The Complexity of Connectivity in Wireless Networks

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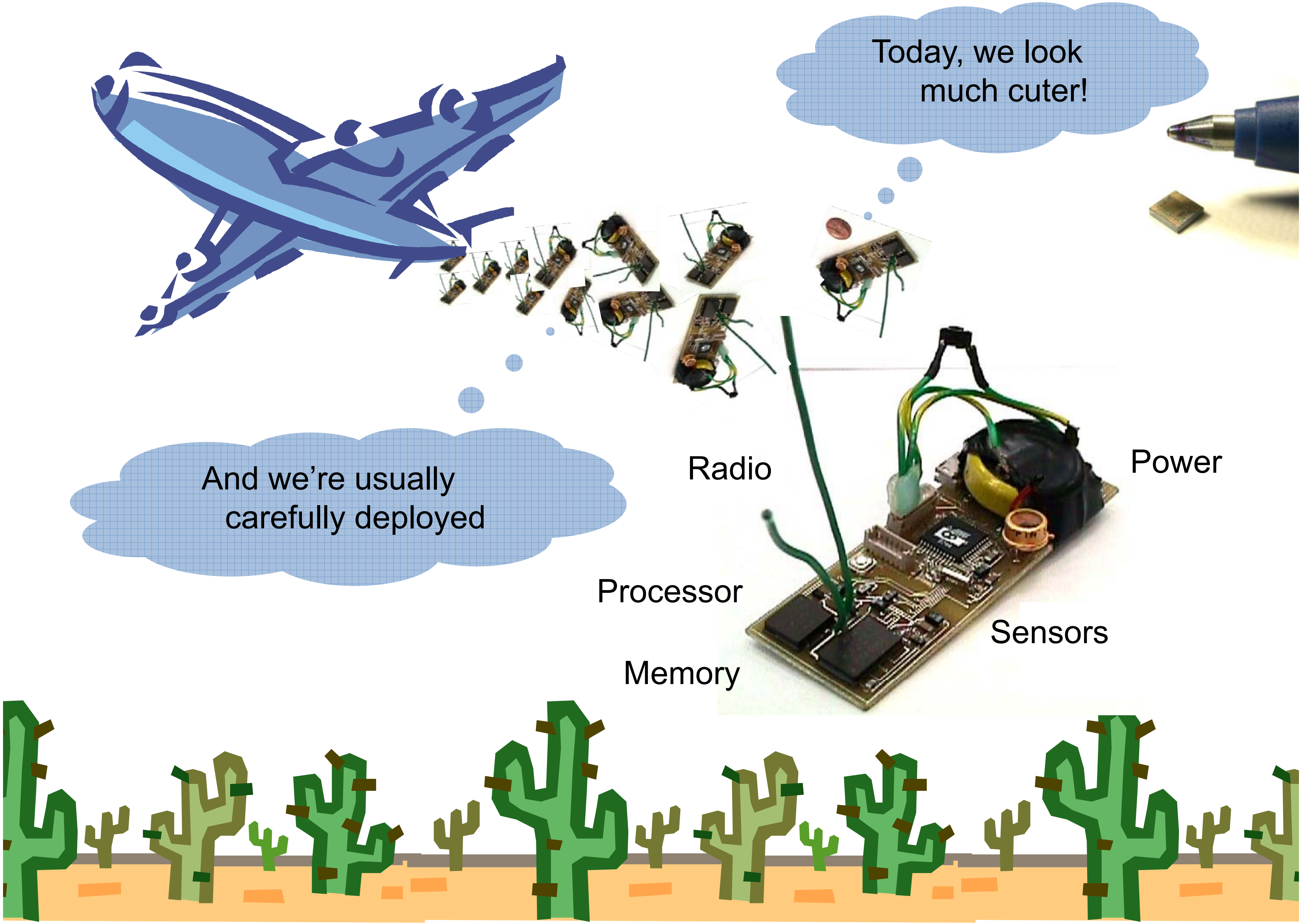
The paper

- Joint work with Thomas Moscibroda
  - Former PhD student of mine
  - Now researcher at Microsoft Research, Redmond
  - Infocom 2006 presentation by Thomas
  - Some slides by Thomas. Thanks!

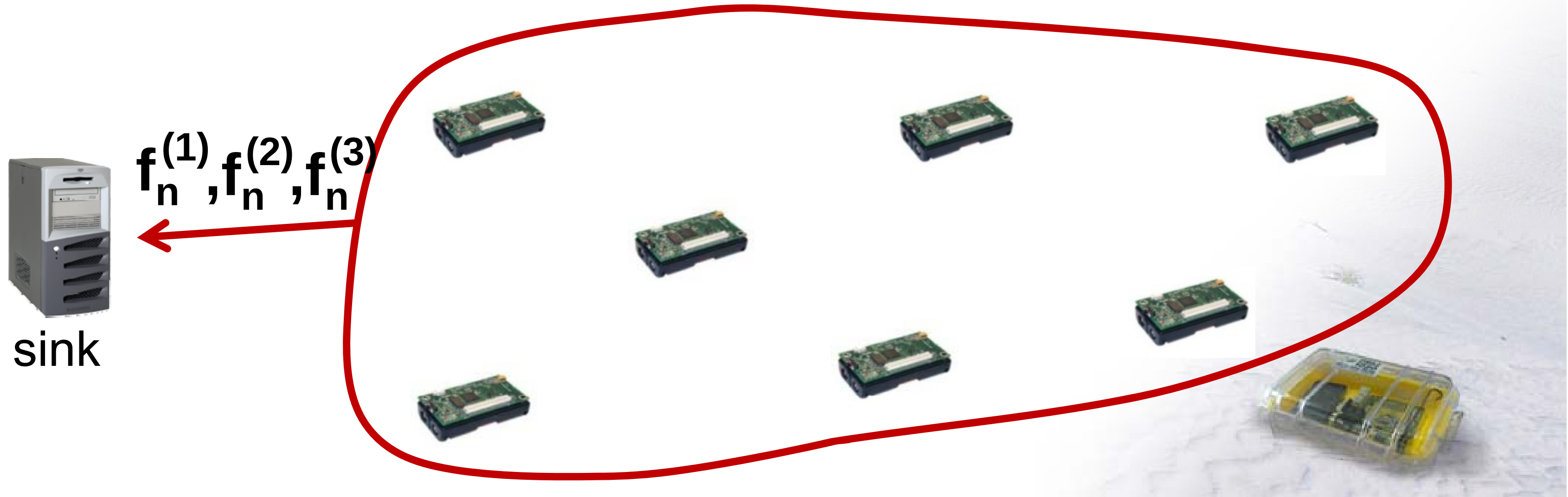
- Paper is about wireless networking in general
  - This talk: new introduction/motivation for sensor networks


