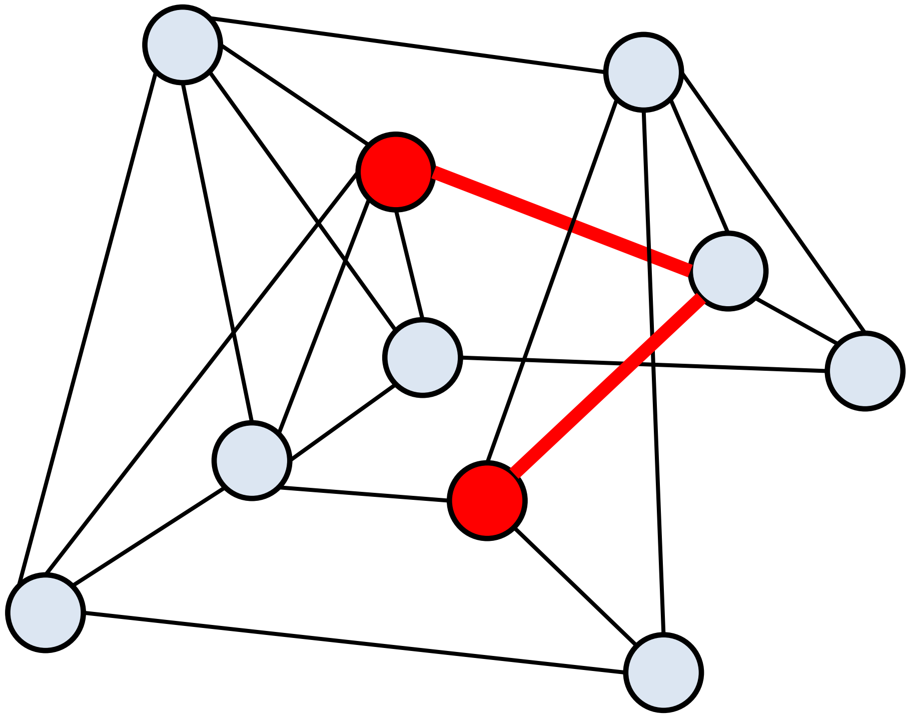

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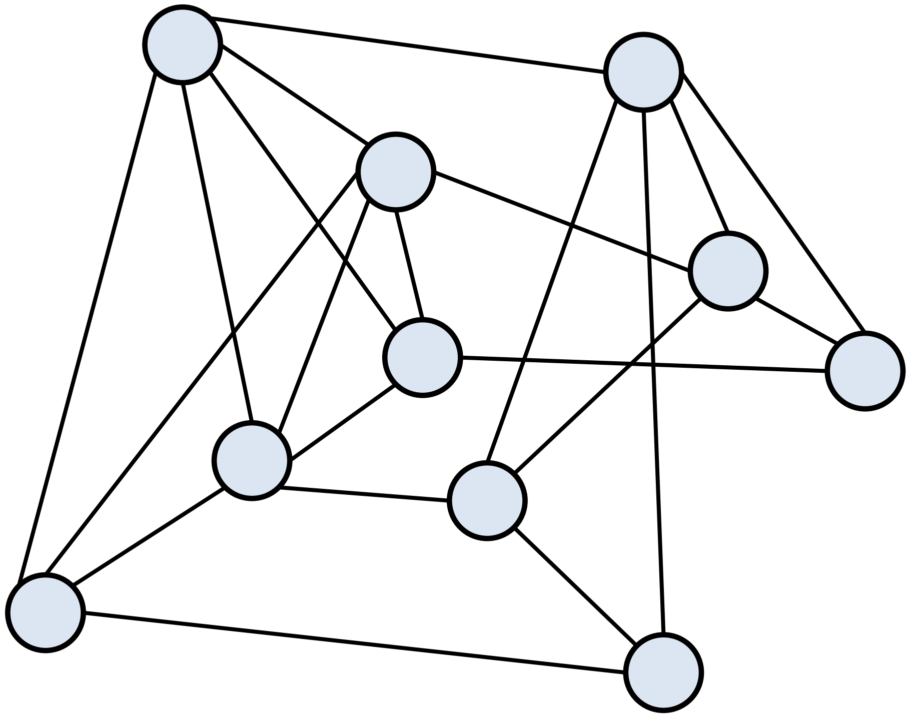
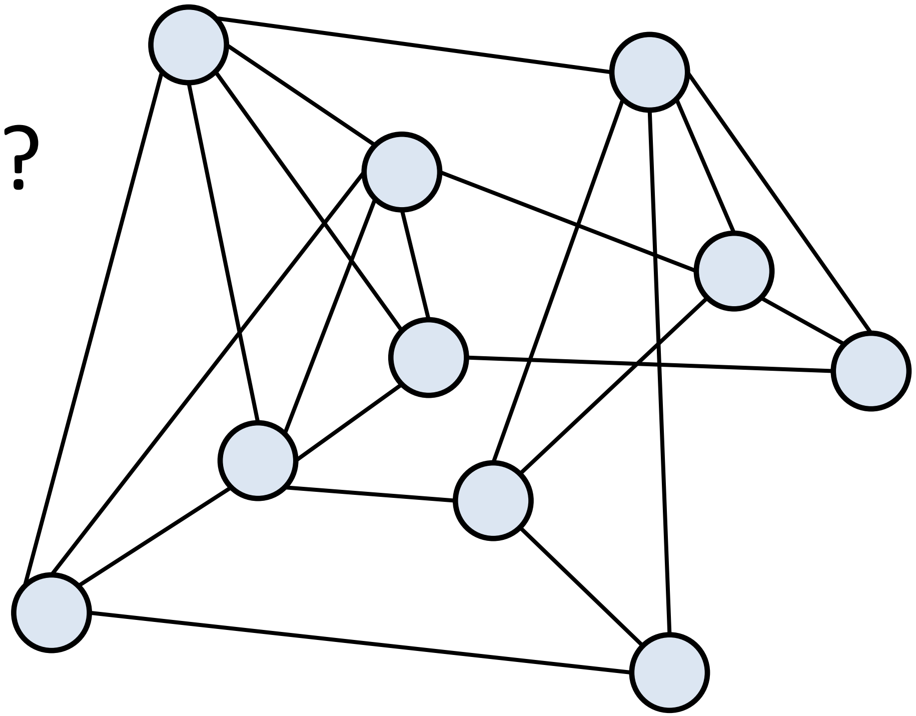


- Distance between two nodes = Number of hops of shortest path
- Diameter of network = Maximum distance, between any two nodes

# Diameter of a network

Diameter of this network?



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# Fundamental graph-problems

- Spanning Tree – Broadcasting, Aggregation, etc
- Minimum Spanning Tree – Efficient broadcasting, etc.
- Shortest path – Routing, etc.
- Steiner tree – Multicasting, etc.
- Many other graph problems.

Thanks for slide to Danupon Nanongkai

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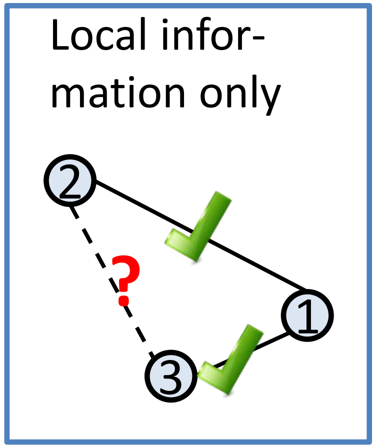
- Global problems: **Ω( D )**



# Fundamental graph-problems

- Maximal Independent Set
- Coloring
- Matching

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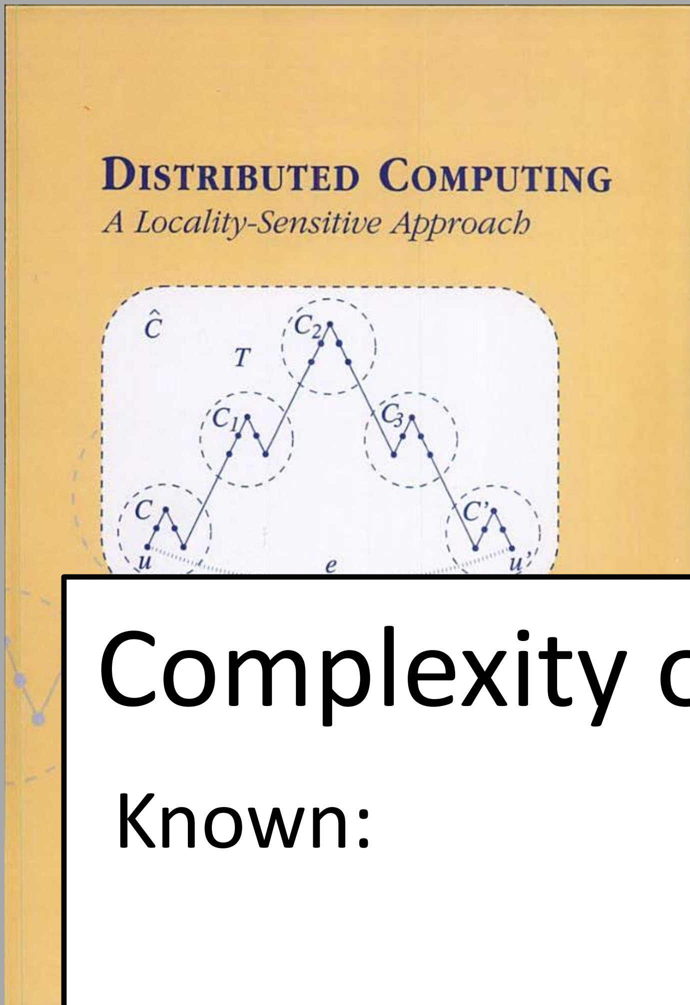
- Maximal Independent Set
- Coloring
- Matching

- Local problems:

**runtime independent of / smaller than D**

**e.g. O(log n)**


- Diameter appears frequently in distributed computing




2.1. The model 17

measuring the distance between $u$ and $w$ looking at $G$ as an unweighted graph, i.e., it is the minimum number of hops necessary to get from $u$ to $w$.

**1. Formal definition?**

*Throughout, we denote* $\Lambda = \lceil \log Diam(G) \rceil$.

*In a weighted graph $G$, let $Diam^{un}(G)$ denote the unweighted diameter of $G$, i.e., the maximum unweighted distance between any two vertices of $G$.*

**Definition 2.1.2 [Radius and center]:** *For a vertex $v \in V$, let $Rad(v,G)$ denote the distance from $v$ to the vertex farthest away from it in the graph $G$:*

$$Rad(v,G) = \max_{w \in V}\{dist_G(v,w)\}.$$

*Let $Rad(G)$ denote the radius of the network, i.e.,*

$$Rad(G) = \min_{v \in V}\{Rad(v,G)\}.$$

*A center of $G$ is any vertex $v$ realizing the radius of $G$ (i.e., such that $Rad(v,G) = Rad(G)$). In order to simplify some of the following definitions, we avoid problems arising from 0-diameter or 0-radius graphs, by defining $Rad(G) = Diam(G) = 1$ for the single-vertex*

# Complexity of computing D?

Known: $\Omega$ (D)

$\approx\Omega(1)$

Even if D = 3

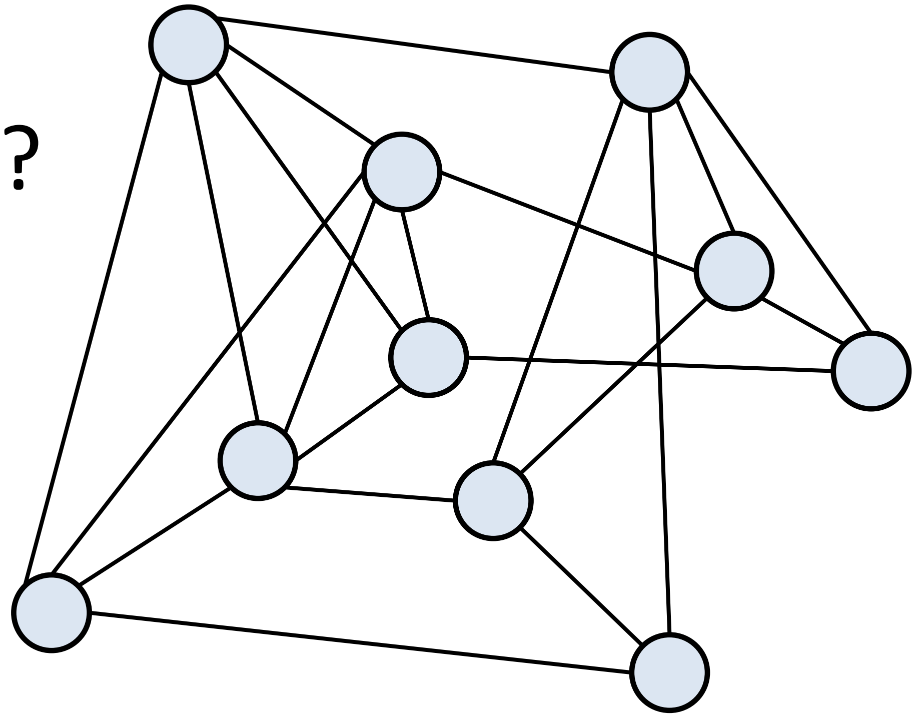
This talk: $\Omega(n)$

computing

Networks cannot compute their diameter in sublinear time!

