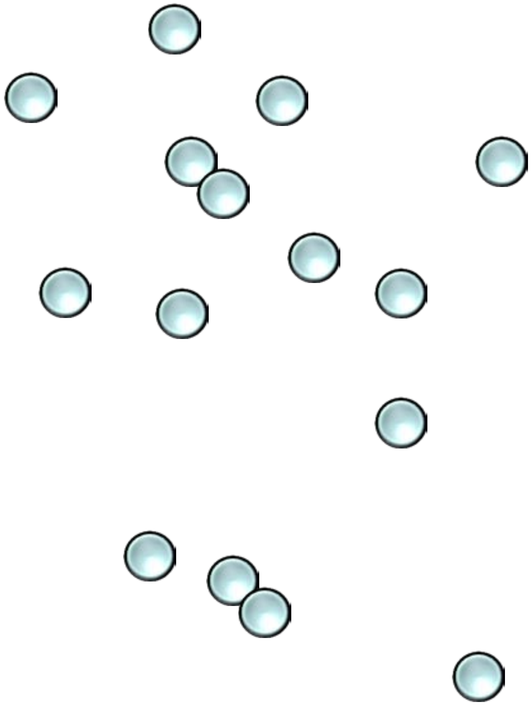
Time-Optimal Information Exchange on Multiple Channels

Stephan Holzer
Yvonne Anne Pignolet
Jasmin Smula
Roger Wattenhofer

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

Problem:

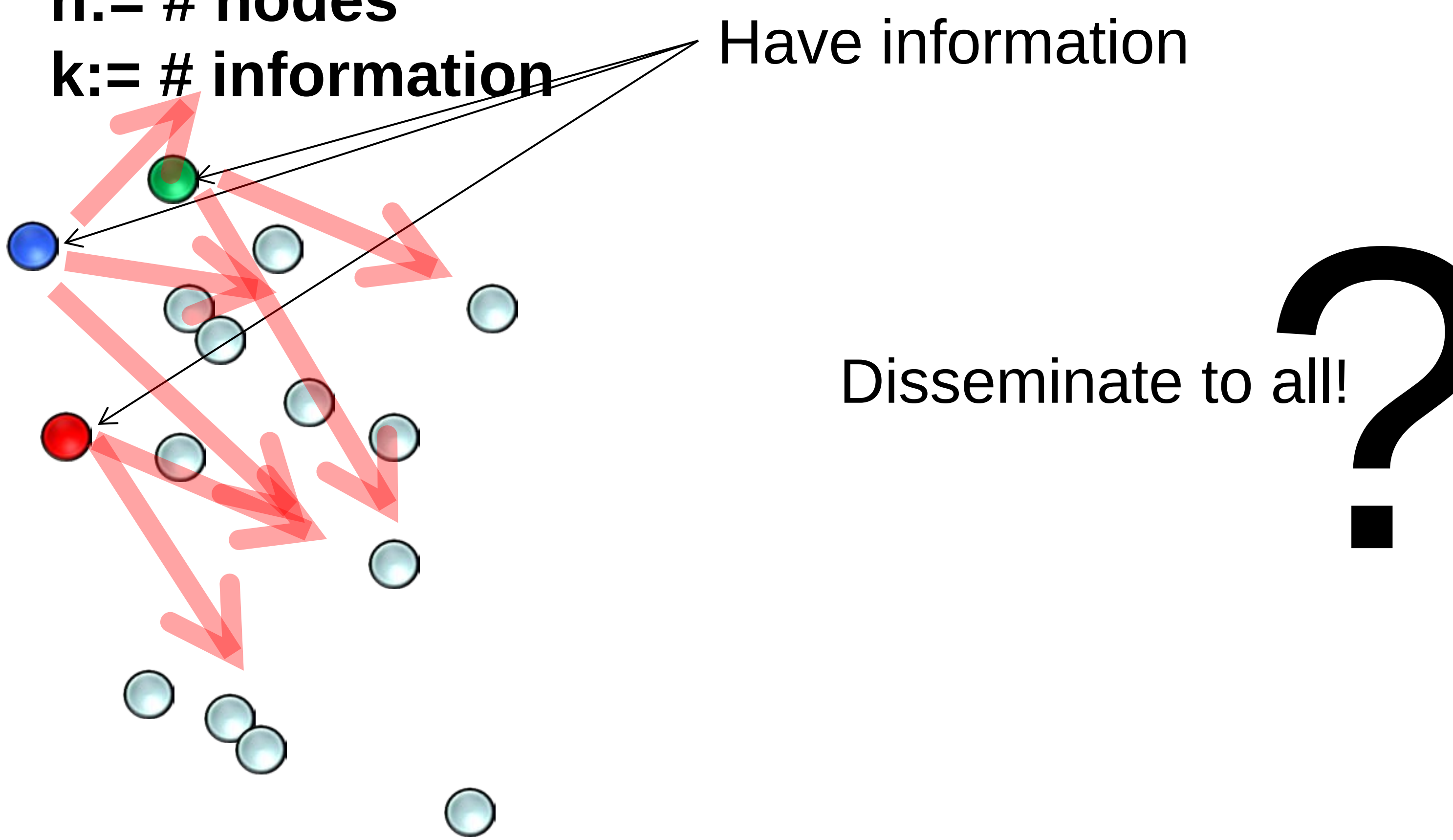


Time-Optimal Information Exchange on Multiple Channels

Problem:

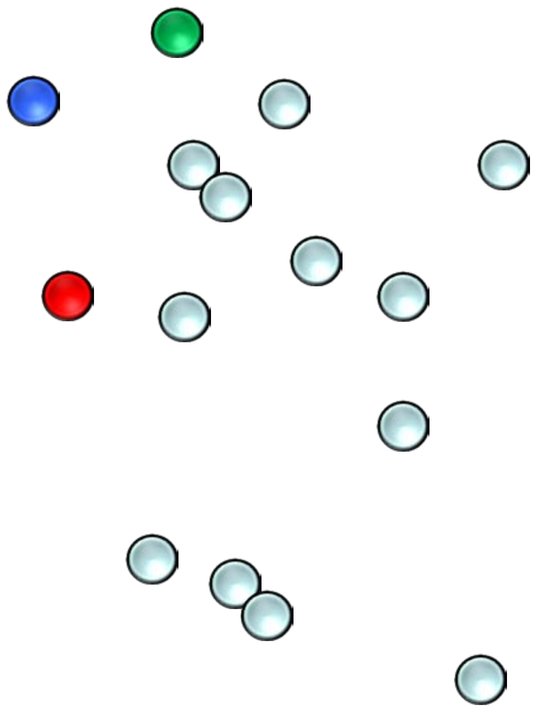
n:= # nodes

k:= # information



Time-Optimal Information Exchange on Multiple Channels

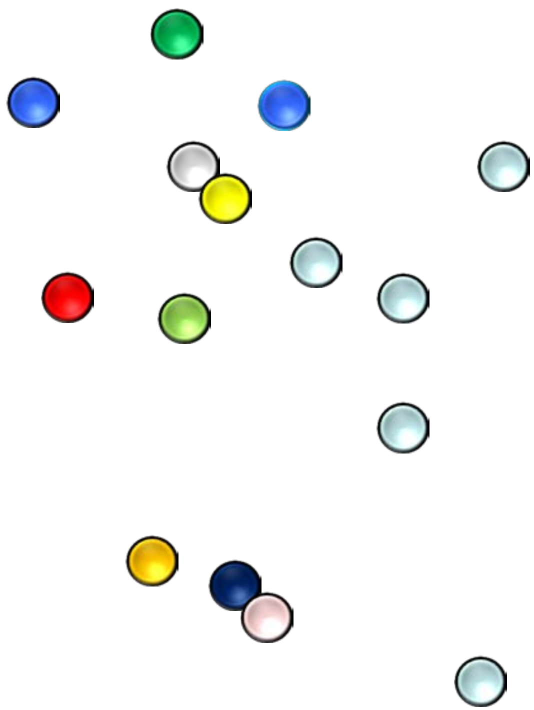
Problem:



Disseminate to all! ?