Today, we look
much cuter!
And we're usually
carefully deployed
Radio
Power
Processor
Sensors
Memory

Data Gathering in Wireless Sensor Networks

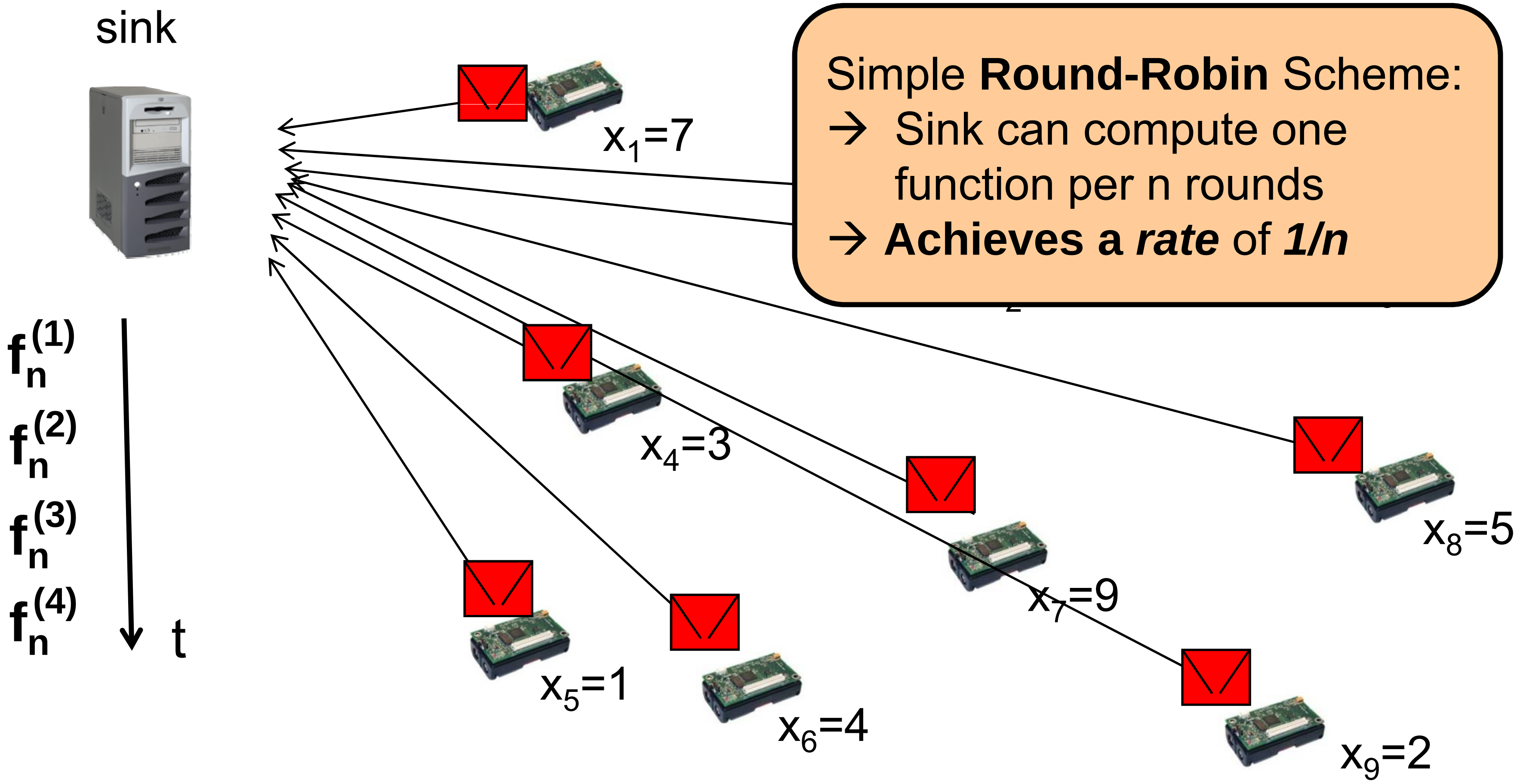
- Data gathering & aggregation
  - Classic application of sensor networks
  - Sensor nodes periodically sense environment
  - Relevant information needs to be transmitted to sink

- Functional Capacity of Sensor Networks
  - Sink peridically wants to compute a function f_n of sensor data
  - At what rate can this function be computed?