# Diameter of a network

Diameter of this network?



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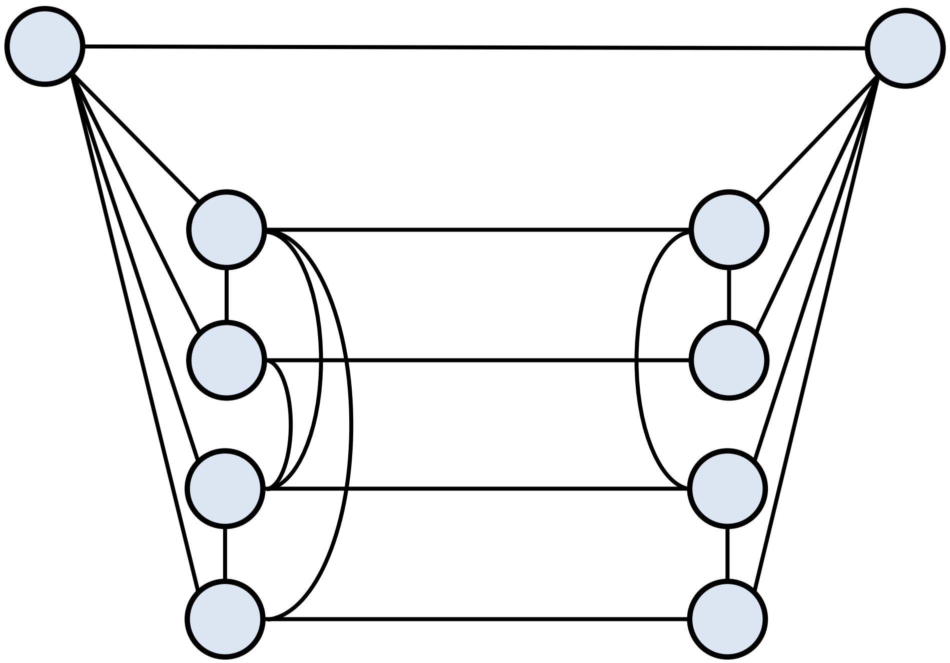

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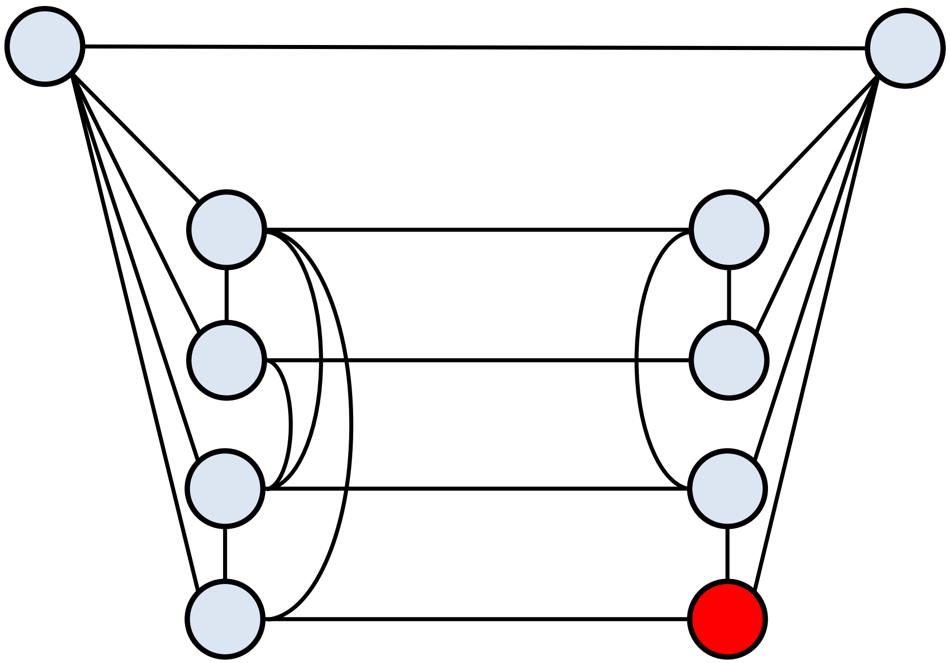

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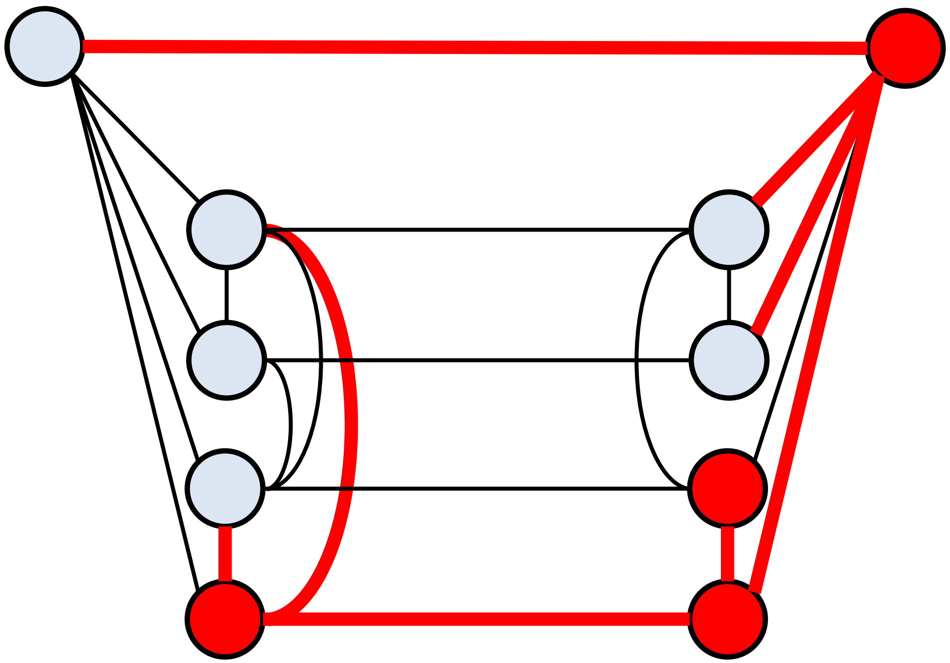

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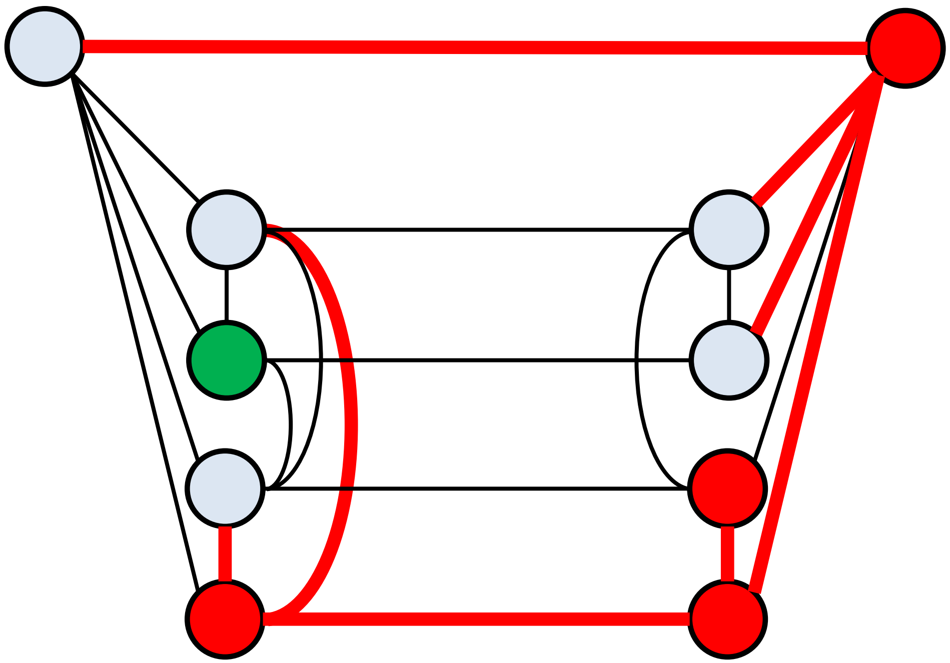

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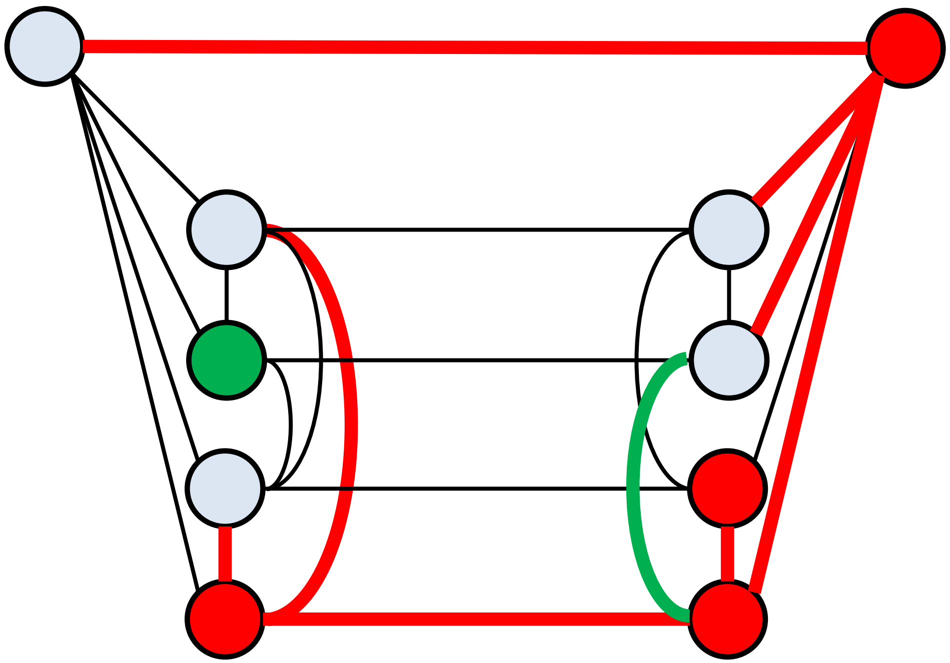

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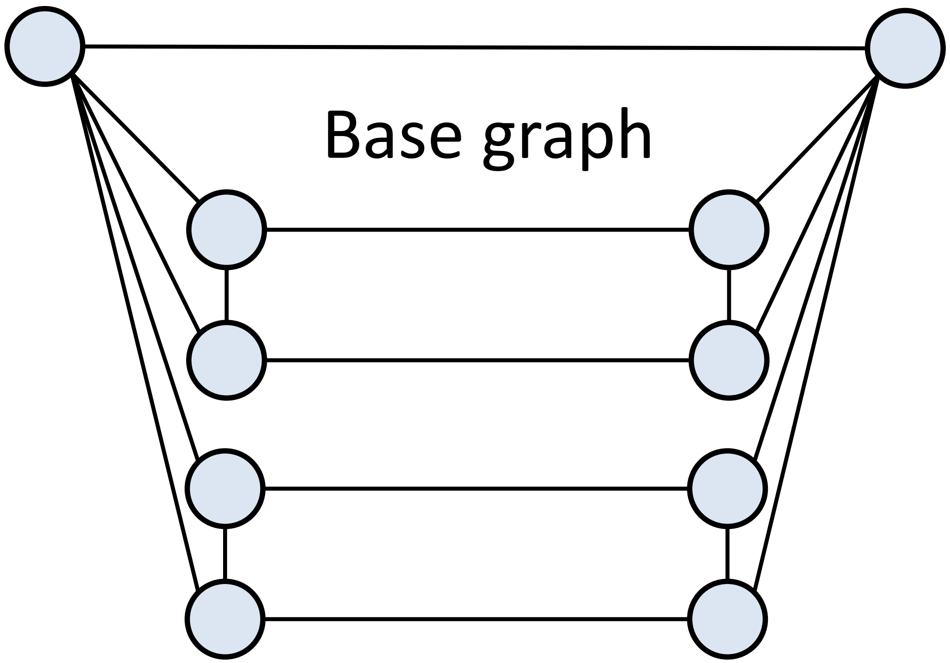