Time-Optimal Information Exchange on Multiple Channels

Problem:



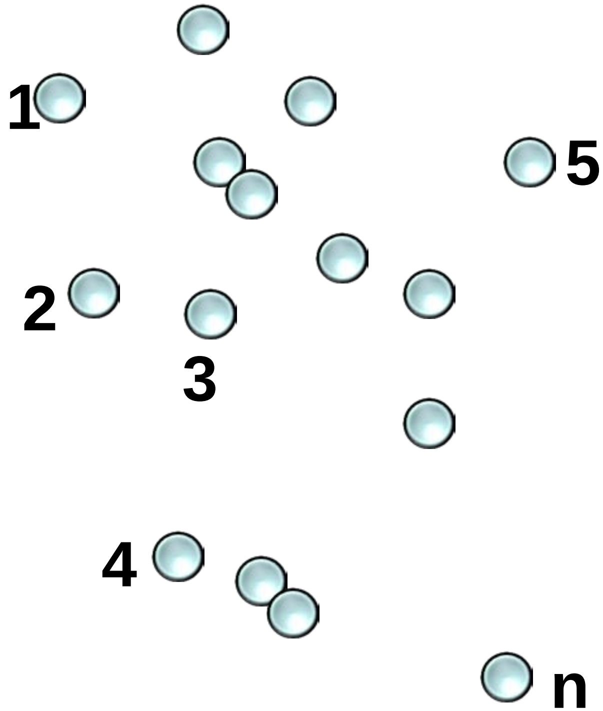
Disseminate to all!

Easy: O(n)

Faster?

Problem:

n:= # nodes



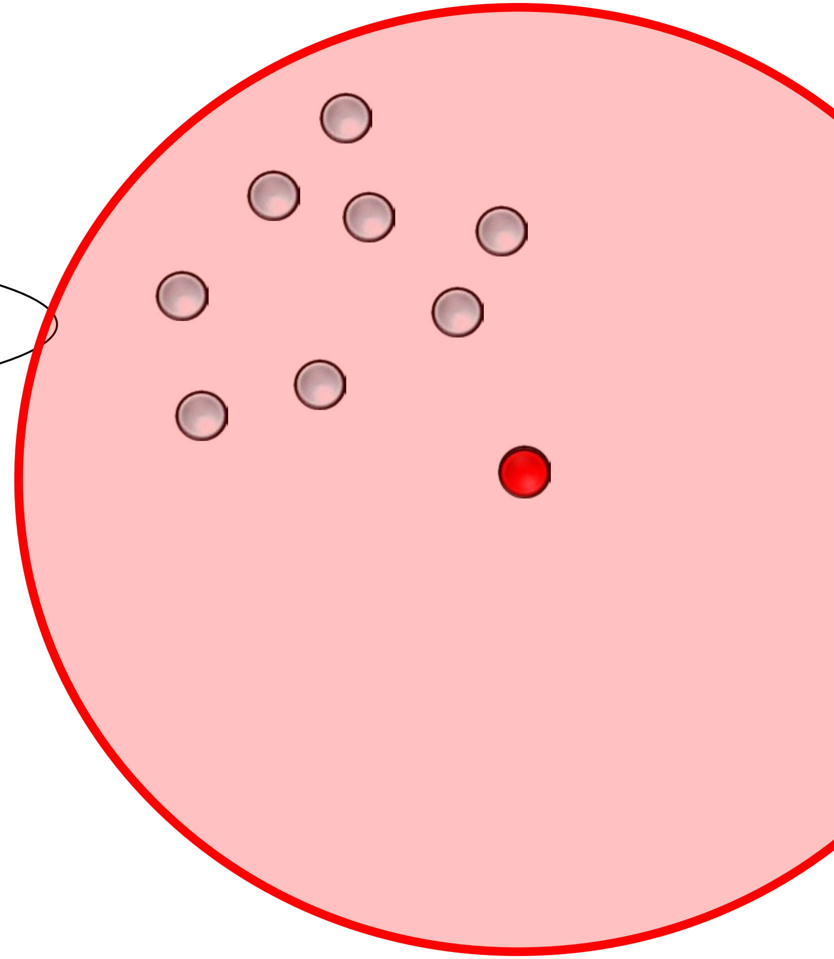
Unique IDs 1…n

Time-Optimal Information Exchange … Channels

I can:

send / receive

reach each node

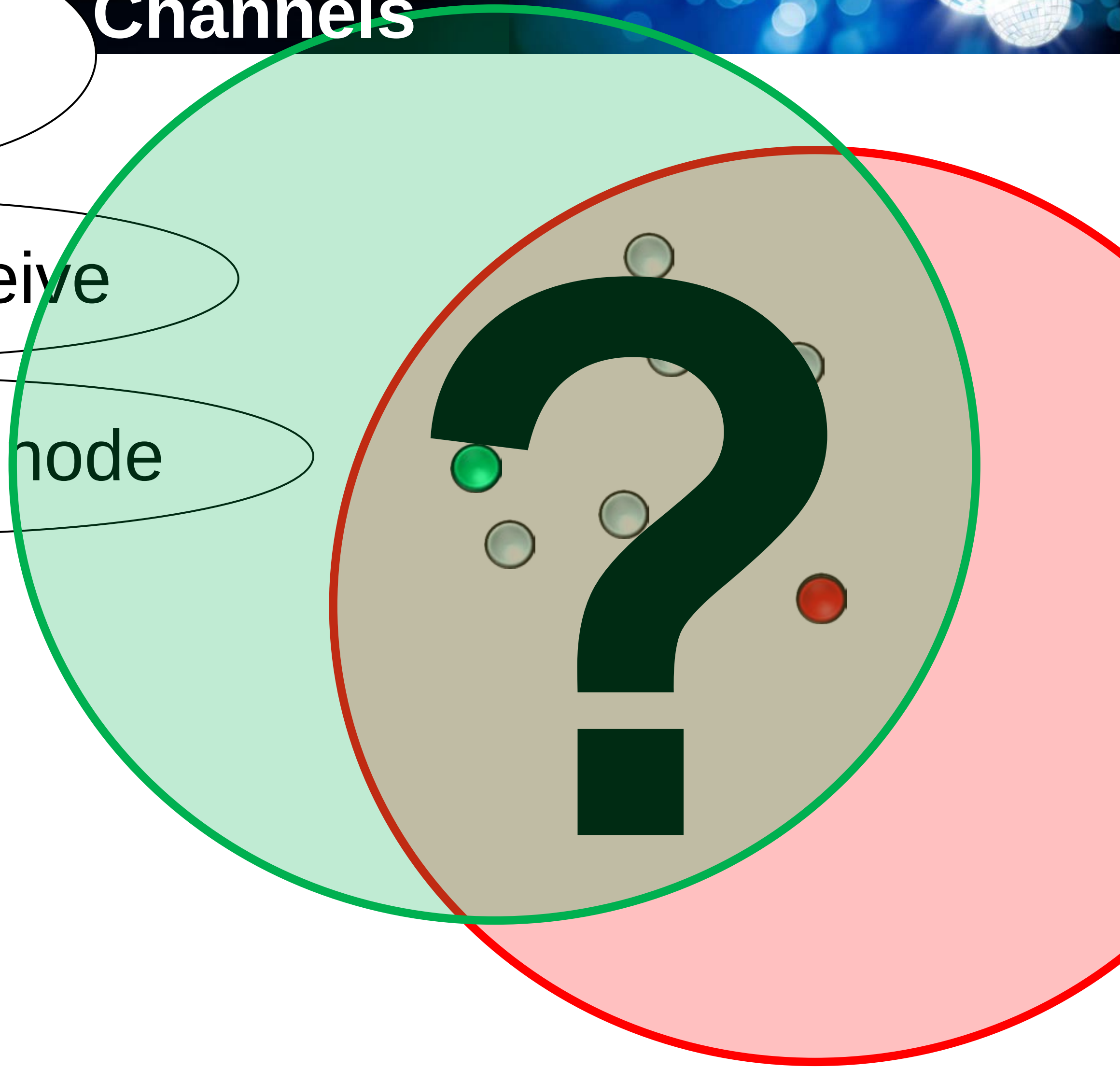


Time-Optimal Information Exchange Channels

I can:

send / receive

reach each node



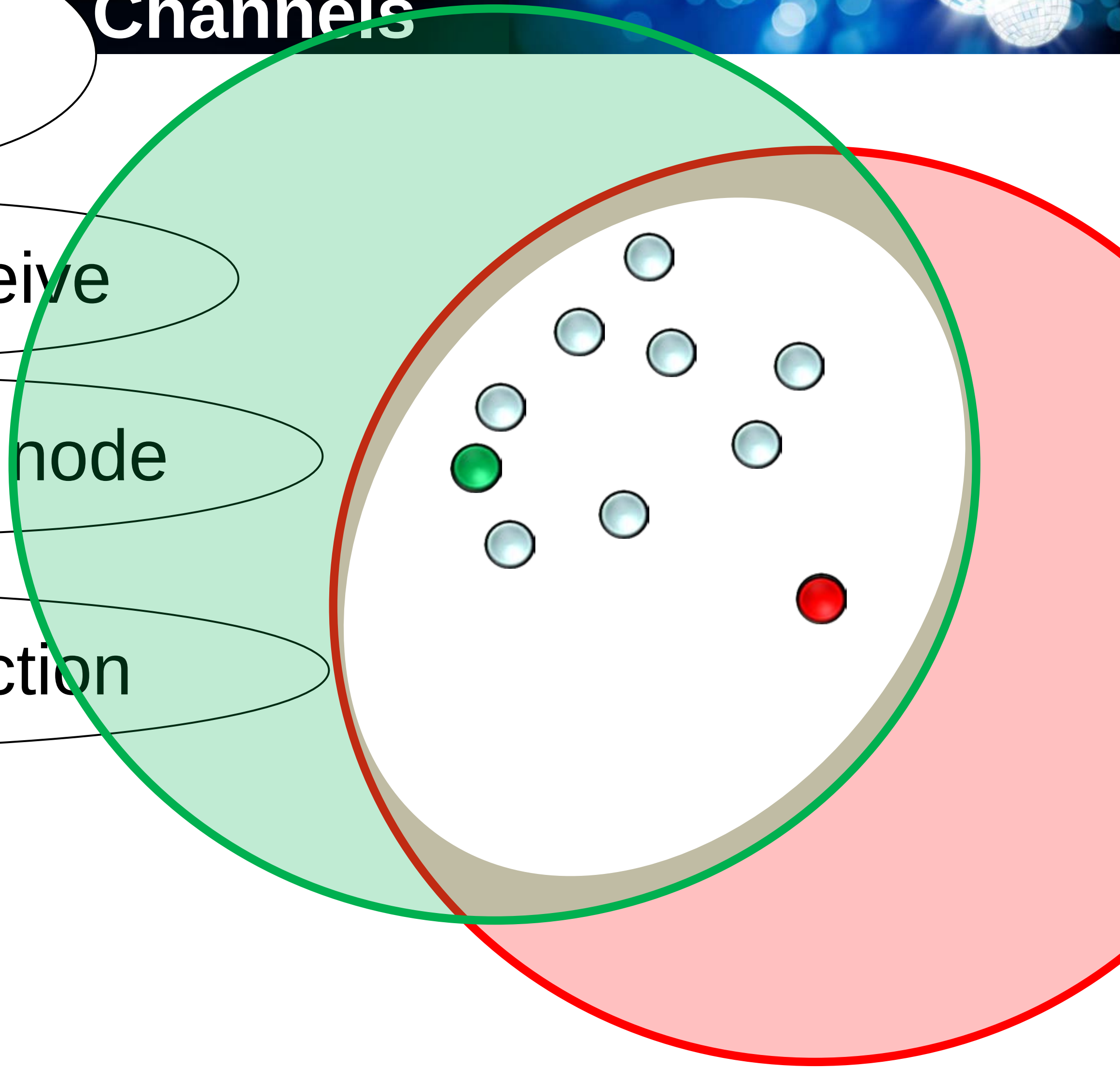
Time-Optimal Information Exchange … Channels

I can:

send / receive

reach each node

no collision detection



Time-Optimal Information Exchange … Channels

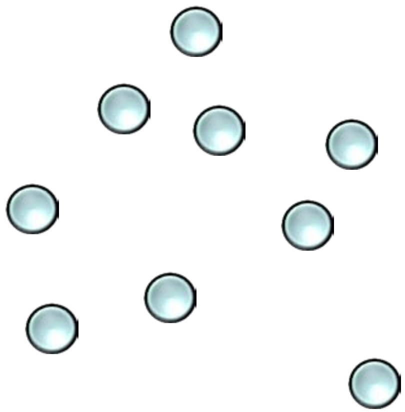
I can:

send / receive

reach each node

no collision detection

switch channels



101 Mhz
117 Mhz
132 Mhz
…

synchronus

Time-Optimal Information Exchange Channels

I can:

send / receive

reach each node

no collision detection

switch channels

complexity

computation: free

radio: time 1

synchronus

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

One Information / log n
bits per message

~~O(1)~~

$\Rightarrow \Omega(k \qquad)$

⌋ ⌋

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Multi channel

$$\boldsymbol{\Theta(k)}$$

?

one channel

$$=> \Omega(k + \cancel{\log n})$$

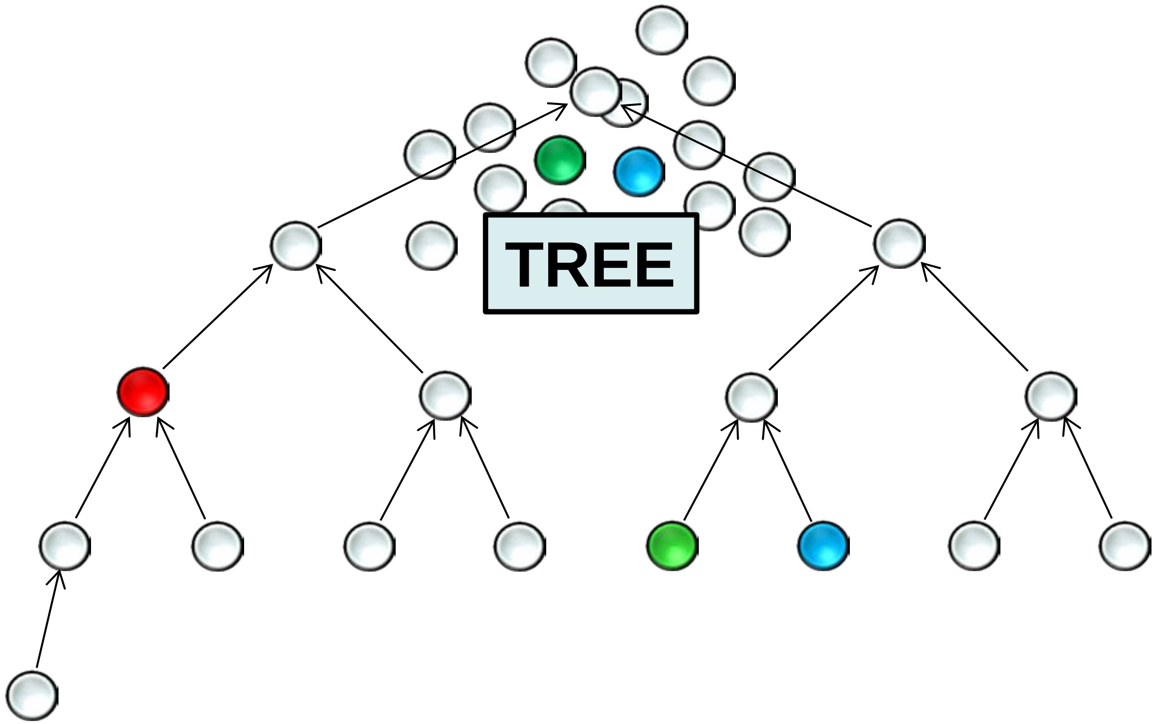
[Kushilevitz, Mansour SIAM JComp 1998]

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

What can I do?