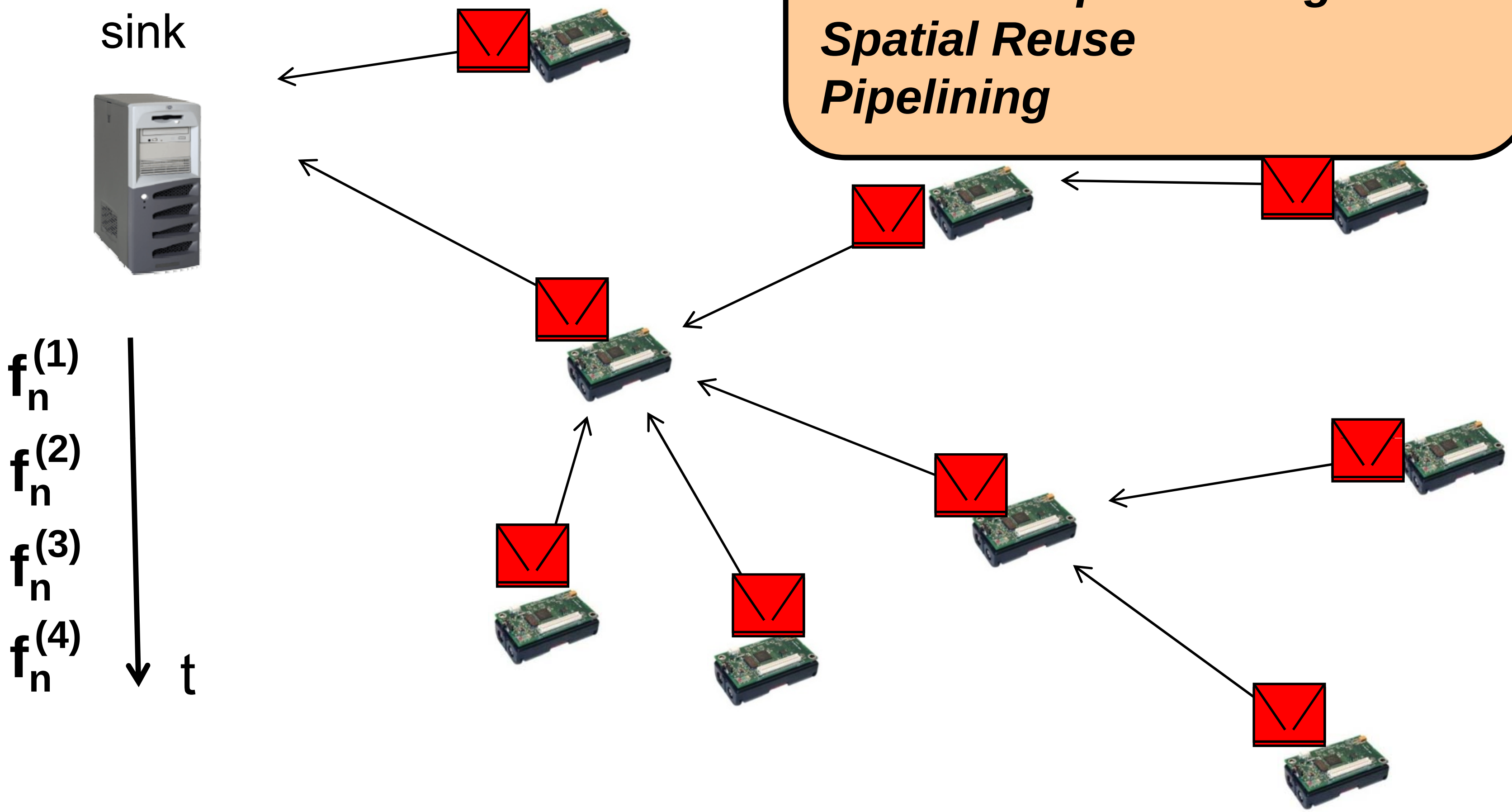
Data Gathering in Wireless Sensor Networks

Example: simple round-robin scheme
→ Each sensor reports its results directly to the root one after another



Simple **Round-Robin** Scheme:
→ Sink can compute one function per n rounds
→ **Achieves a *rate* of *1/n***

Data Gathering in Wireless Sensor Networks



There are better schemes using
Multi-hop relaying
In-network processing
Spatial Reuse
Pipelining

Capacity in Wireless Sensor Networks

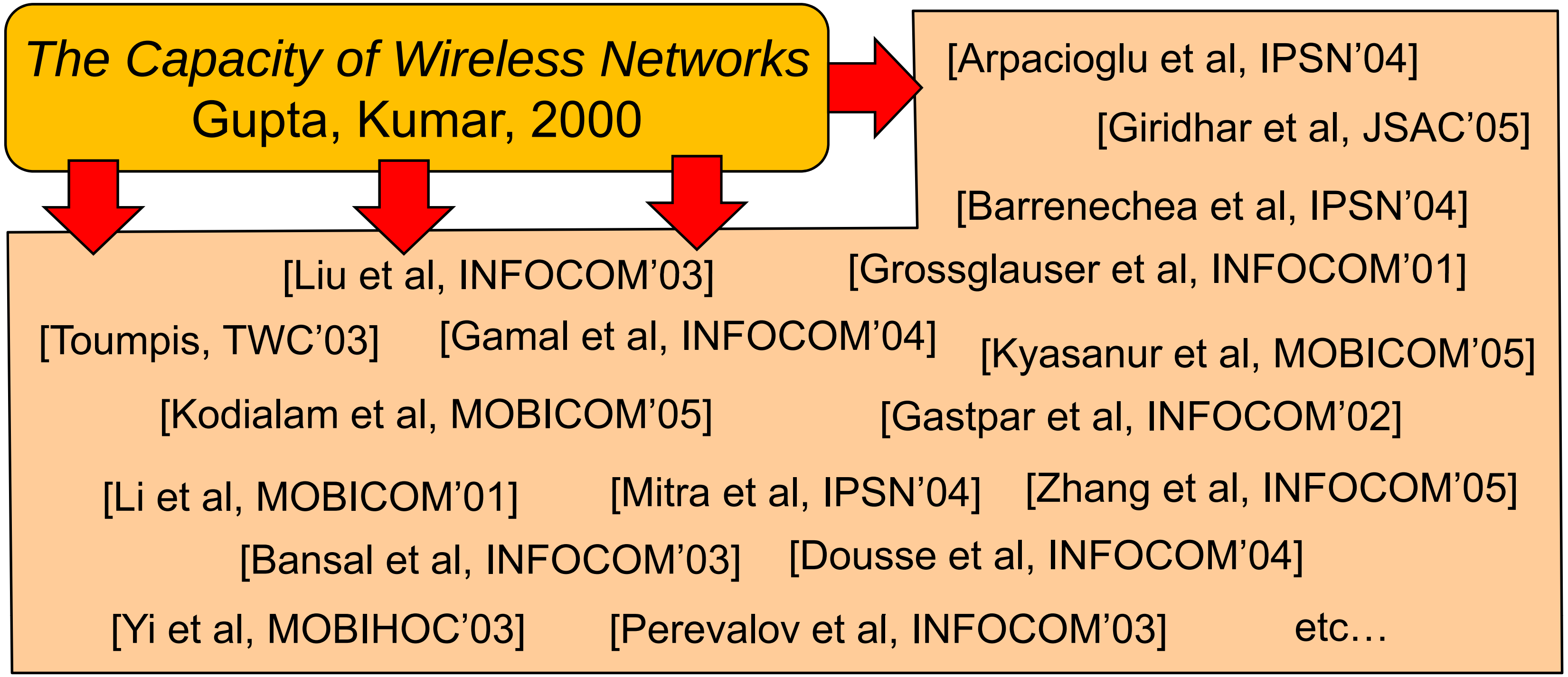
At what ***rate*** can sensors transmit data to the sink?
Scaling-laws → how does rate decrease as *n* increases…?