Base graph

has diameter 3

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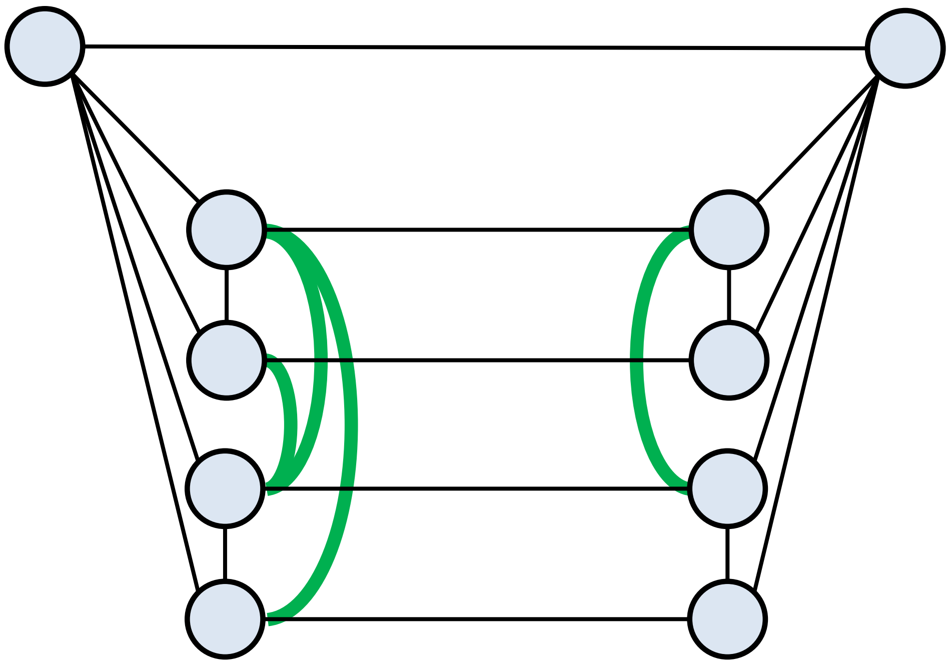
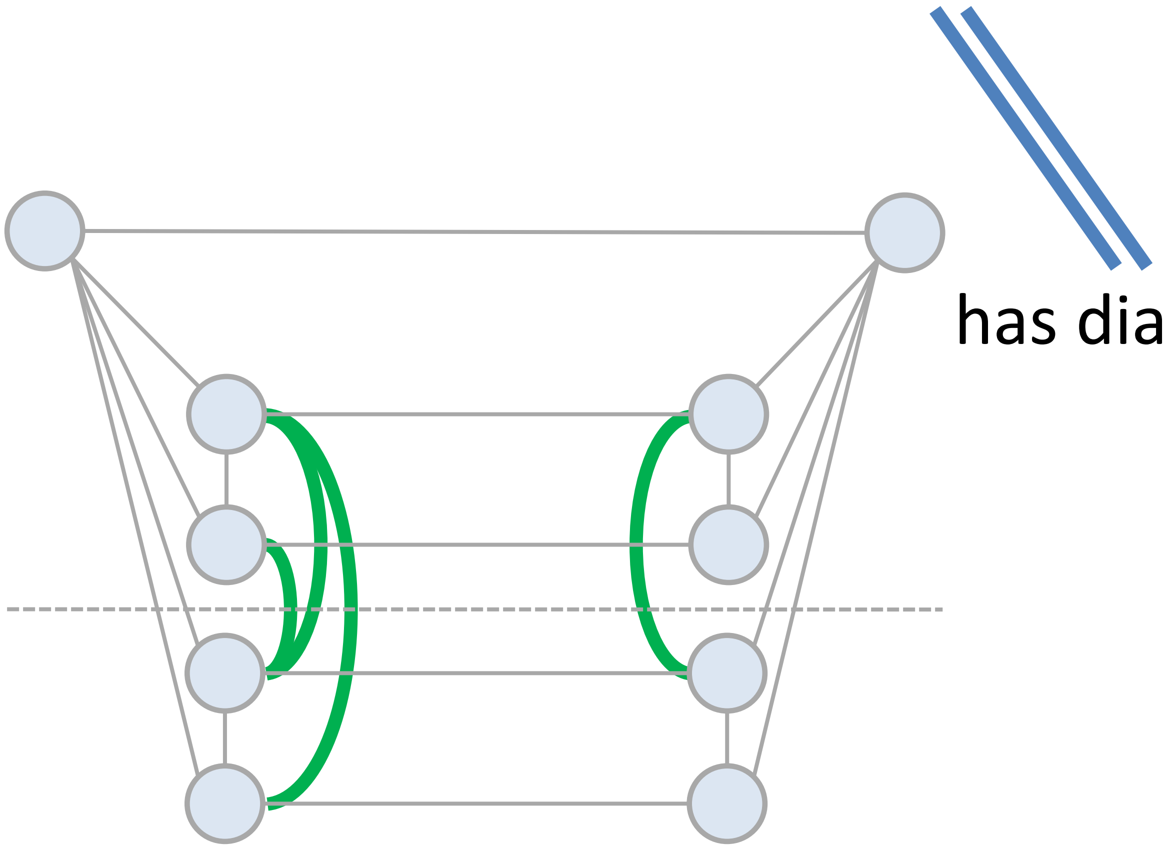


has diameter ~~3~~
2?

# Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?



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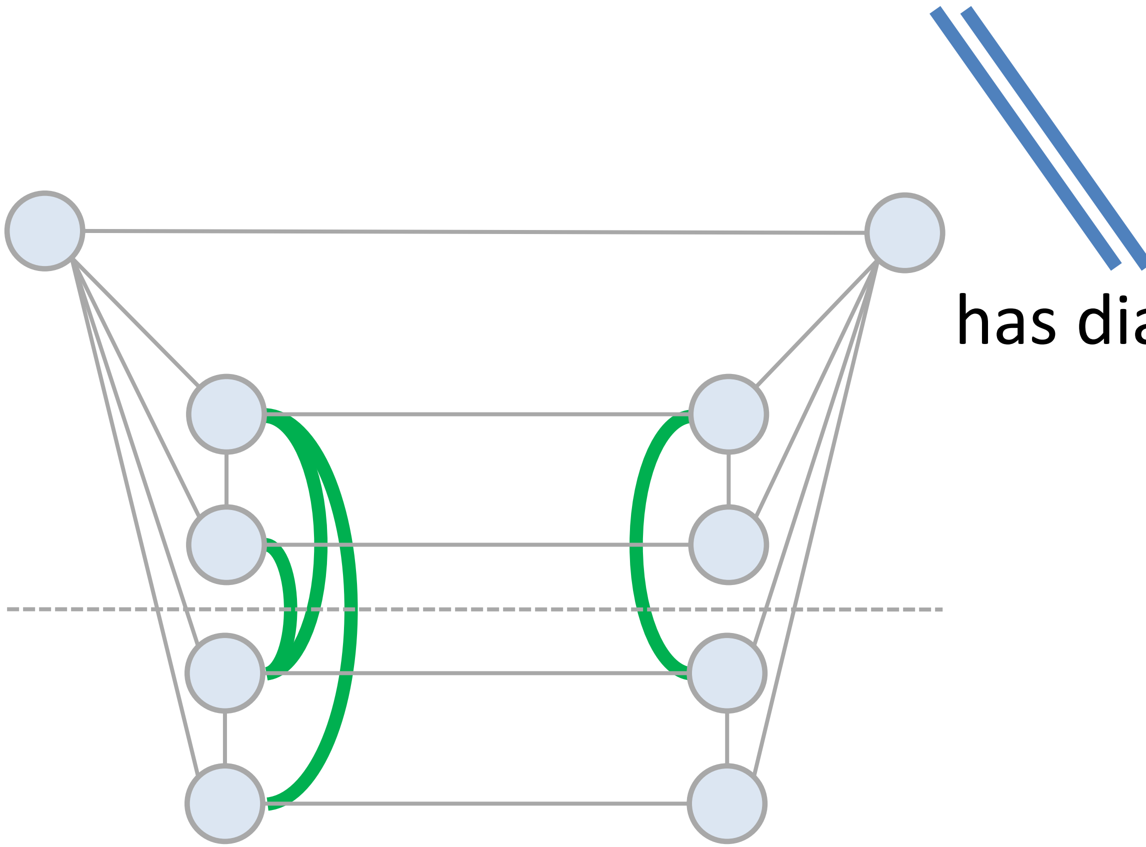

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**Now: slightly more formal**

# Networks cannot compute their diameter in sublinear time!

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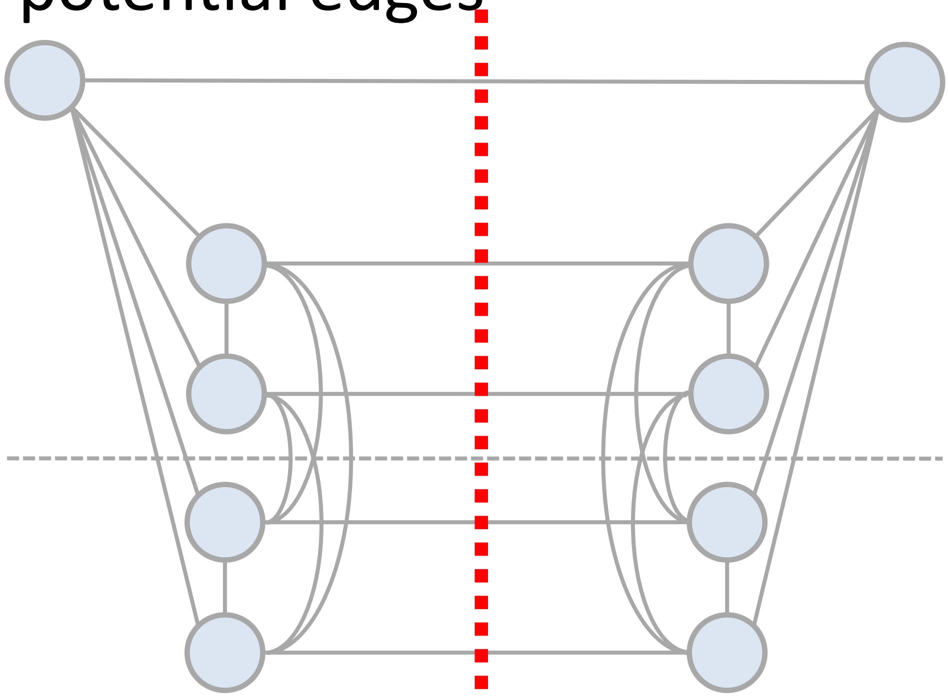
Label potential edges



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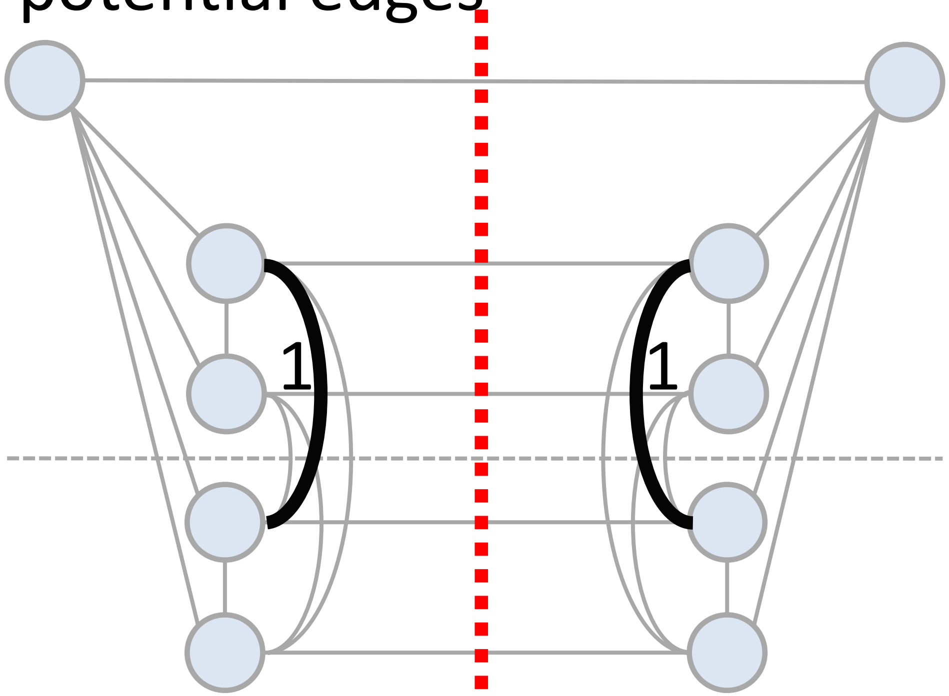

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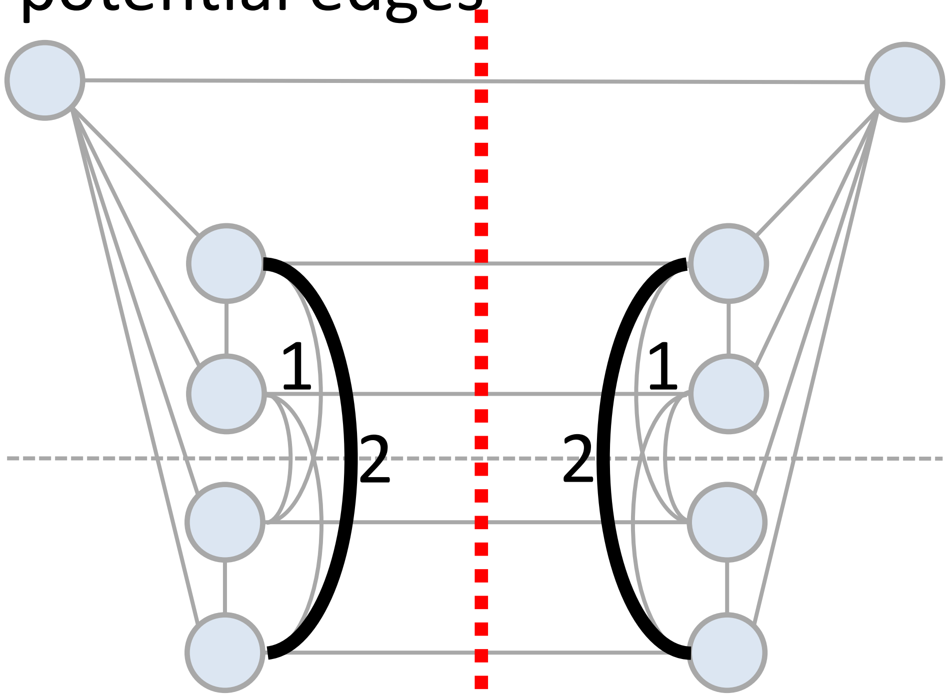