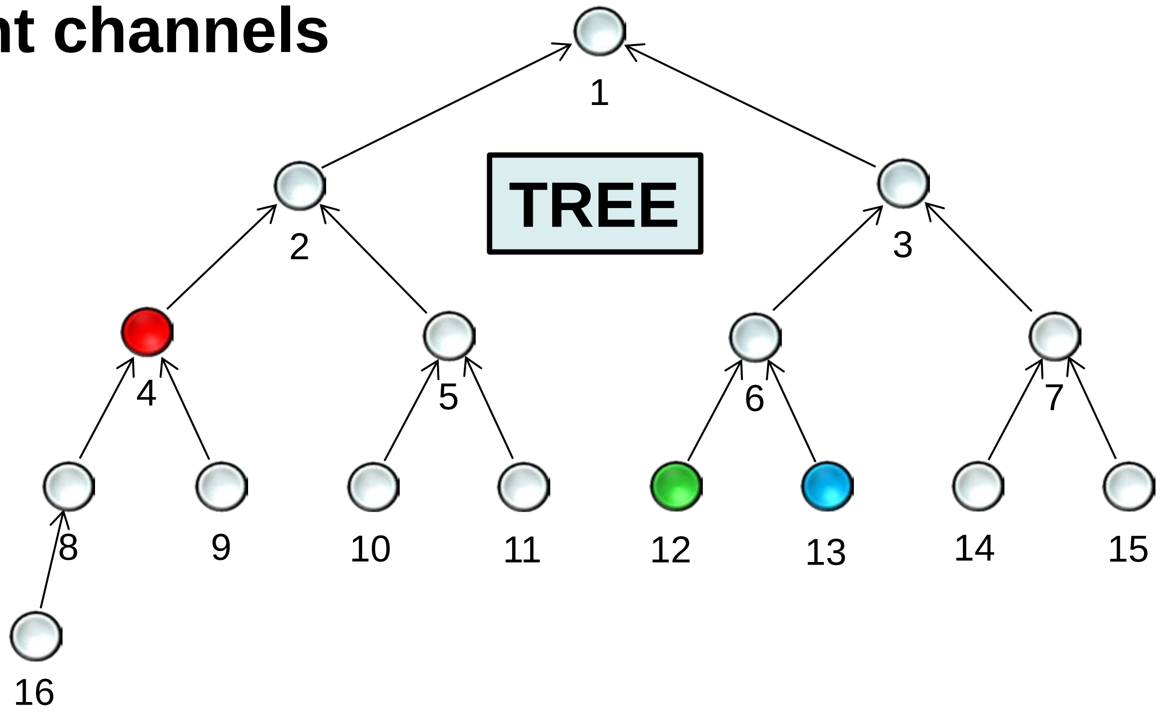


Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Communicate in parallel on different channels

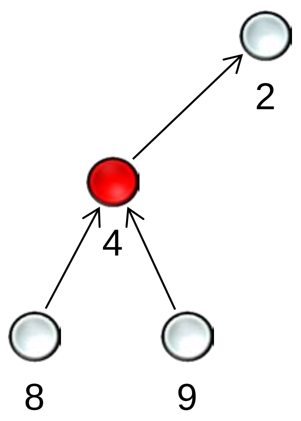


Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Communicate in parallel
on different channels