$\Theta(1/n)$ | $\Theta(1/\sqrt{n})$ | $\Theta(1/\log n)$ | $\Theta(1)$

Answer depends on:

- Function to be computed → Only perfectly compressible functions (max, min, avg,…)
- Coding techniques → No fancy coding techniques
- Network topology
- Wireless communication model

"Classic" Capacity…

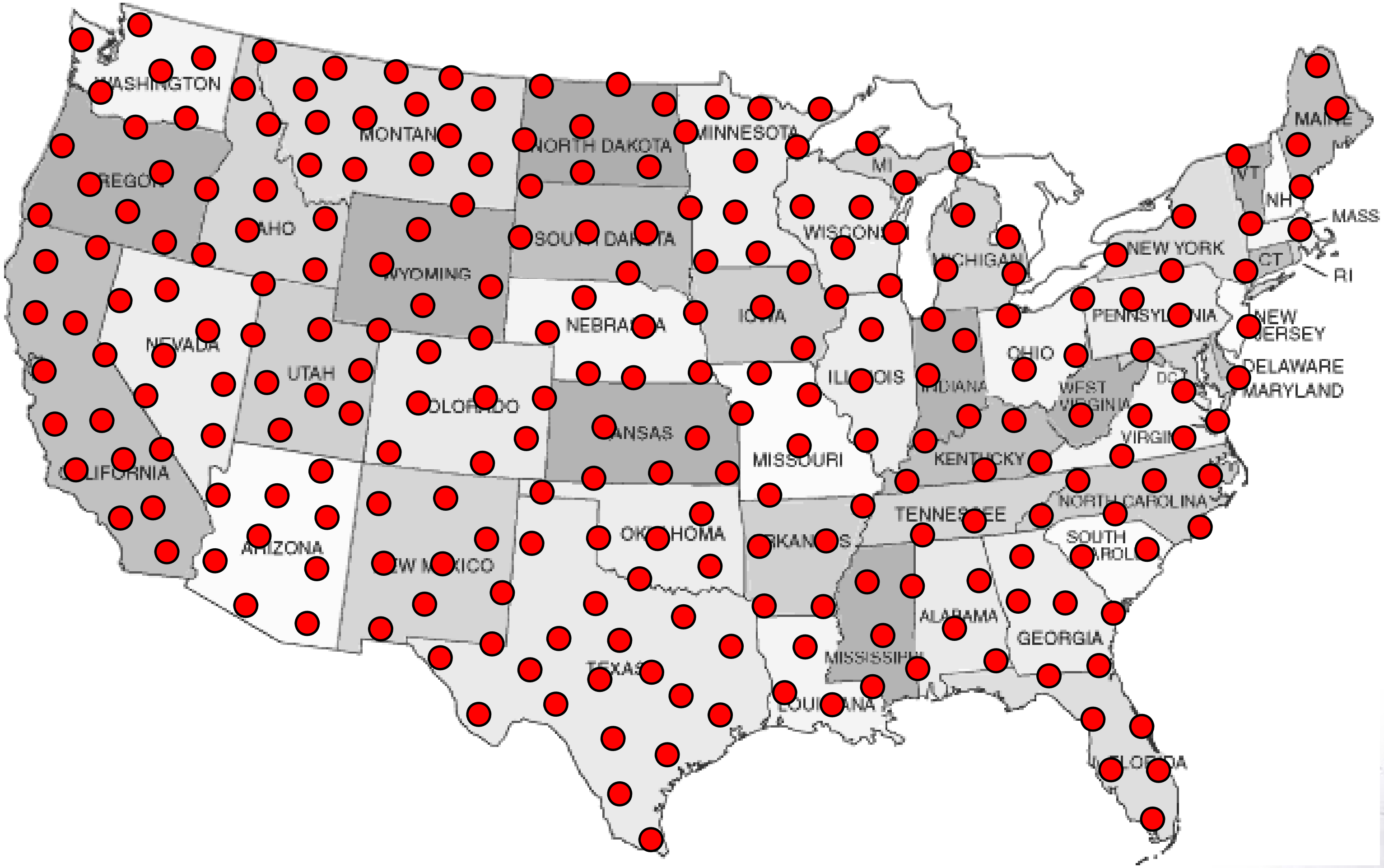


Worst-Case Capacity

- Capacity studies so far make very strong assumptions on node deployment, topologies
  - randomly, uniformly distributed nodes
  - nodes placed on a grid
  - etc...

What if a network looks differently…?