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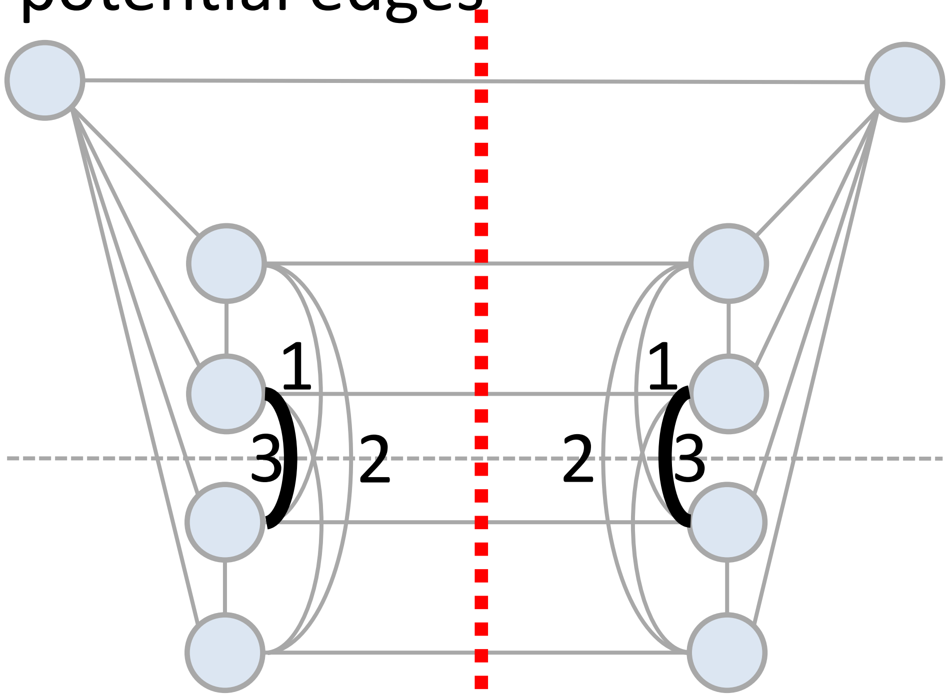

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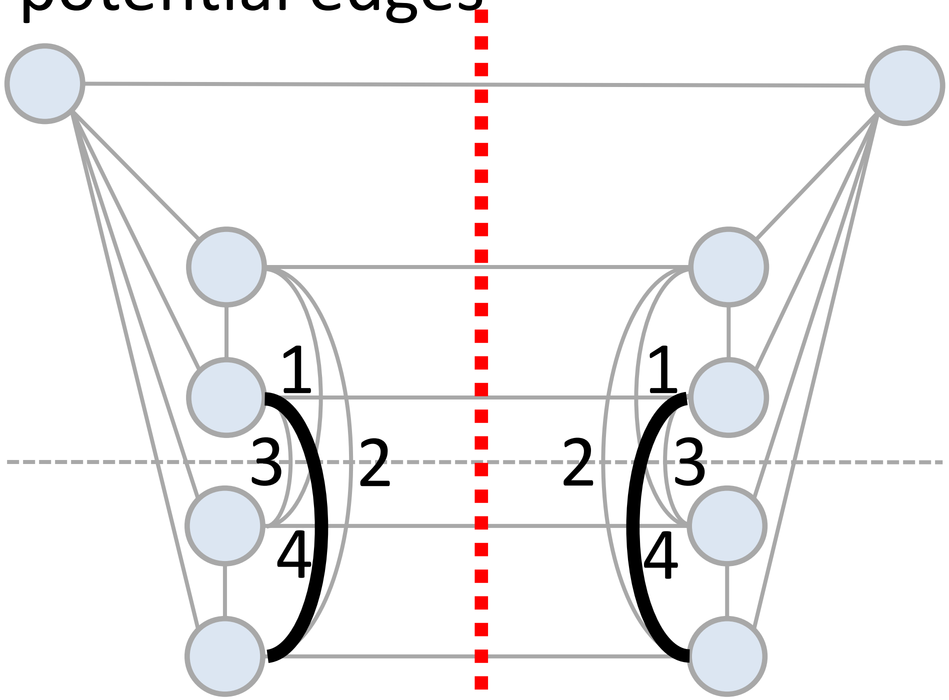

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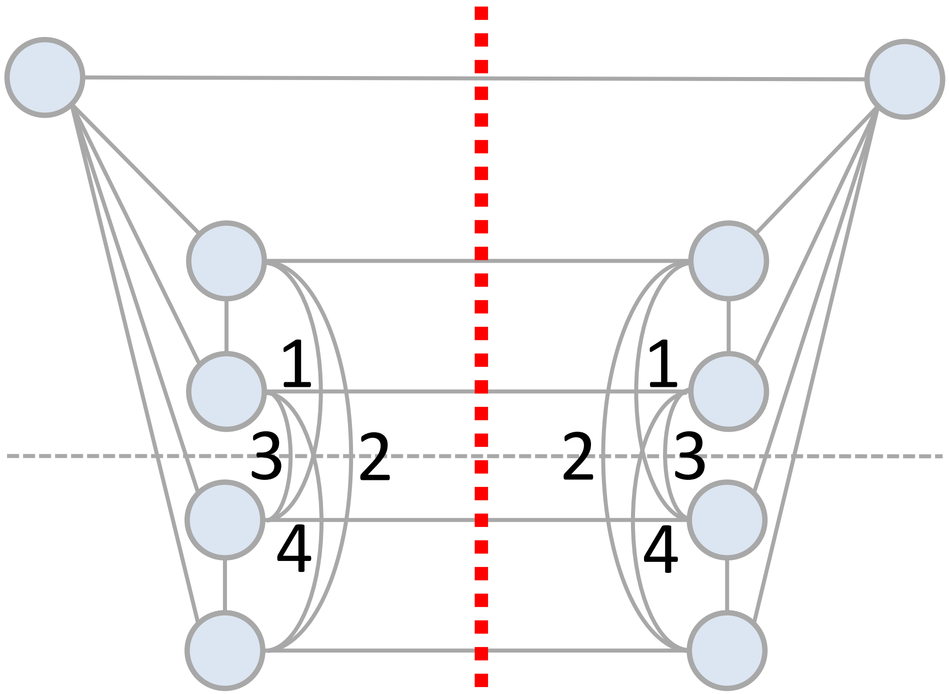

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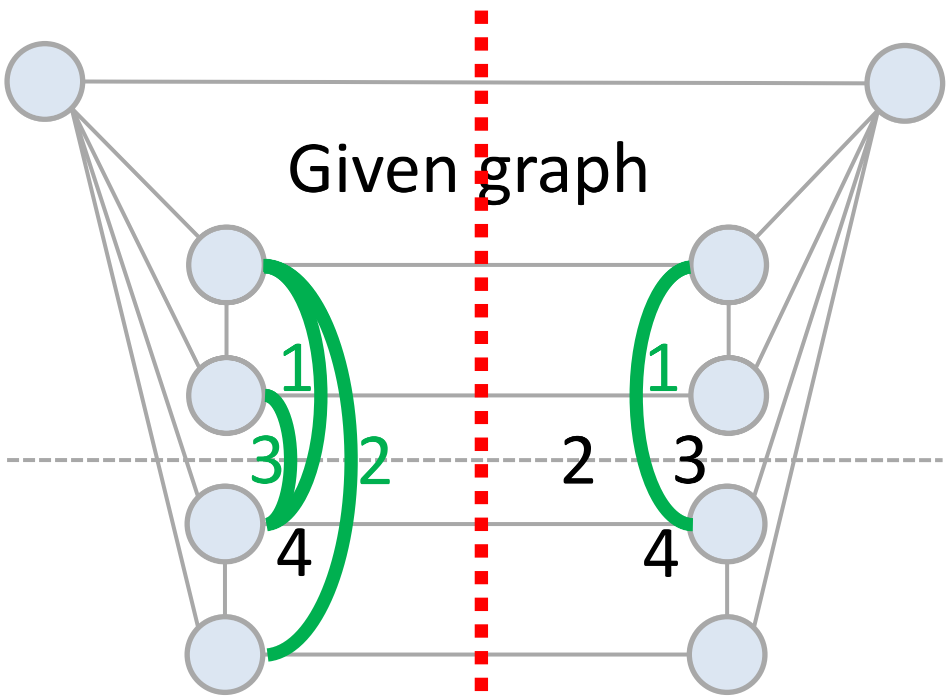

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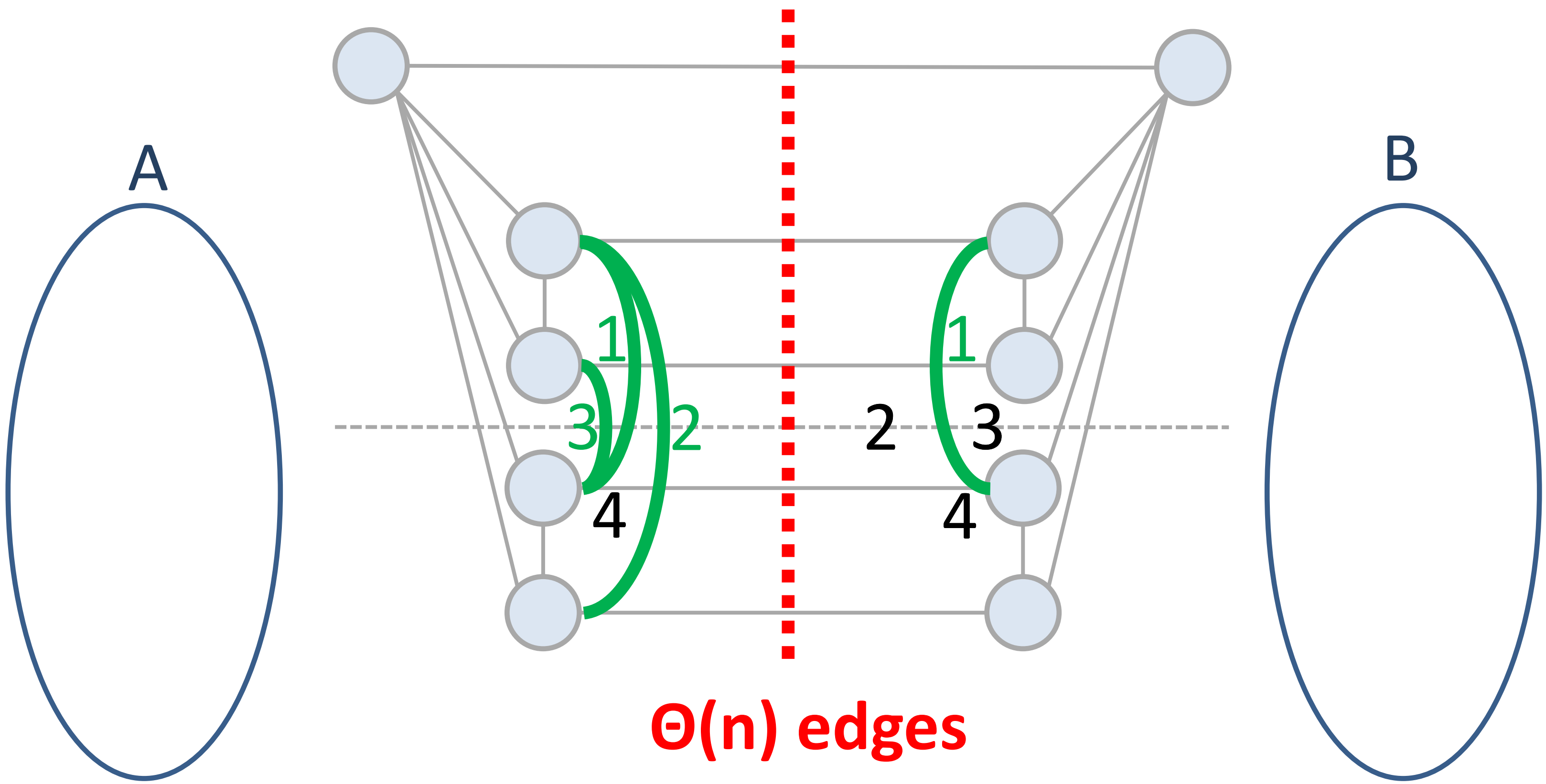