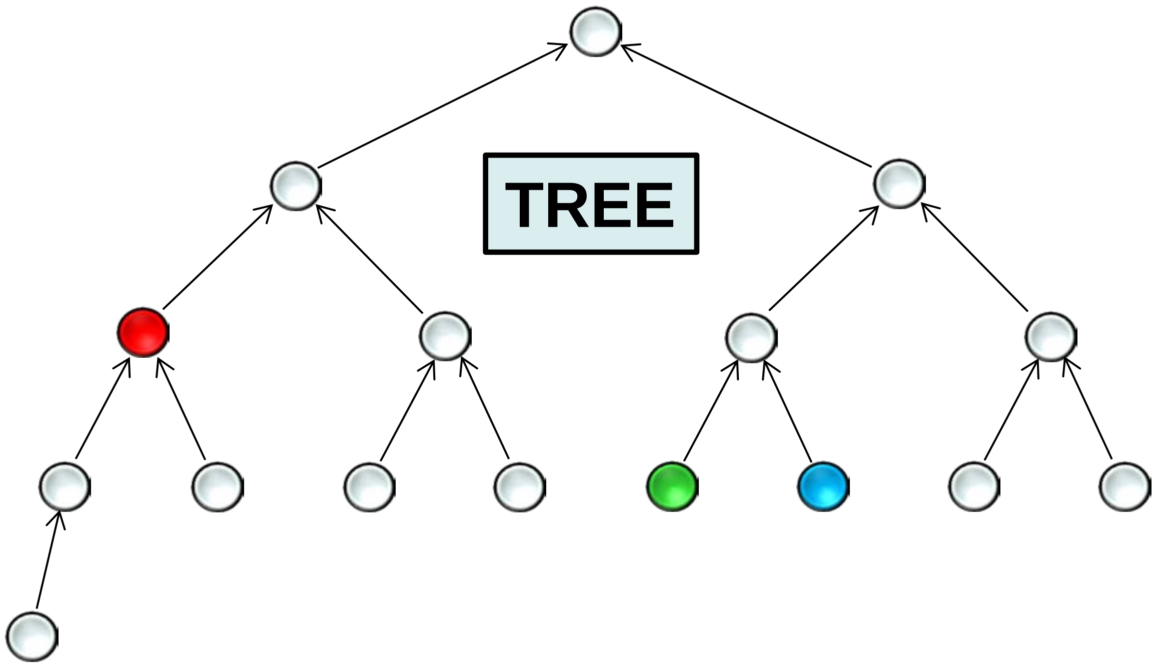


TREE

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

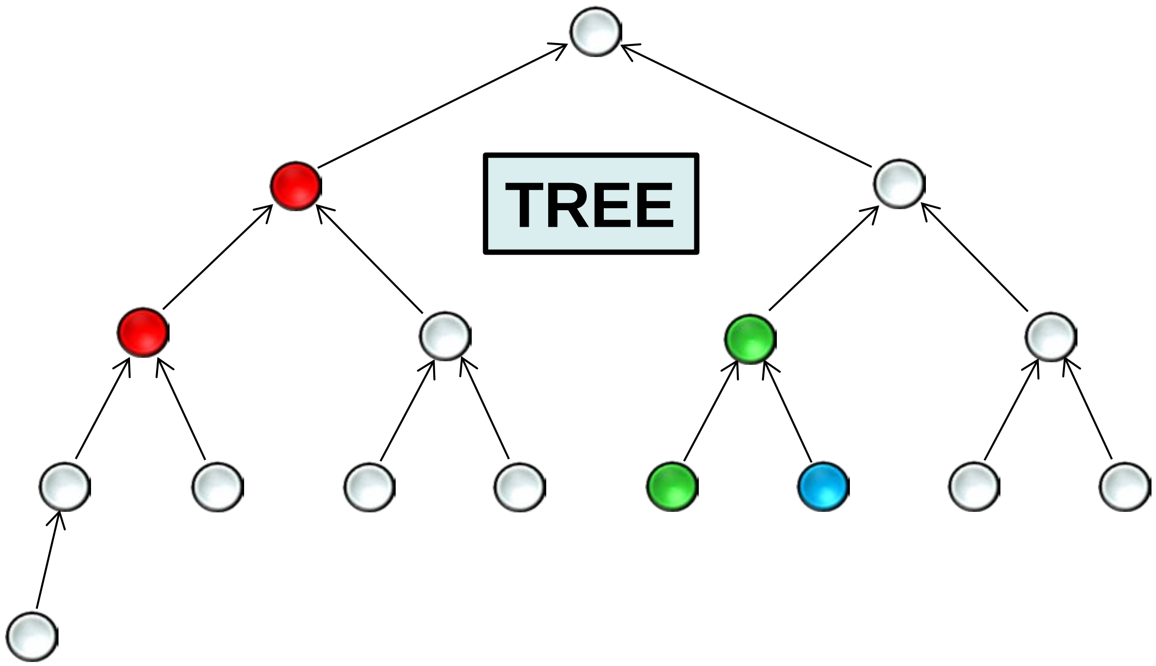
k:= # information



Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

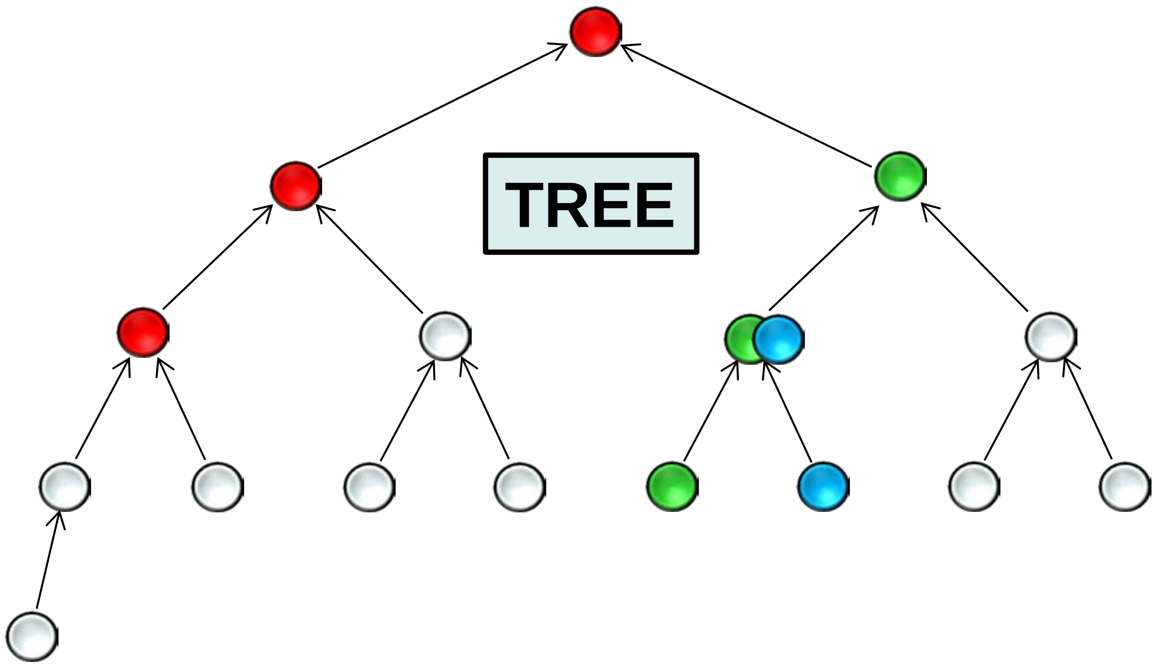
k:= # information



Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

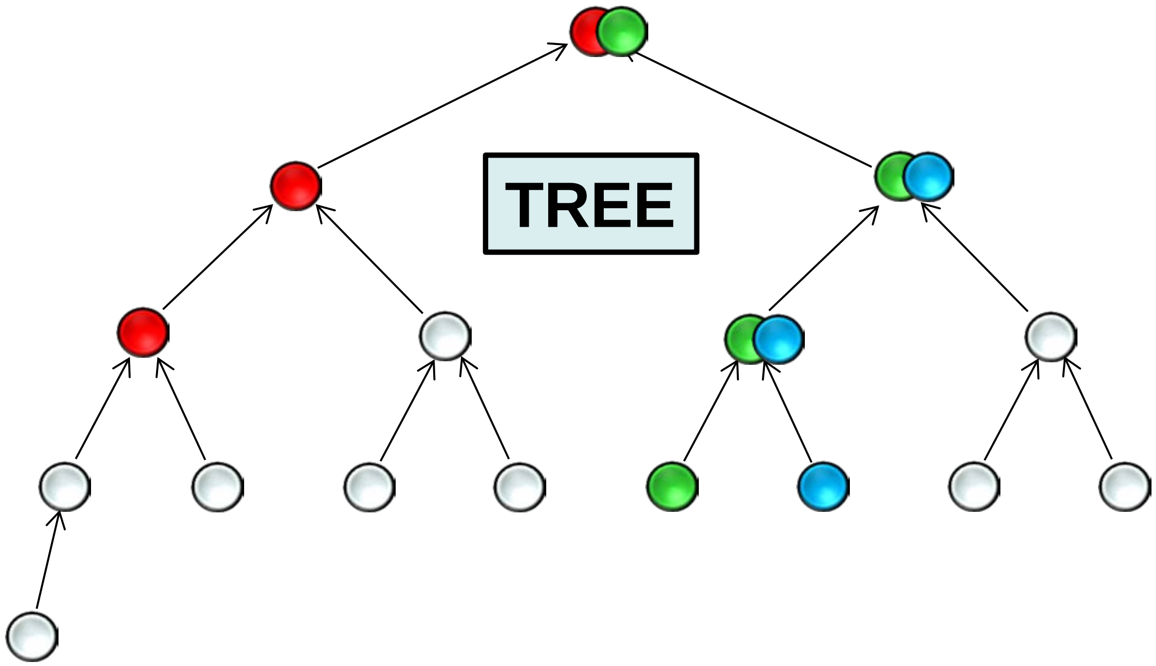
k:= # information



Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

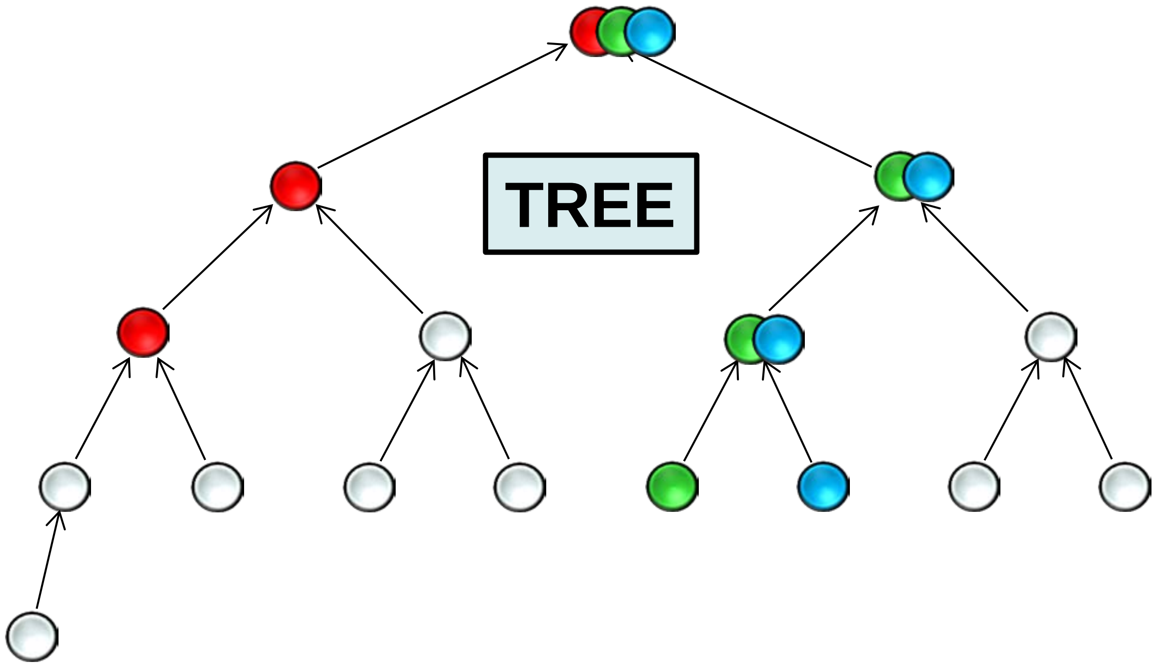
k:= # information



Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

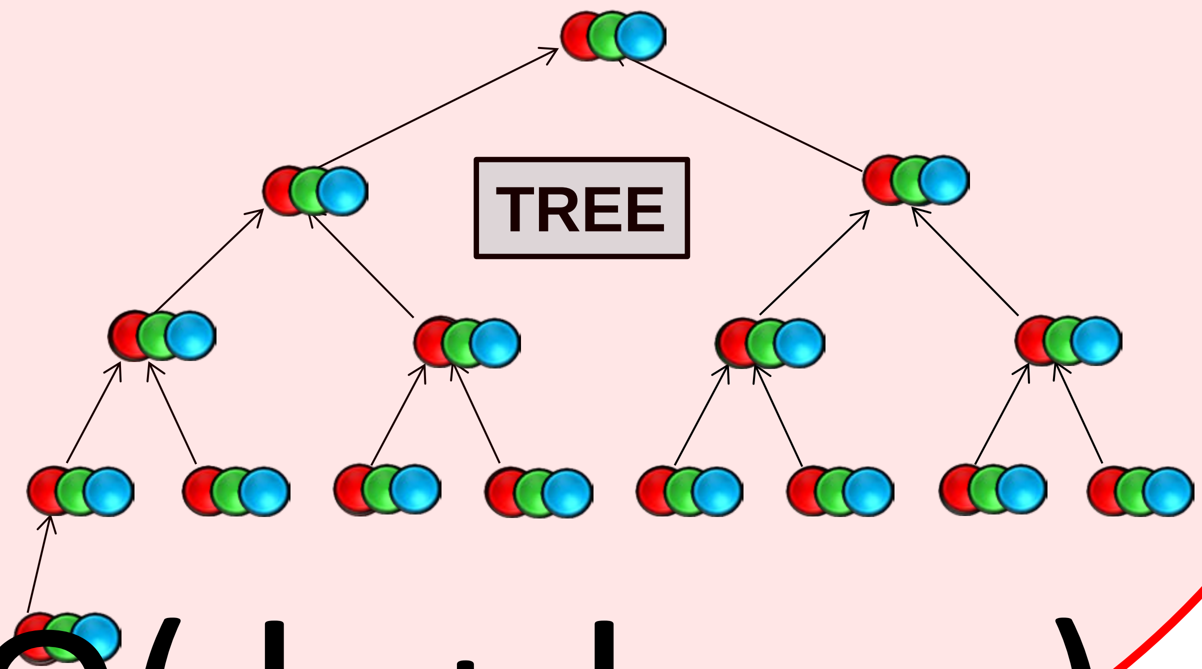
k:= # information



Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information



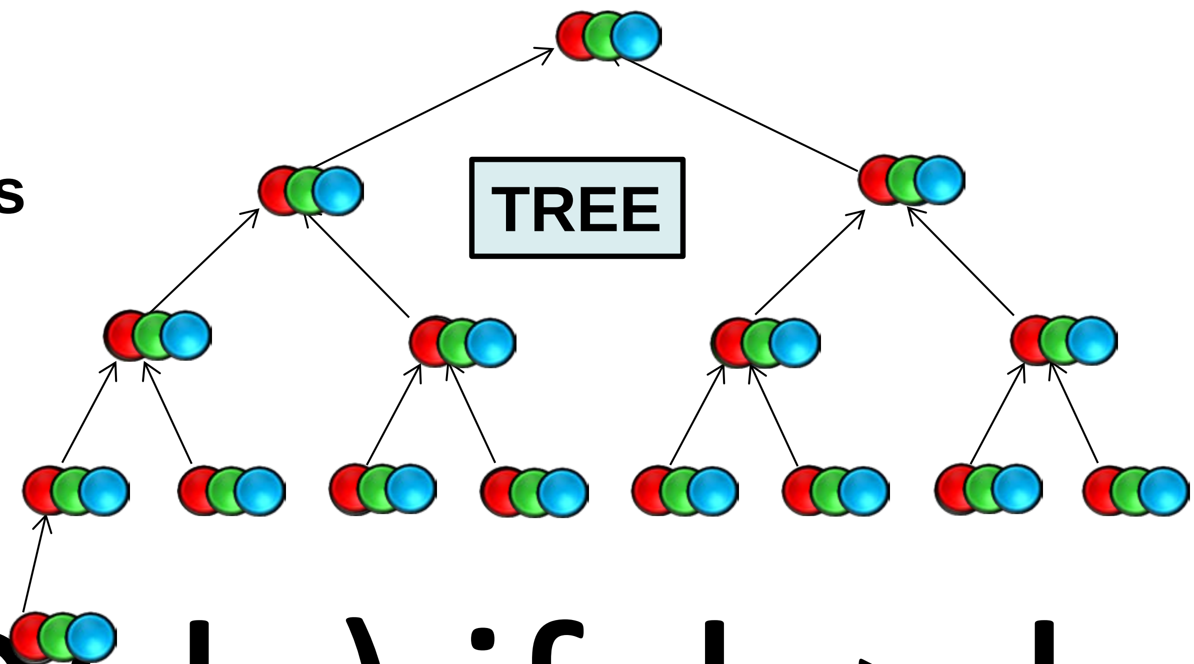
$O(k + \log n)$