Like this?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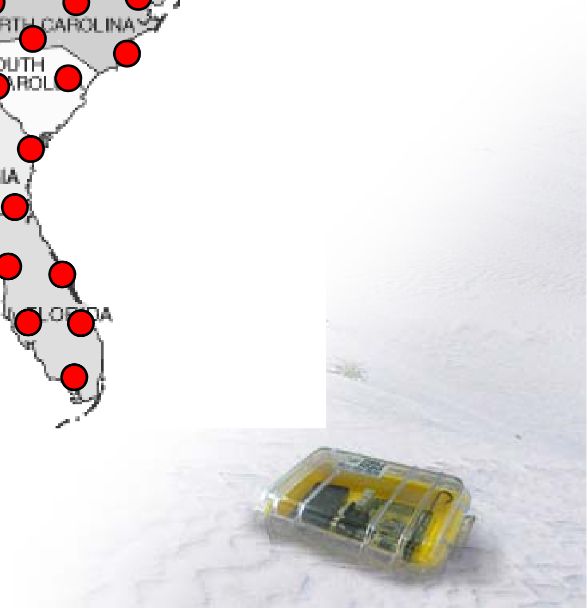
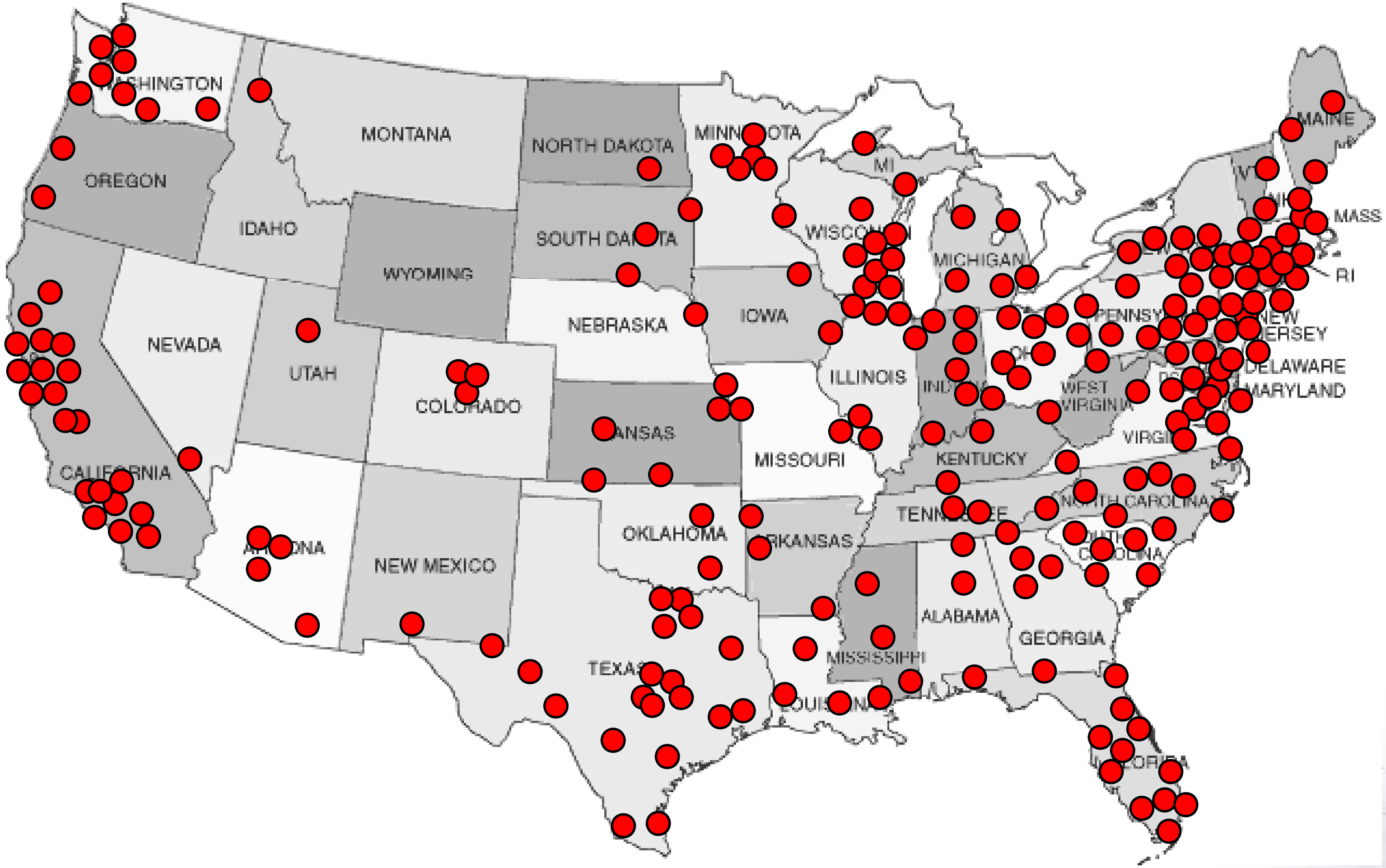


Or rather like this?



Worst-Case Capacity

- Capacity studies so far have made very strong assumptions on node deployment, topologies
  - randomly, uniformly distributed nodes
  - nodes placed on a grid
  - etc...

What if a network looks differently…?

We assume arbitrary node distribution

worst-case topologies

Classic Capacity ⟷ **Worst-Case Capacity**

How much information can be transmitted in nice, well-behaving networks

How much information can be Transmitted in any network

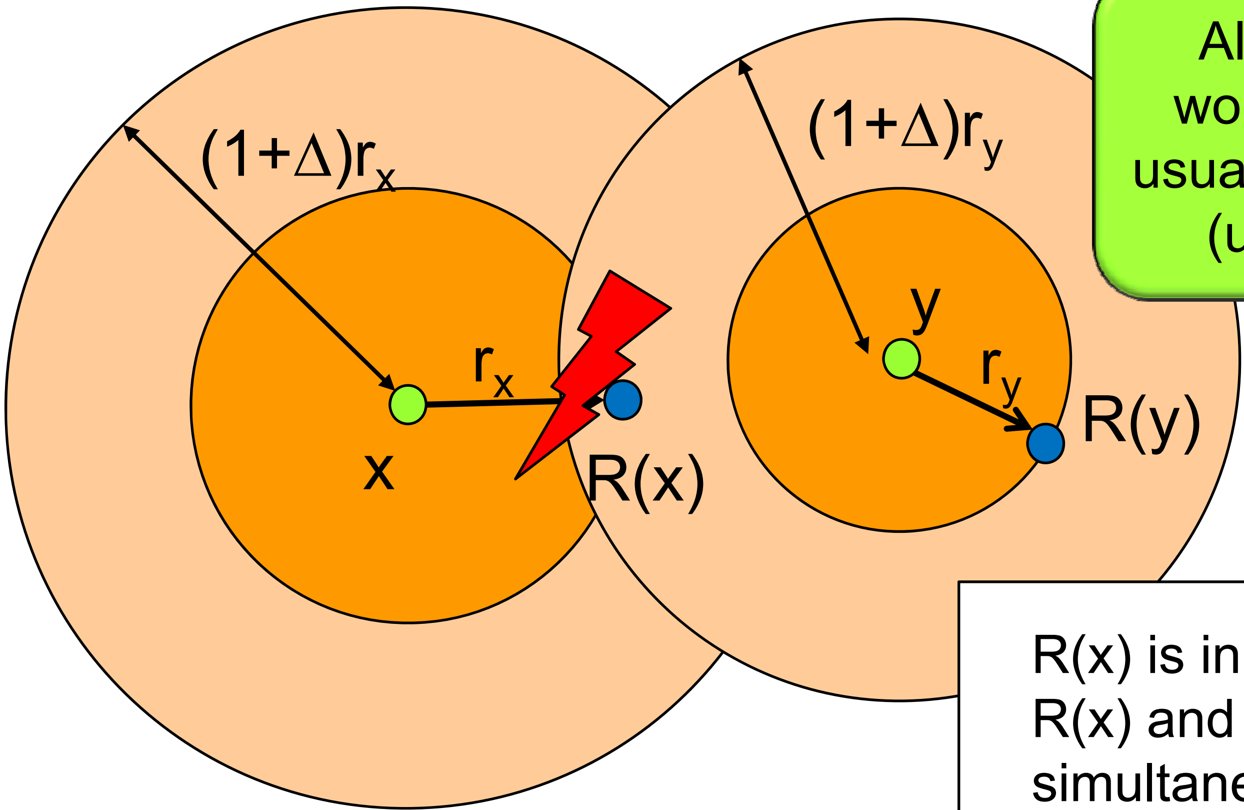
Models

- Two standard models in wireless networking

Protocol Model (graph-based, simpler) ⟷ Physical Model (SINR-based, more realistic)