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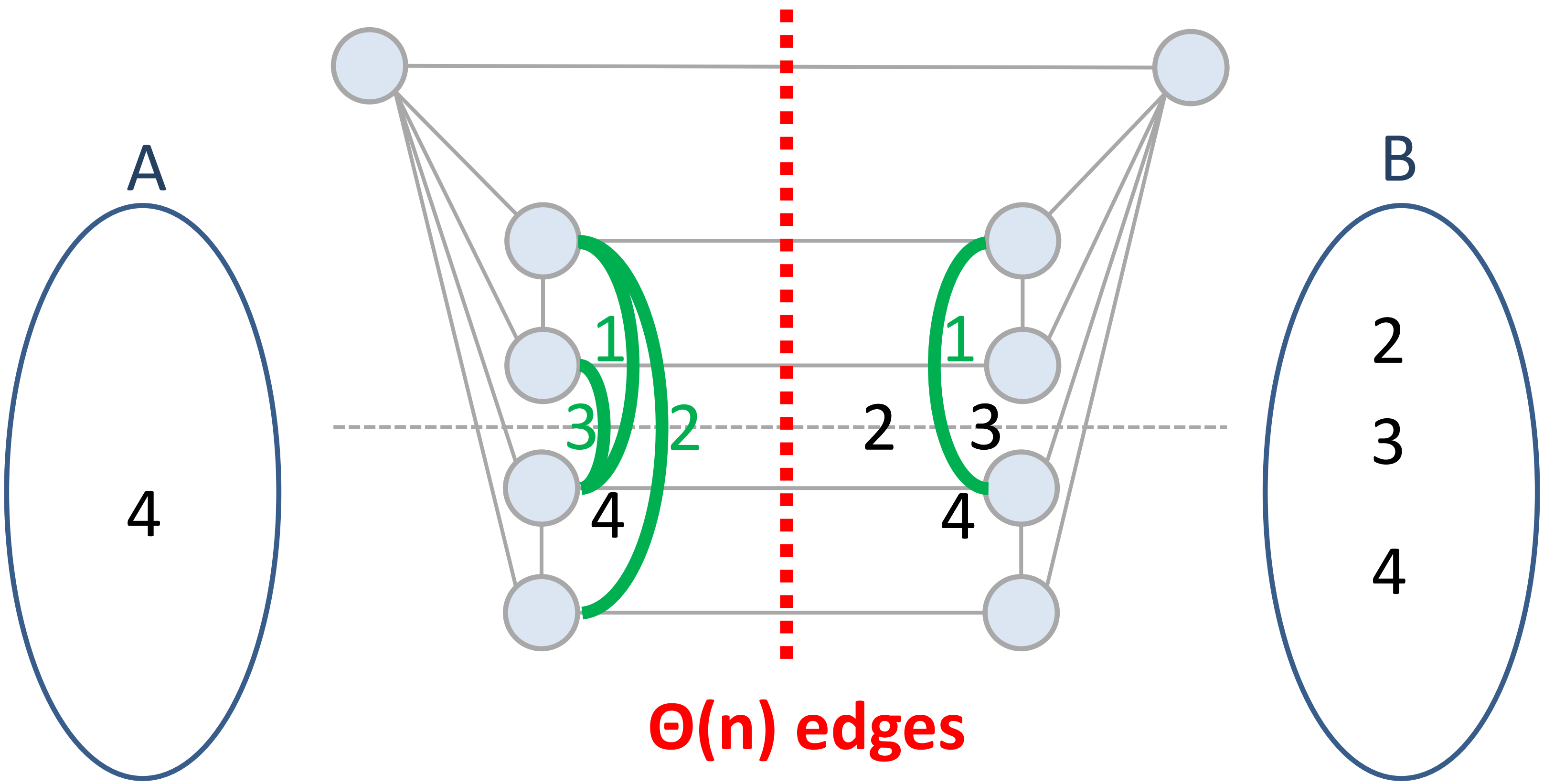
Pair of nodes not connected on both sides?

Same as "A and B not disjoint?"



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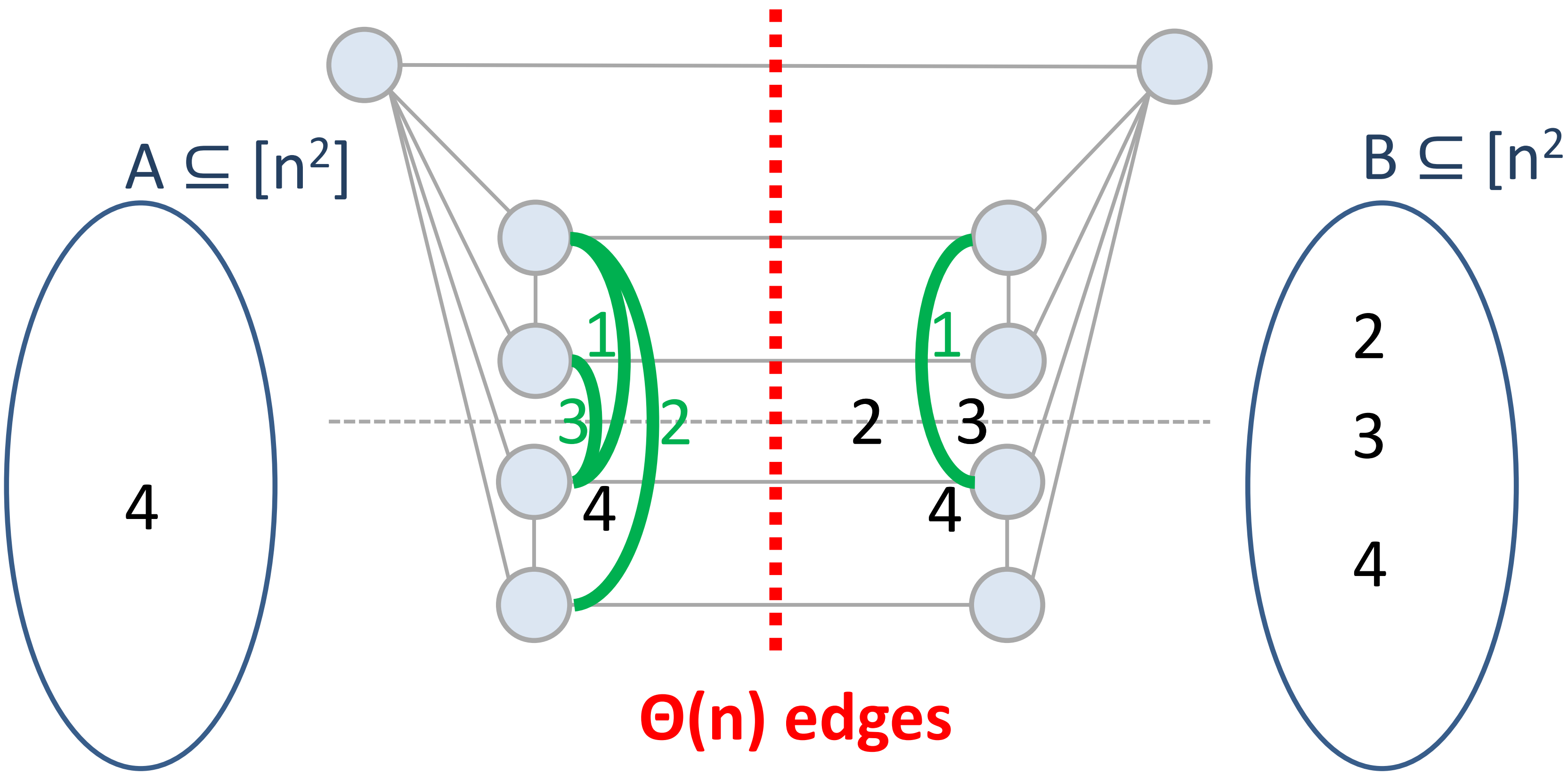


# Networks cannot compute their diameter in sublinear time!

“A and B not disjoint?”

$A \subseteq [n^2]$

4

$B \subseteq [n^2]$

2

3

4

# Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

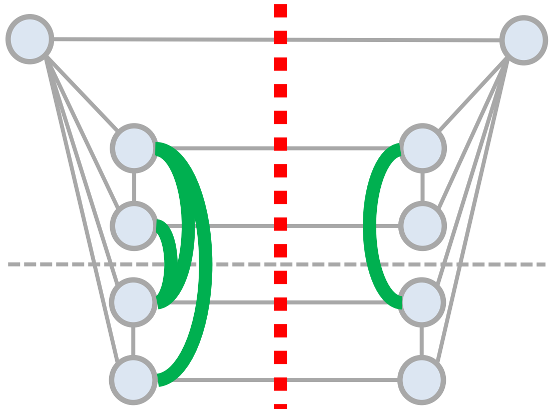
Same as "A and B not disjoint?"

Communication Complexity

randomized: $\Omega(n^2)$ bits


$A \subseteq [n^2]$

4



$B \subseteq [n^2]$

2

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4

**$\Omega(n)$ time**

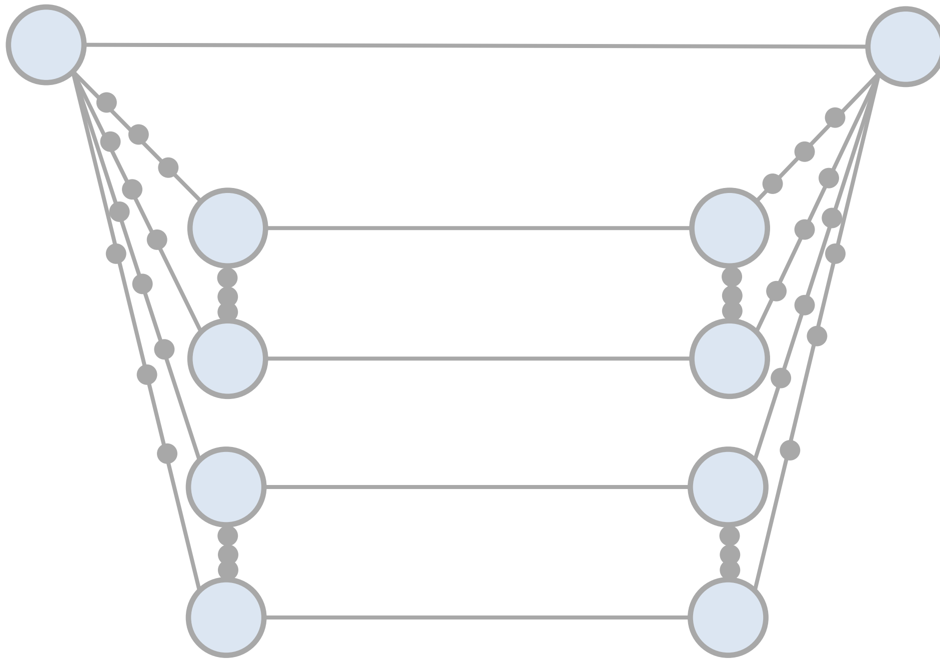
**$\Theta(n)$ edges**

# Diameter Approximation

# 3/2-ε approximating the diameter takes $\Omega(n^{1/2})$

**Extend**

2 vs. 3



D = 9 base graph

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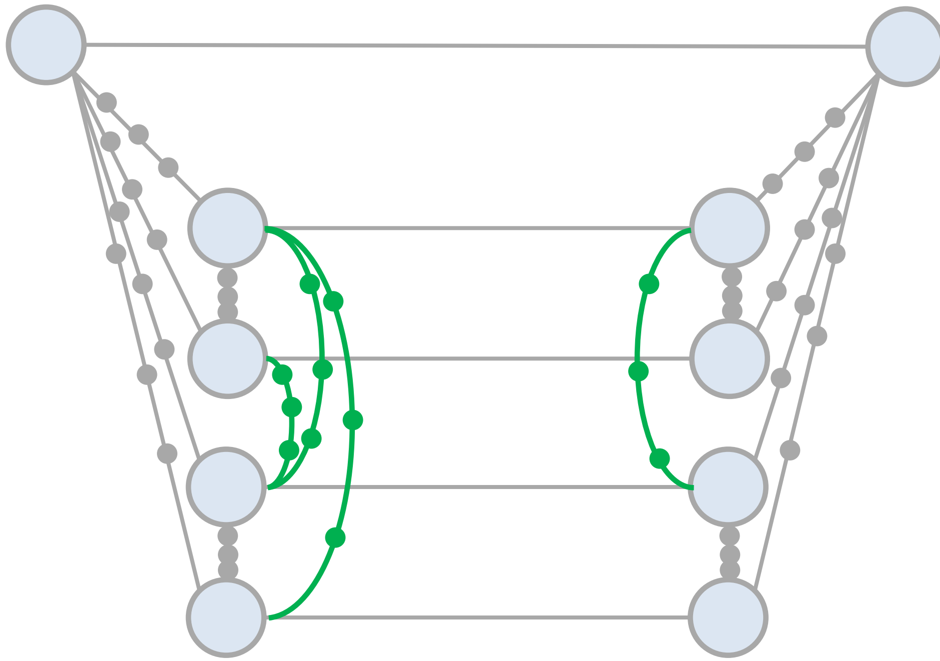