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information

What if k < log n?

n channels

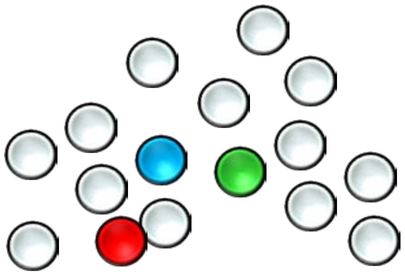


O(k) if k > log n

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information

What if k < log n?

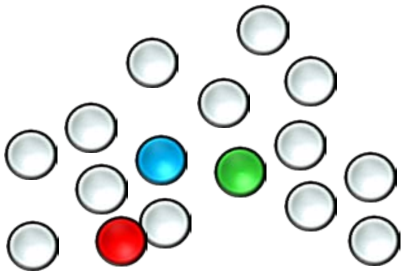


Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

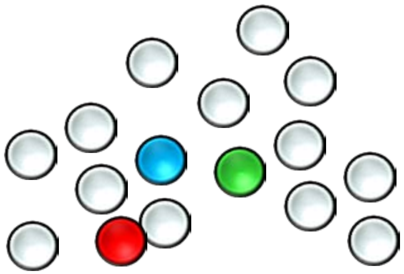


Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known



Balls into Bins

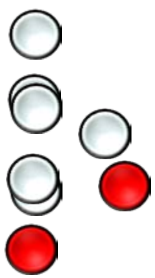
Time-Optimal Information Exchange on Multiple Channels

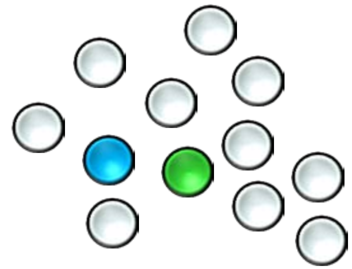
n:= # nodes

k:= # information

Assume: k known

ID=1…2^k





Balls into Bins

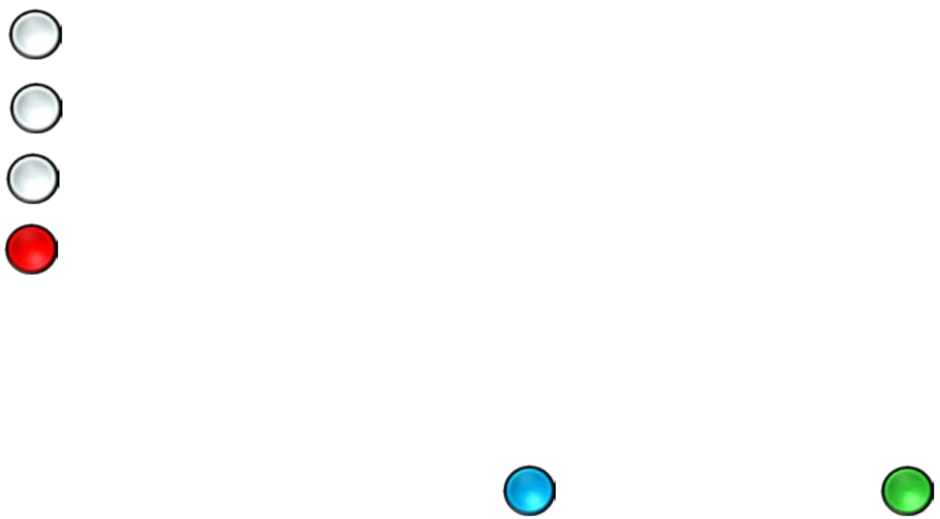
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k