Protocol Model

- Based on graph-based notion of interference
- Transmission range and interference range

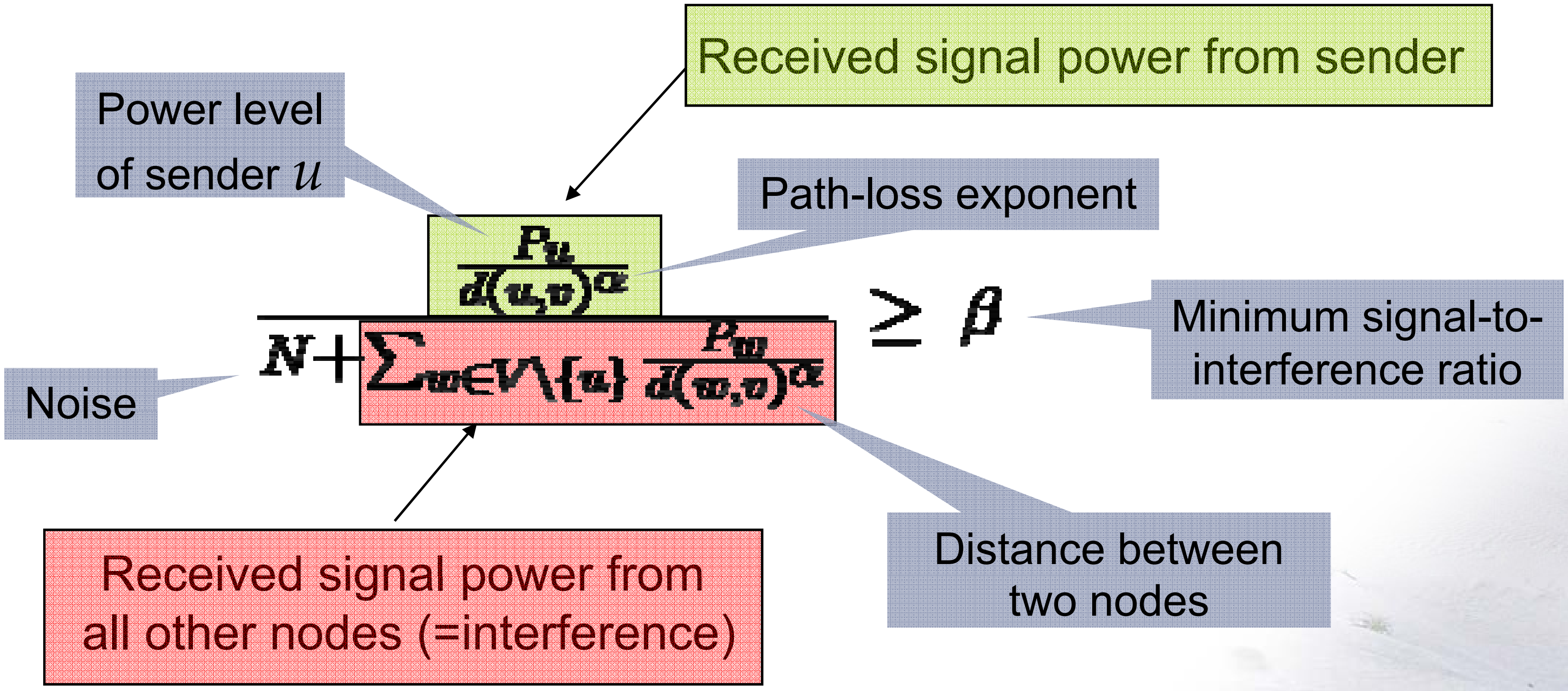


Algorithmic work on worst-case topologies usually in protocol models (unit disk graph,…)

R(x) is in interference range of y
R(x) and R(y) cannot simultaneously receive!

Physical Model

- Based on signal-to-noise-plus-interference (SINR)
- Simplest case:
  → packets can be decoded if SINR is larger than β at receiver



Models

- Two standard models of wireless communication

Protocol Model (graph-based, simpler) ⟷ Physical Model (SINR-based, more realistic)

- Algorithms typically designed and analyzed in protocol model

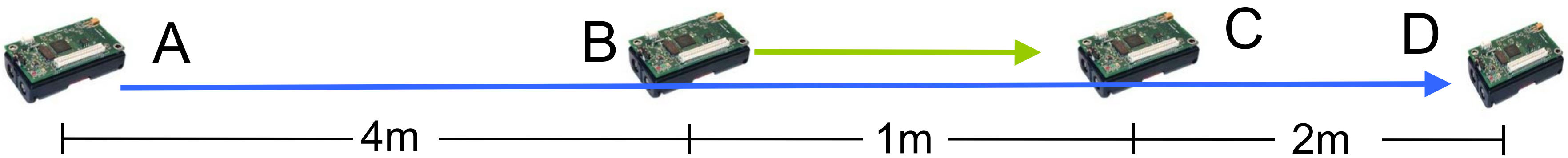
Premise: Results obtained in protocol model do not divert too much from more realistic model!

Justification:

Capacity results are typically (almost) the same in both models (e.g., Gupta, Kumar, etc...)

Example: Protocol vs. Physical Model

A sends to D, B sends to C



Assume a single frequency (and no fancy decoding techniques!)

Is spatial reuse possible?

- NO — Protocol Model
- YES — Physical Model
- YES — In Reality!