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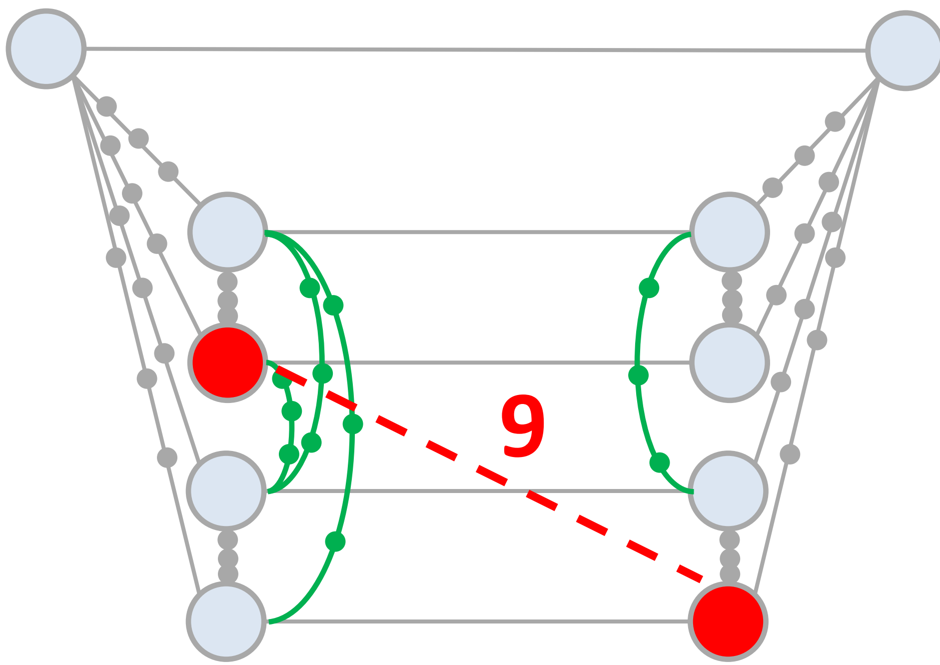


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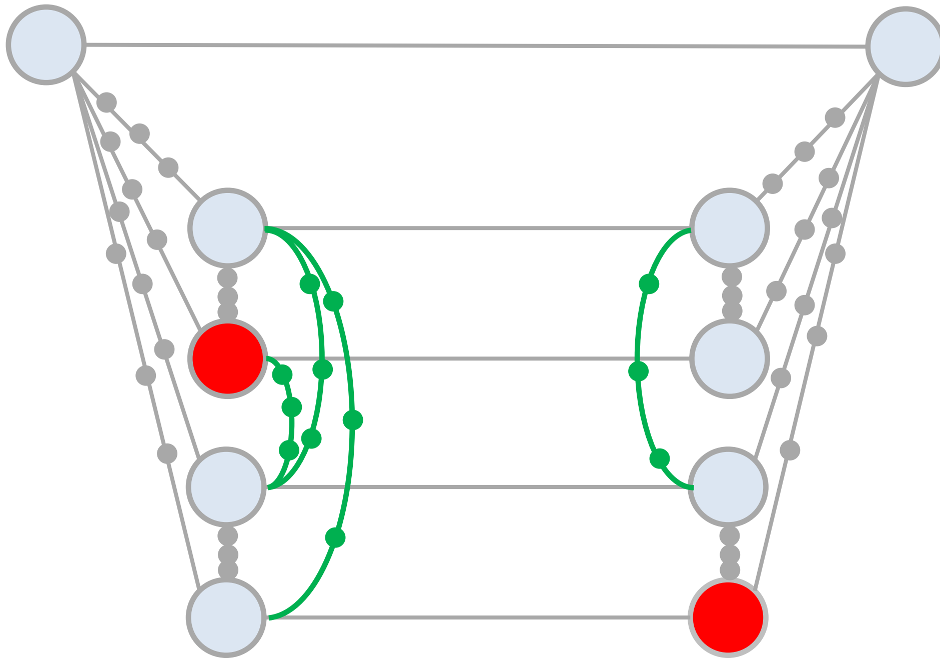

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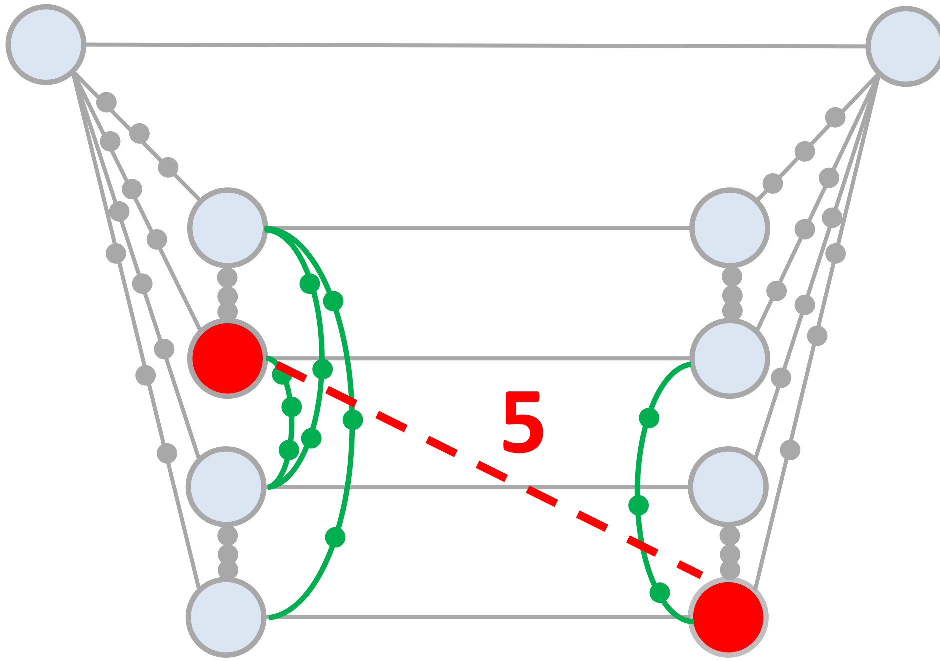


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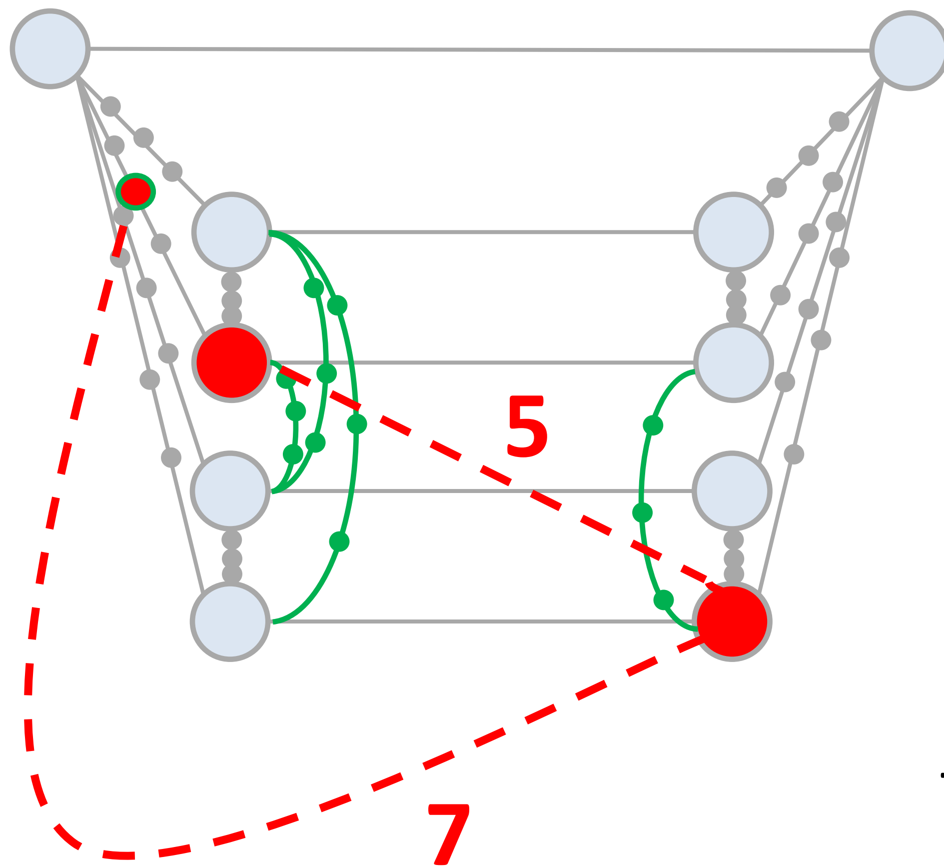


D = 9 base graph

D = 7 good graph

-------------

Factor: 3/2

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**Extend**

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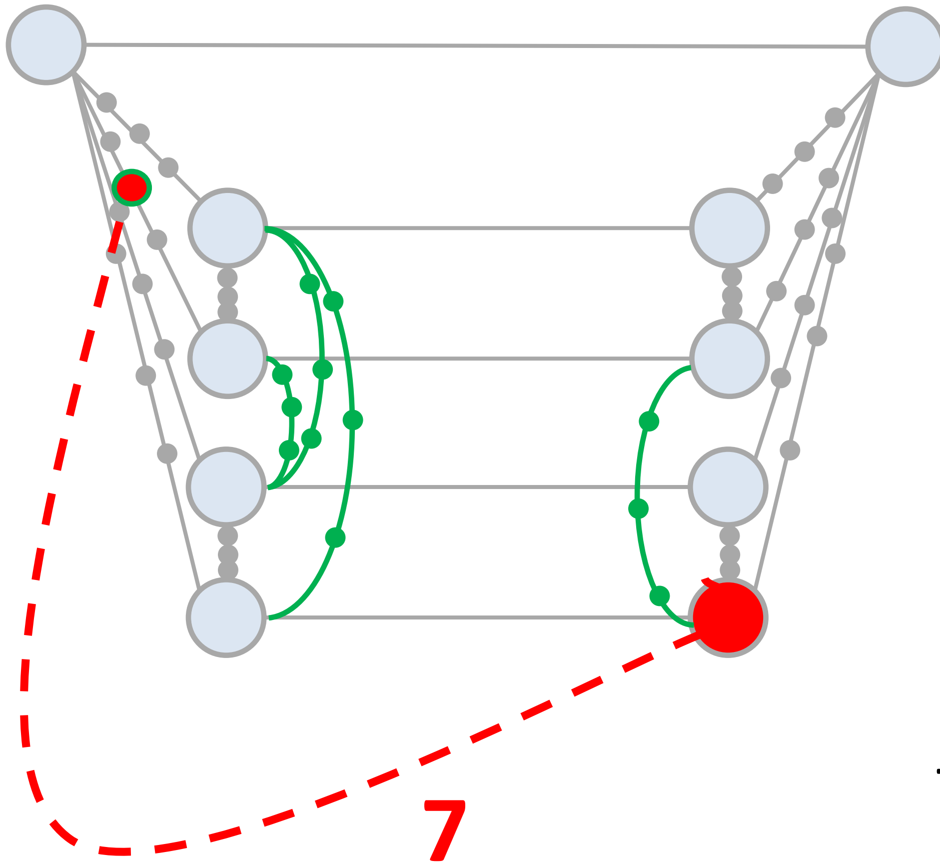


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Factor: 3/2 -ε

# Technique is general

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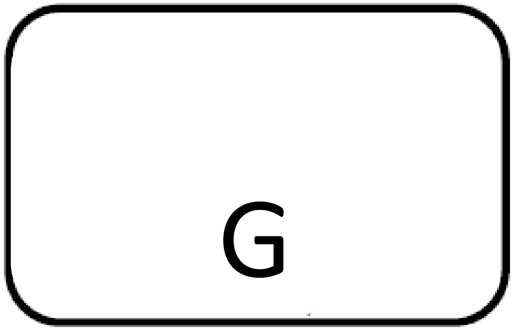

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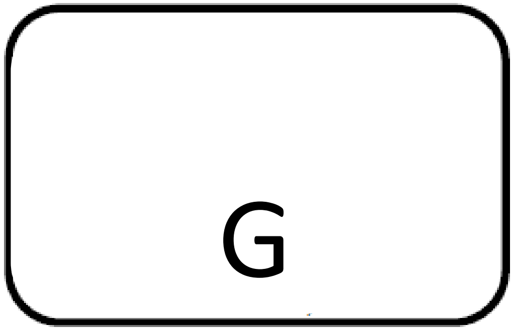


$f(G\ \ )$

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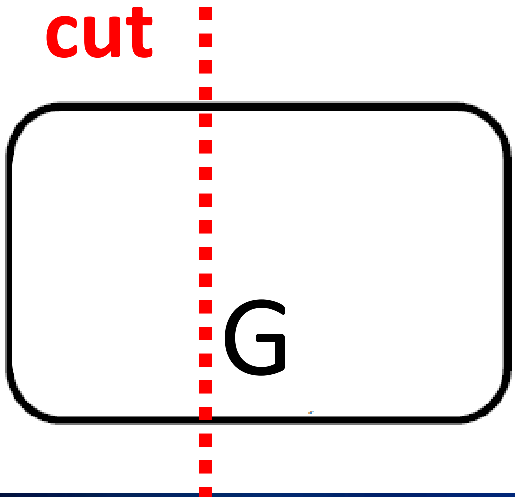