Balls into Bins

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k

- **Listens on channel 1**
- **Listens on channel 2**
- **Listens on channel 3**
- **Listens on channel 4**

Balls into Bins

Send on random channel 1…2^k

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k

Listens on channel 1

Listens on channel 2

Listens on channel 3

Listens on channel 4

Balls into Bins

Send on random channel 1…2^k

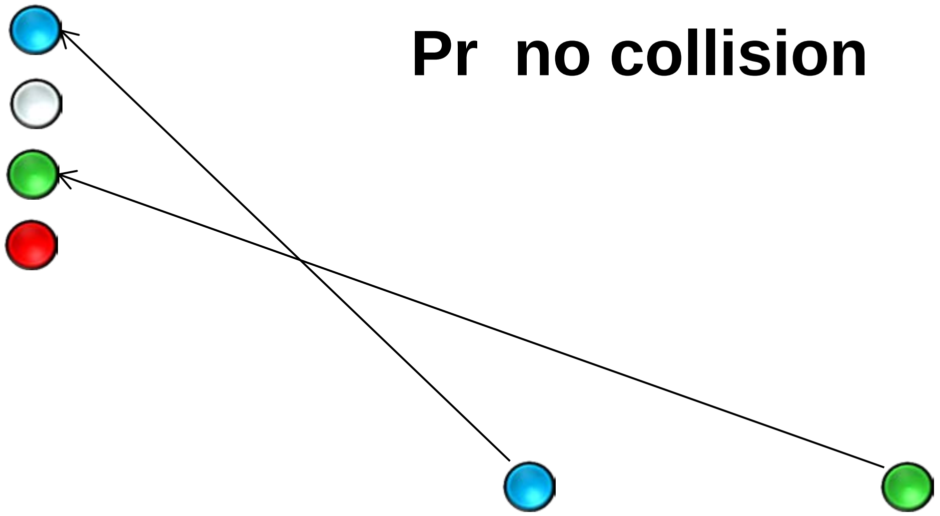
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k



Pr no collision

Balls into Bins

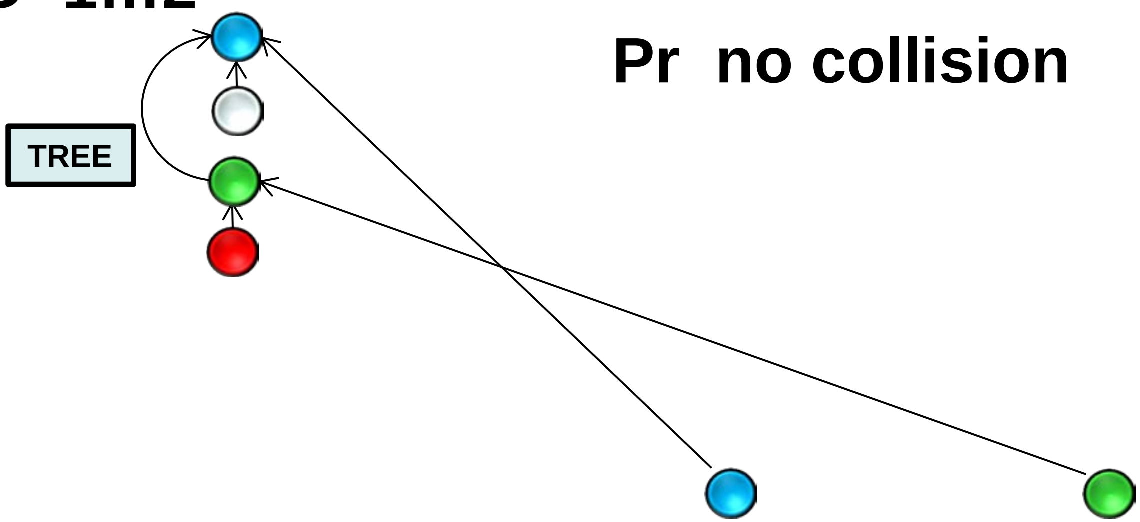
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k



Pr no collision

Balls into Bins

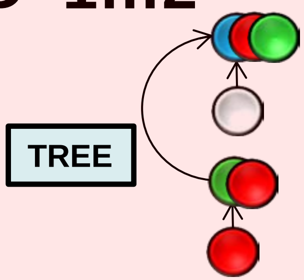
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k



Pr no collision

Balls into Bins

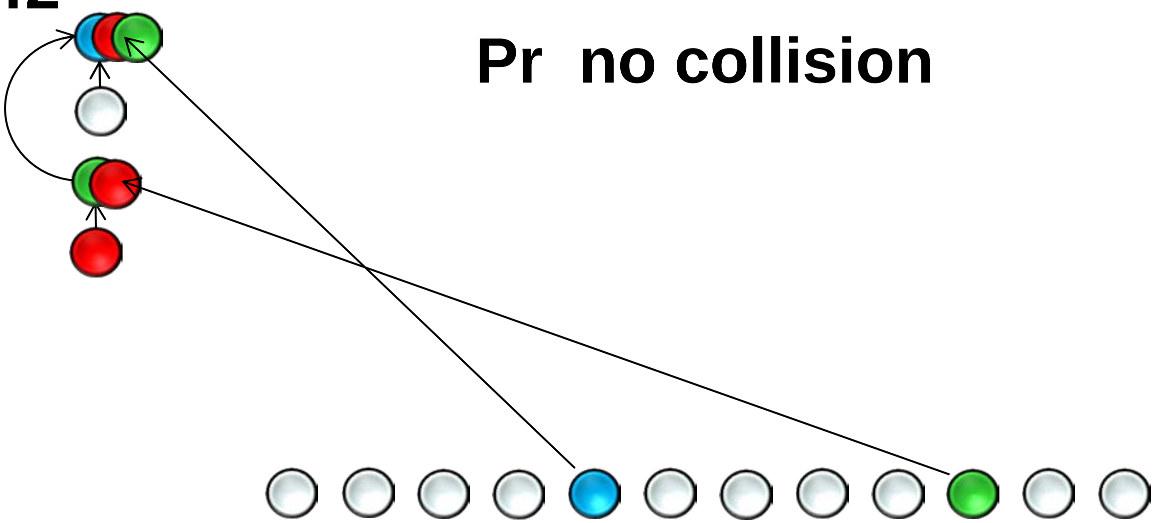
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

ID=1…2^k



Repeat k times

Pr no collision

Balls into Bins

Time-Optimal Information Exchange on Multiple Channels

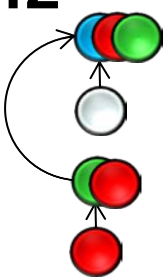
n:= # nodes

k:= # information

Assume: k known

What if $k < \sqrt{\log n}$?

ID=1…2^k



Repeat k times

Pr [no collision] > 1- $\frac{1}{n^{-c}}$

If $k > \sqrt{\log n}$

2^k channels

Balls into Bins

TREE

O(k) if $k > \sqrt{\log n}$

Time-Optimal Information Exchange on Multiple Channels

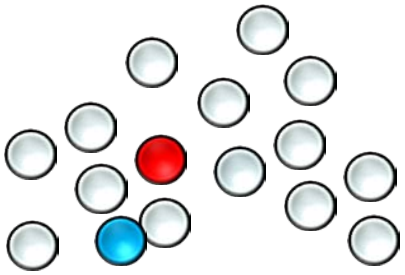
n:= # nodes

k:= # information

Assume: k known

What if $\boldsymbol{k} < \sqrt{\boldsymbol{log\ n}}$?