Let α=3, β=3, and N=10nW

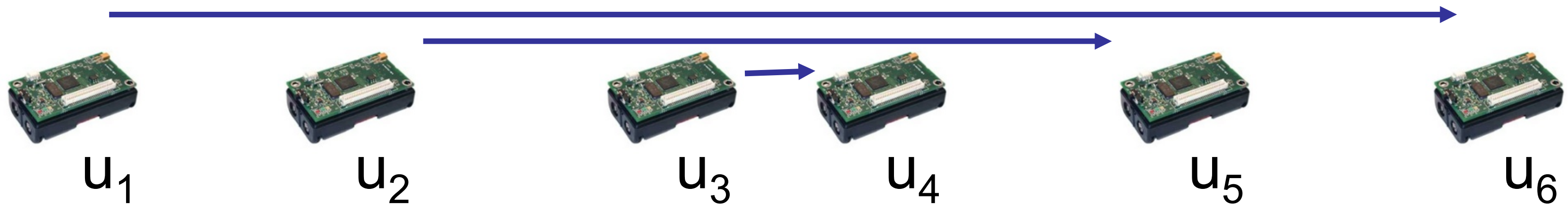
Transmission powers: P_B= -15 dBm and P_A= 1 dBm

SINR of A at D: $$\frac{1.26mW/(7m)^3}{0.01\mu W+31.6\mu W/(3m)^3} \approx 3.11 \geq \beta$$

SINR of B at C: $$\frac{31.6\mu W/(1m)^3}{0.01\mu W+1.26mW/(5m)^3} \approx 3.13 \geq \beta$$

This works in practice!

- We did measurements using standard mica2 nodes!
- Replaced standard MAC protocol by a (tailor-made) „SINR-MAC“
- Measured for instance the following deployment...



- Time for successfully transmitting 20‘000 packets:

	Time required	
	standard MAC	“SINR-MAC”
Node u_1	$721s$	$267s$
Node u_2	$778s$	$268s$
Node u_3	$780s$	$270s$

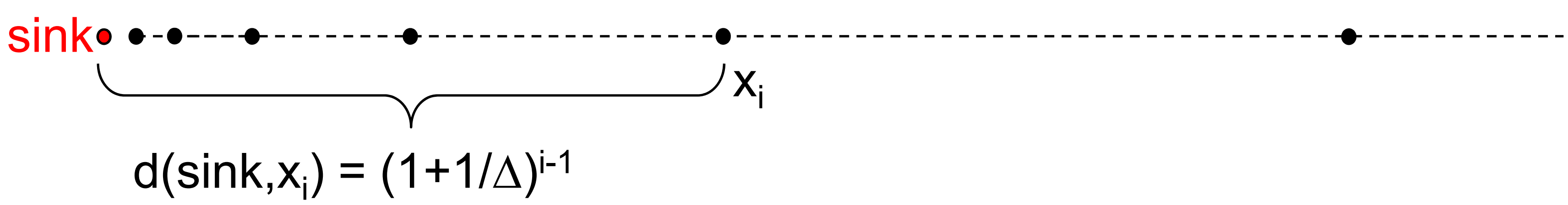
	Messages received	
	standard MAC	“SINR-MAC”
Node u_4	19999	19773
Node u_5	18784	18488
Node u_6	16519	19498

Speed-up is almost a factor 3

[Moscibroda, Wattenhofer, Weber, Hotnets’06]

Upper Bound Protocol Model

- There are networks, in which at most one node can transmit!
  → like round-robin
- Consider exponential node chain
- Assume nodes can choose arbitrary transmission power



- Whenever a node transmits to another node
  → All nodes to its left are in its interference range!
  → Network behaves like a single-hop network

In the **protocol model**, the achievable rate is $\Theta(1/n)$.

Lower Bound Physical Model

- Much better bounds in SINR-based physical model are possible (exponential gap)
- Paper presents a scheduling algorithm that achieves a rate of $\Omega(1/\log^3 n)$

In the **physical model**, the achievable rate is $\Omega(1/\text{polylog}\, n)$.

- Algorithm is centralized, highly complex → not practical
- But it shows that high rates are possible even in worst-case networks

- Basic idea: Enable spatial reuse by exploiting SINR effects.

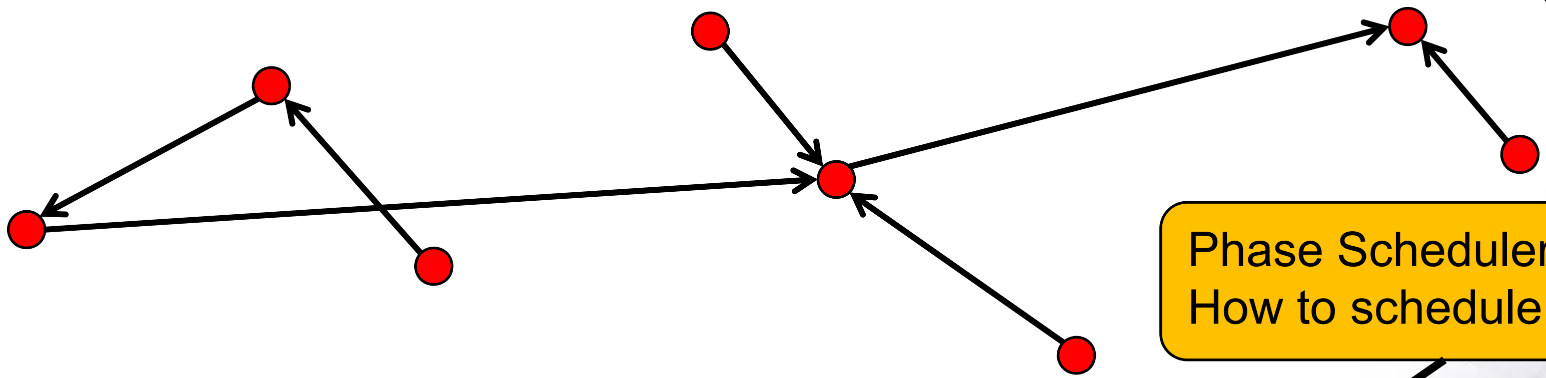
Scheduling Algorithm – High Level Procedure

- High-level idea is simple
- Construct a hierarchical tree T(X) that has desirable properties