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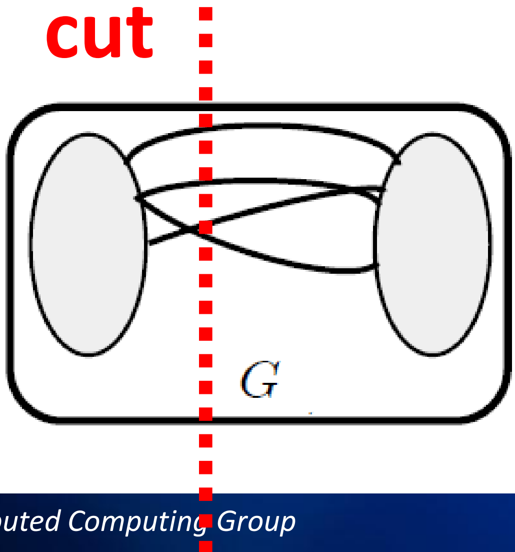


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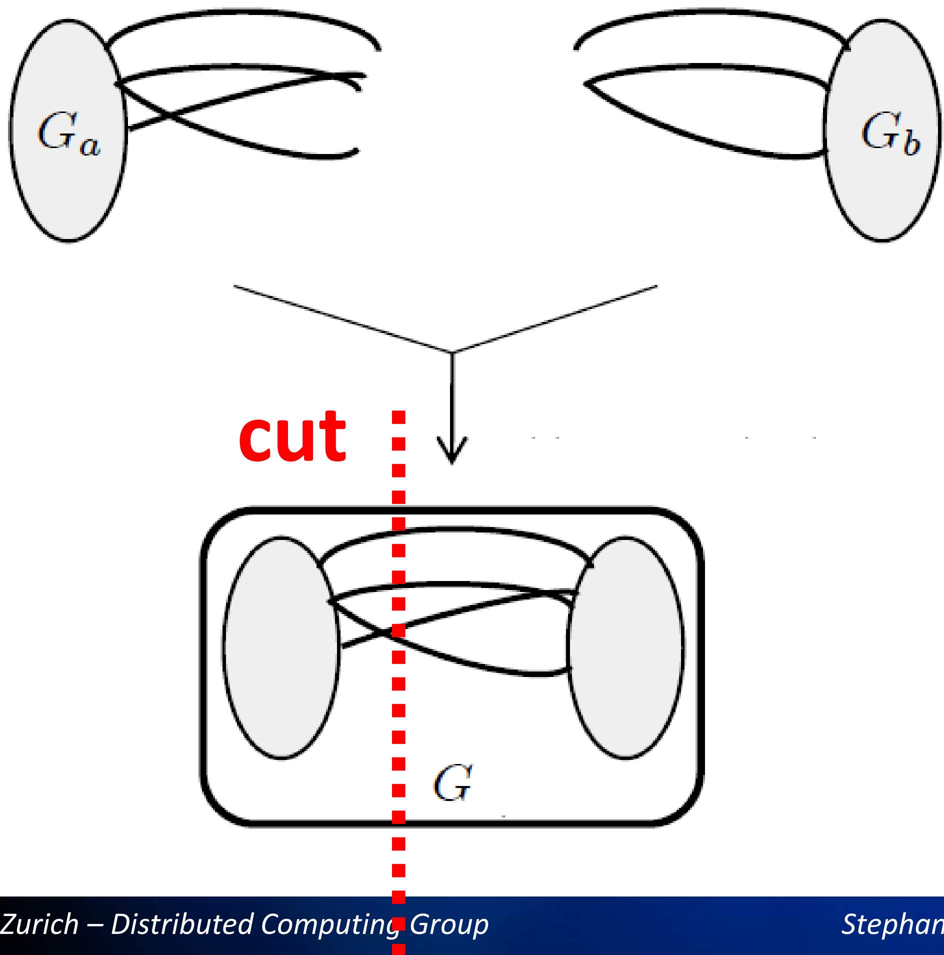


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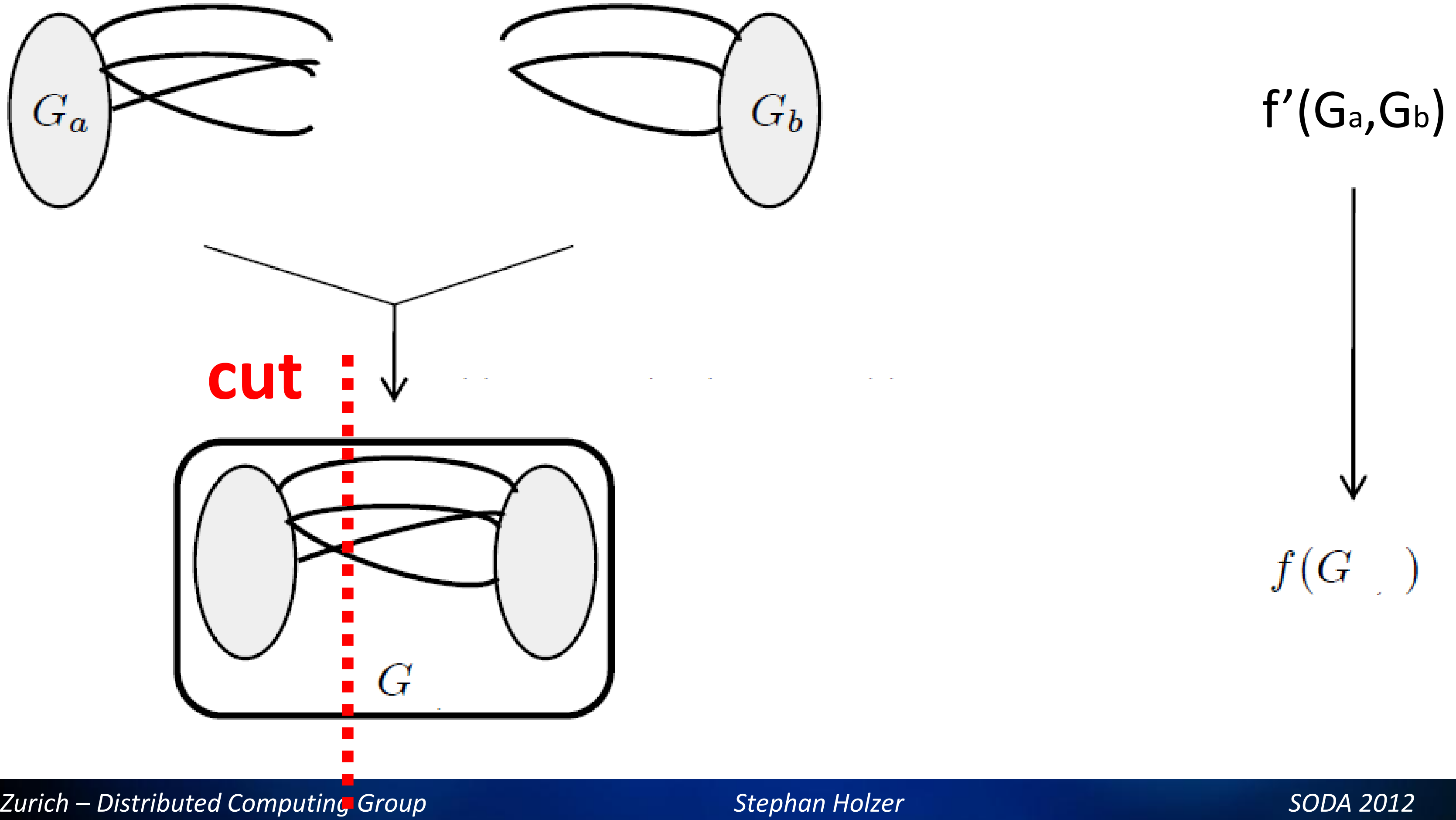
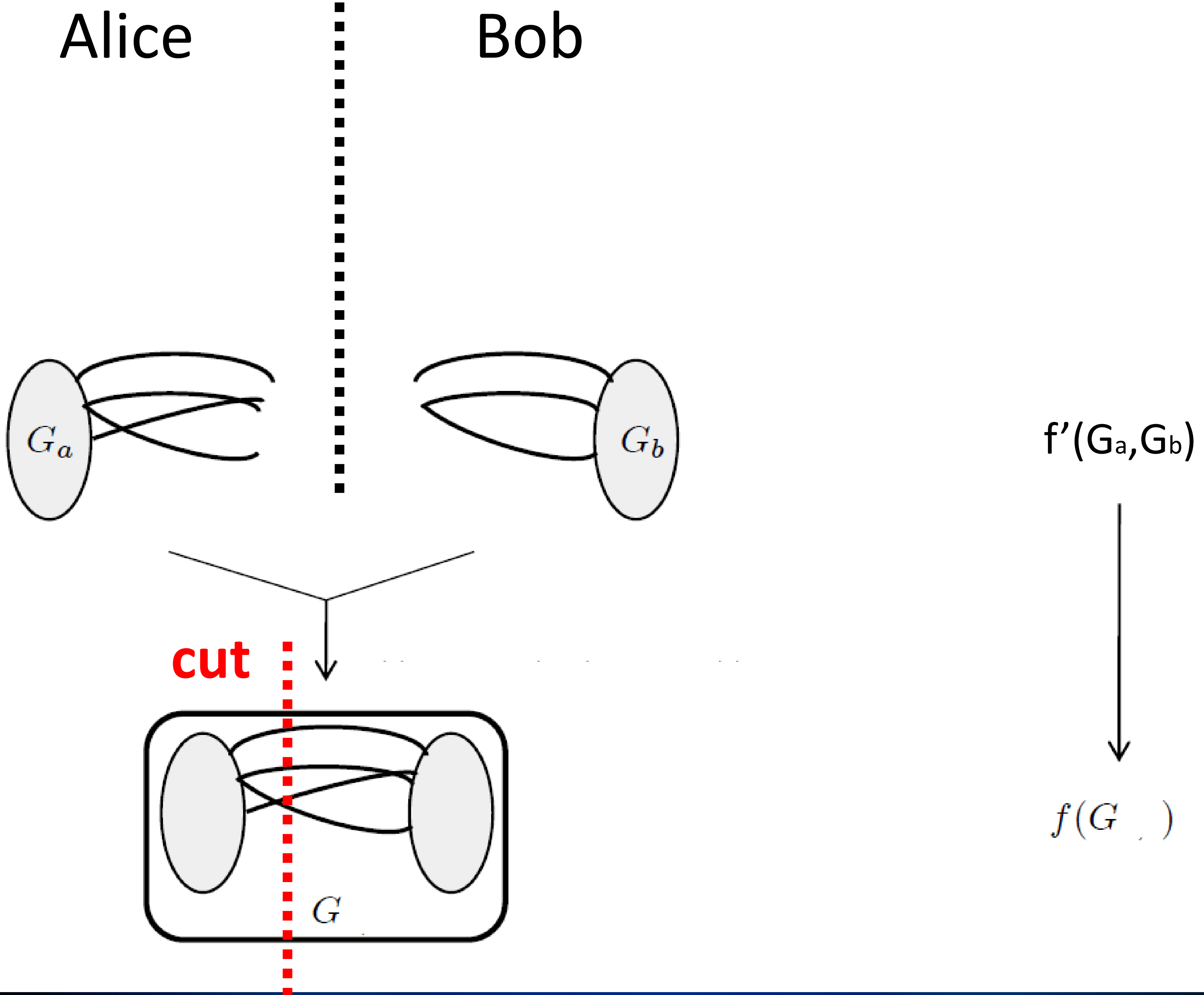
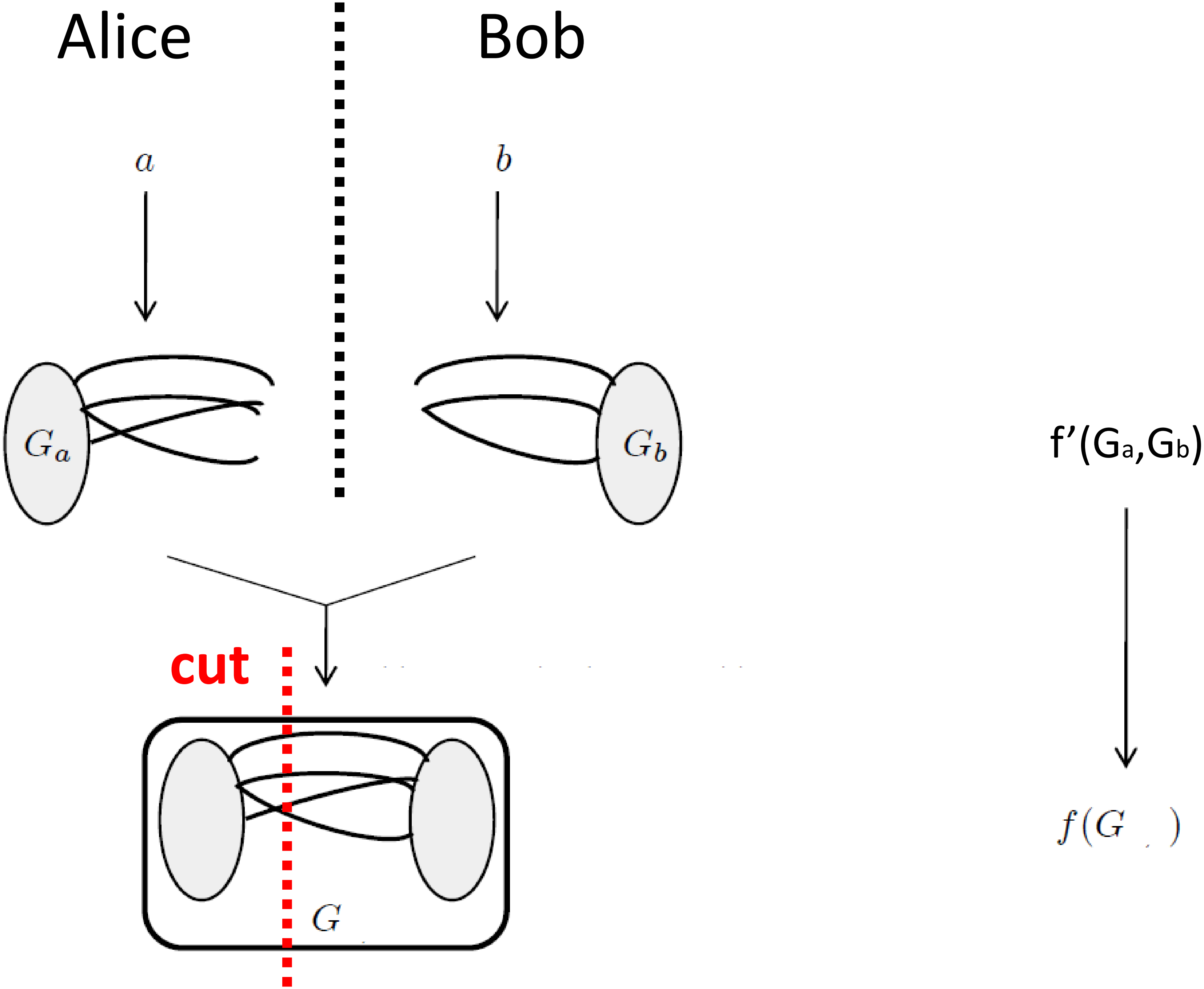
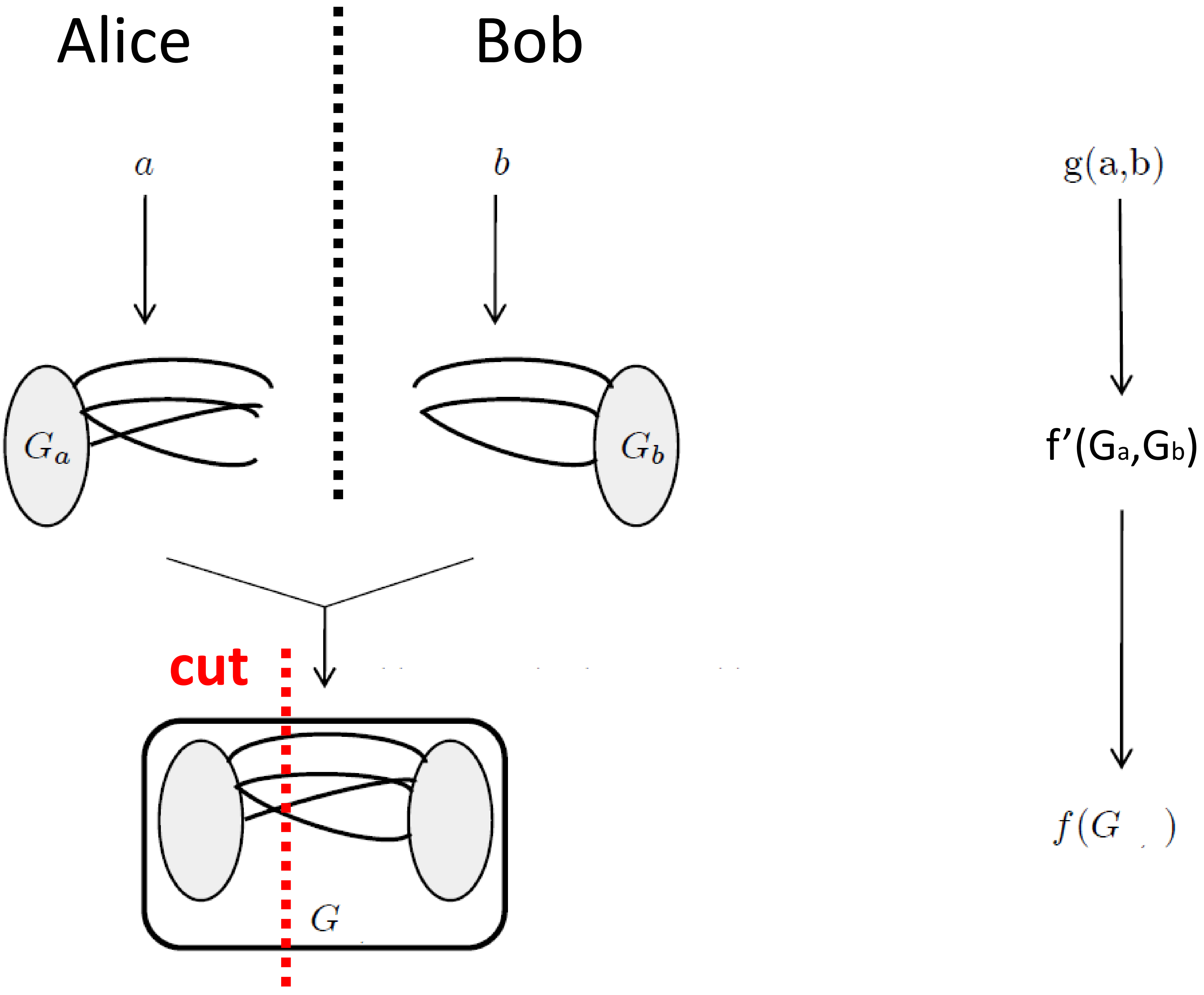