1) Initially, each node is active
2) Each node connects to closest active node
3) Break cycles → yields forest
4) Only root of each tree remains active

loop until no active nodes

Can be adjusted if transmission power limited

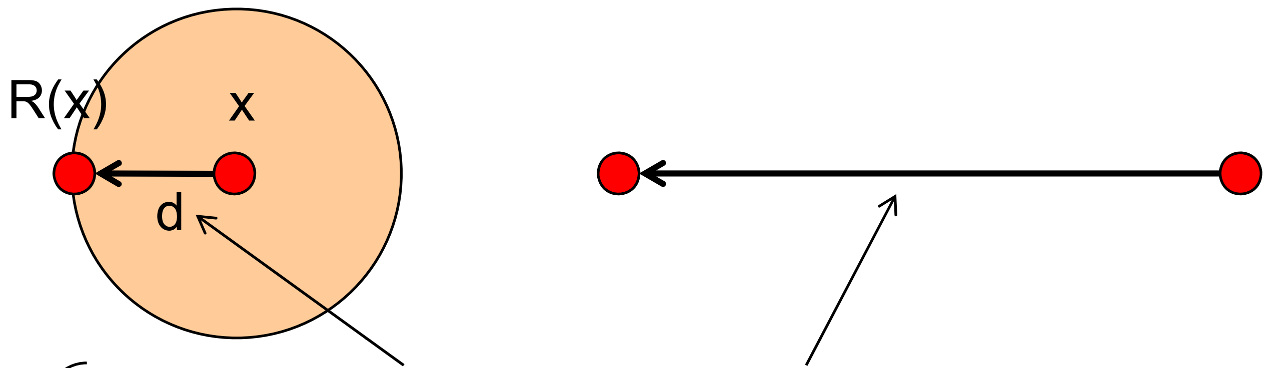


Phase Scheduler:
How to schedule T(X)?

The resulting structure has some nice properties
→ If each link of T(X) can be scheduled at least once in L(X) time-slots
→ Then, a rate of 1/L(X) can be achieved

Scheduling Algorithm – Phase Scheduler

- How to schedule T(X) efficiently
- We need to schedule links of different magnitude simultaneously!
- Only possibility:
  senders of small links must overpower their receiver!



If we want to schedule both links…

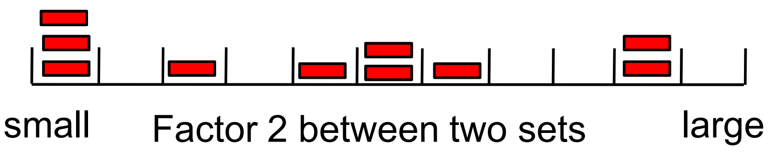
1) … R(x) must be overpowered
→ Must transmit at power more than ~d^{α}

2) If senders of small links overpower their receiver…
… their “safety radius” increases (spatial reuse smaller)

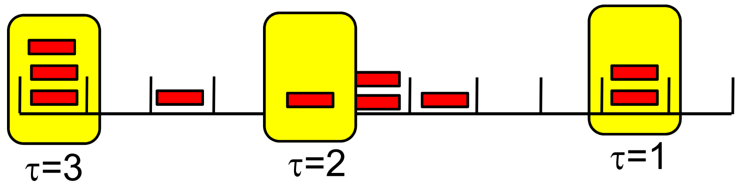
Subtle balance is needed!

Scheduling Algorithm – Phase Scheduler

1) Partition links into sets of similar length



2) Group sets such that links a and b in two sets in the same group have at least $d_a \geq (\xi\beta)^{\xi(\tau a-\tau b)} \cdot d_b$



→ Each link gets a τ_{ij} value → Small links have large τ_{ij} and vice versa

→ Schedule links in these sets in one outer-loop iteration

→ Intuition: Schedule links of similar length or very different length

3) Schedule links in a group → Consider in order of decreasing length
(I will not show details because of time constraints.)

Together with structure of T(x) → $\Omega(1/\log^3 n)$ bound

Worst-Case Capacity in Wireless Networks

Worst-Case Capacity

Traditional Capacity

Model \ Networks	Max. rate in arbitrary, worst-case deployment	Max. rate in random, uniform deployment
Protocol Model	$\Theta(1/n)$	$\Theta(1/\log n)$
Physical Model	$\Omega(1/\log^3 n)$	$\Omega(1/\log n)$

[Giridhar, Kumar, 2005]

Exponential gap between protocol and physical model!

<u>The Price of Worst-Case Node Placement</u>

- Exponential in protocol model
- Polylogarithmic in physical model (almost no worst-case penalty!)

Conclusions

- Introduce worst-case capacity of sensor networks
  → How much data can periodically be sent to data sink

- Complements existing capacity studies
- Many novel insights

1) Possibilities and limitations of wireless communication
2) Fundamentals of wireless communication models
3) How to devise efficient scheduling algorithms, protocols

Sensor Networks Scale!
Efficient data gathering is possible in every (even worst-case) network!

Protocol Model Poor!
Exponential gap between protocol and physical model!

Efficient Protocols!
Must use SINR-effects and power control to achieve high rate!

Overview of results so far

- Moscibroda, Wattenhofer, Infocom 2006
  - First paper in this area, O($\log^3 n$) bound for connectivity, and more
  - This is essentially the paper I presented on the previous slides

- Moscibroda, Wattenhofer, Zollinger, MobiHoc 2006
  - First results beyond connectivity, namely in the topology control domain
- Moscibroda, Wattenhofer, Weber, HotNets 2006
  - Practical experiments, ideas for capacity-improving protocol
- Moscibroda, Oswald, Wattenhofer, Infocom 2007
  - Generalizion of Infocom 2006, proof that known algorithms perform poorly
- Goussevskaia, Oswald, Wattenhofer, MobiHoc 2007
  - Hardness results & constant approximation for constant power
- Chafekar, Kumar, Marathe, Parthasarathy, Srinivasan, MobiHoc 2007
  - Cross layer analysis for scheduling and routing
- Moscibroda, IPSN 2007
  - Connection to data gathering, improved O($\log^2 n$) result
- Locher, von Rickenbach, Wattenhofer, ICDCN 2008
  - Still some major open problems

Main open question in this area

- Most papers so far deal with special cases, essentially scheduling a number of links with special properties. The general problem is still wide open:

- A communication request consists of a source and a destination, which are arbitrary points in the Euclidean plane. Given *n* communication requests, assign a color (time slot) to each request. For all requests sharing the same color specify power levels such that each request can be handled correctly, i.e., the SINR condition is met at all destinations. The goal is to minimize the number of colors.

- E.g., for arbitrary power levels not even hardness is known…

Thank You!

Questions & Comments?