$G_a$
$G_b$
cut
$G$
f'(Ga,Gb)
$f(G\ \ )$

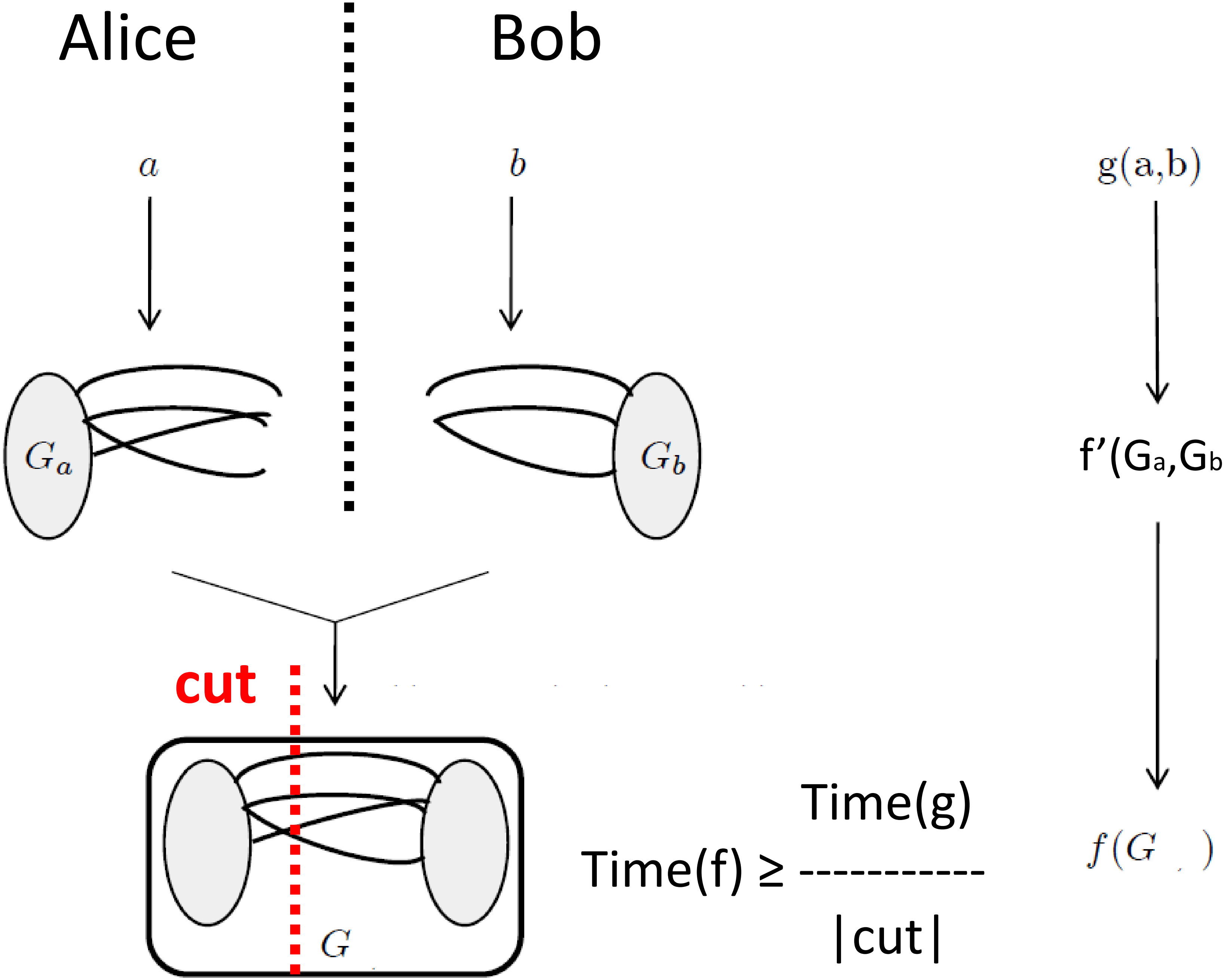


Alice
Bob
$G_a$
$G_b$
f'(Ga,Gb)
cut
$G$
$f(G\ )$


Alice
Bob
a
b
$G_a$
$G_b$
f'(Ga,Gb)
cut
$G$
$f(G)$

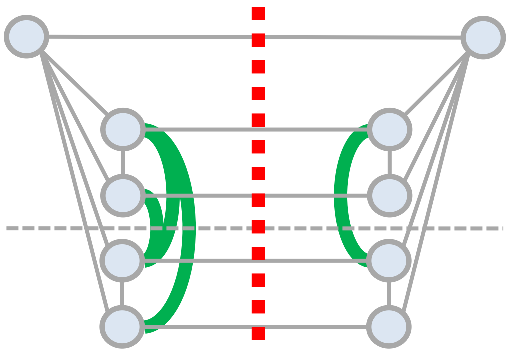


Alice
Bob
$a$
$b$
$G_a$
$G_b$
cut
$G$
g(a,b)
f'(Ga,Gb)
$f(G\ )$

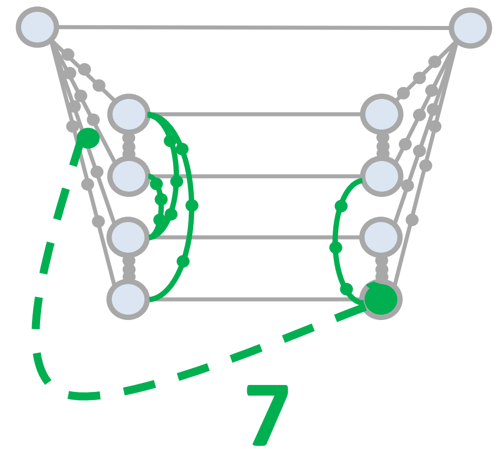



$$\text{Time}(f) \geq \frac{\text{Time}(g)}{|\text{cut}|}$$

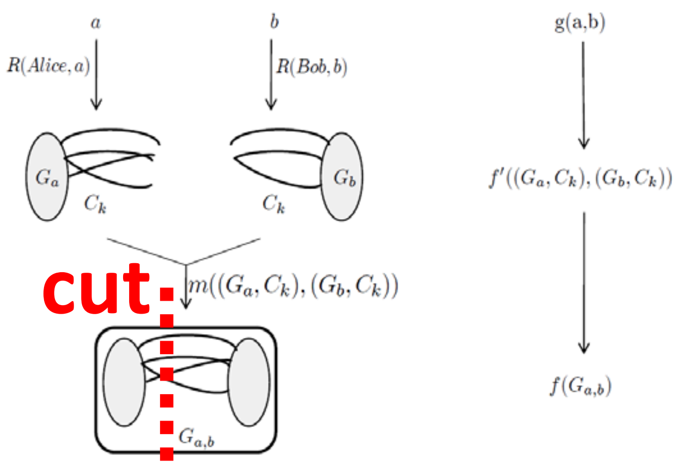
# Summary



Diameter Ω(n)




3/2-eps approximation takes $\Omega(n^{1/2})$




general technique

# Thanks!