$Size: \sqrt{n \log n}$



Time: O(k)

Unique Subset

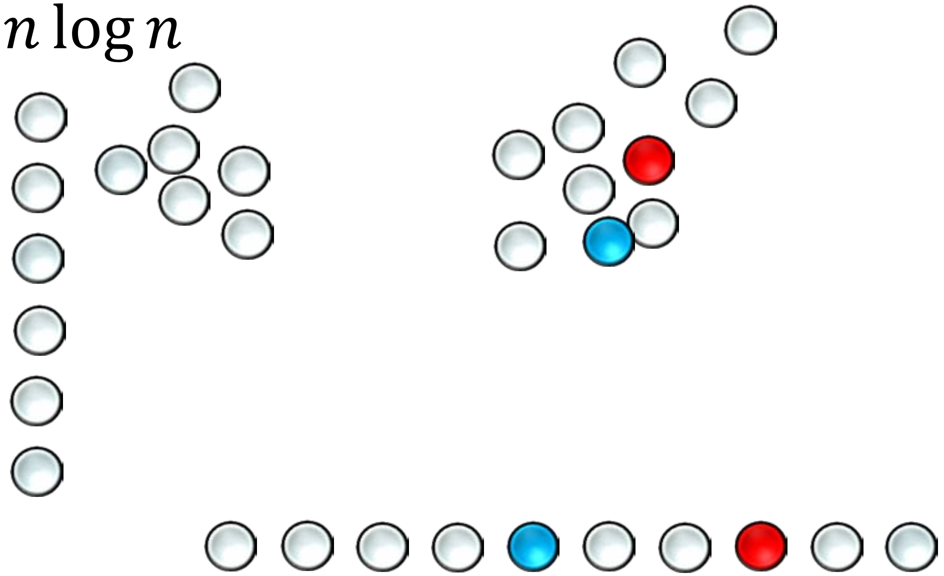
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

$Size\colon \sqrt{n \log n}$



Unique Subset

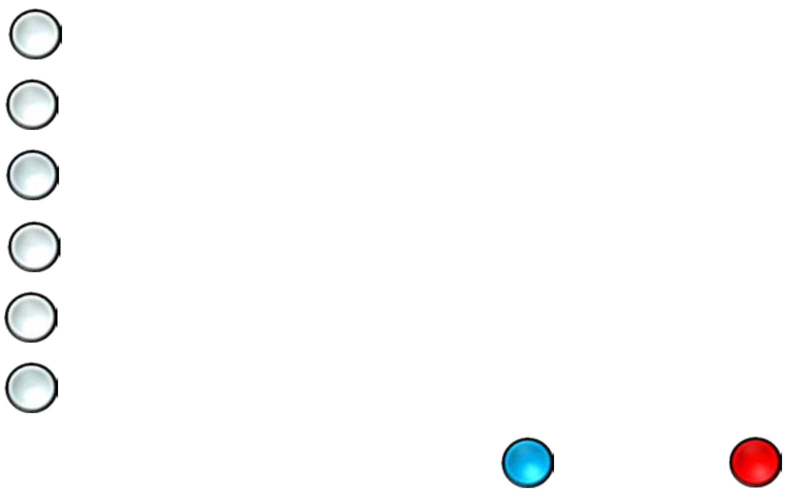
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

$$Size: \sqrt{n \log n}$$



Unique Subset

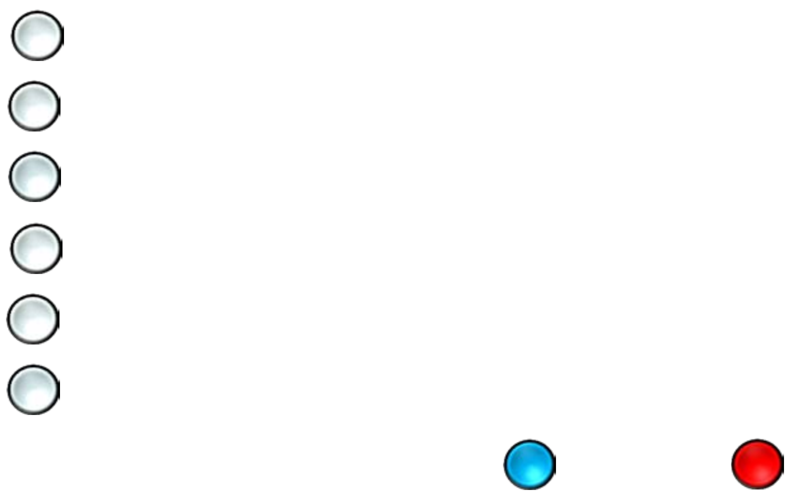
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

$Size: \sqrt{n \log n}$



Unique Subset

Send on random channel $\in \{1, \ldots, 2^{\frac{\log n}{2k}}\}$.

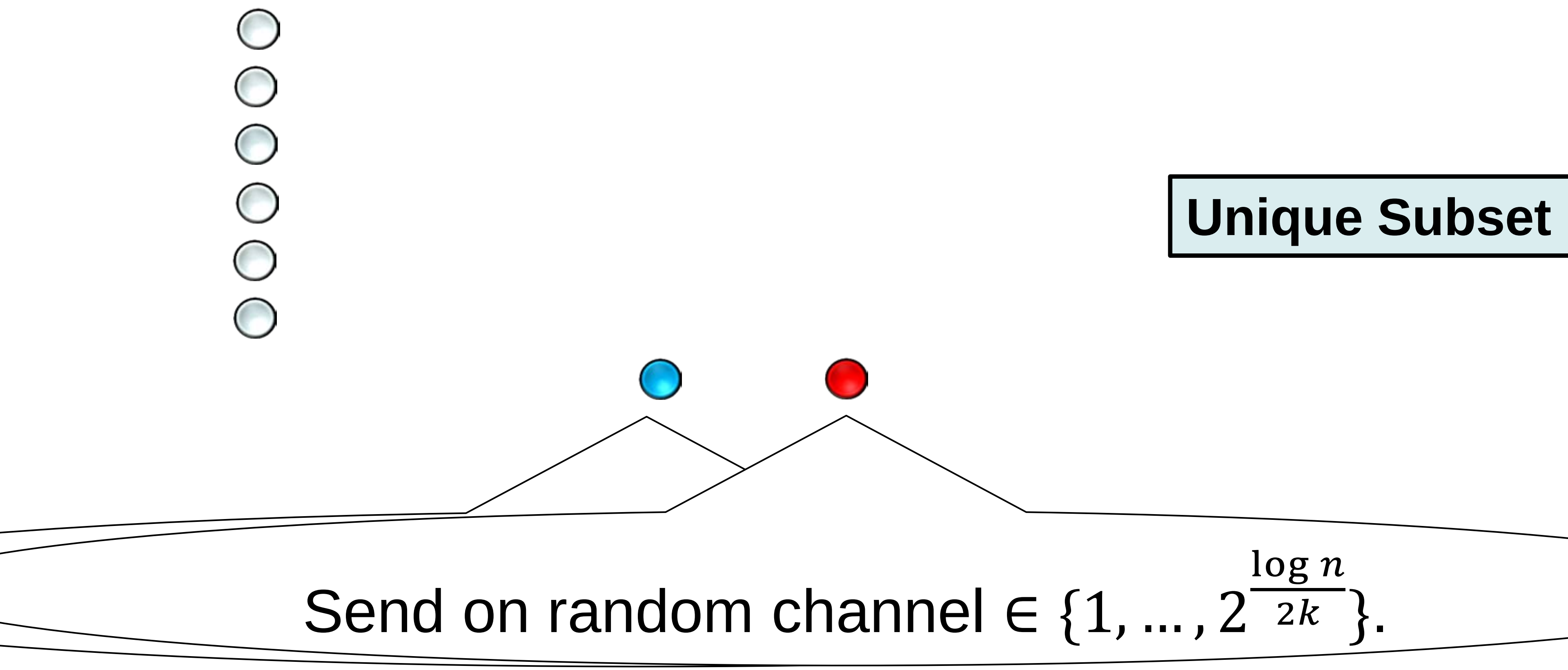
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

$Size: \sqrt{n \log n}$



Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

$Size: \sqrt{n \log n}$

$$map: \{1, \ldots, \sqrt{n \log n}\} \longrightarrow subsets\ of\ \left\{1, \ldots, 2^{\frac{\log n}{2k}}\right\}$$

$of\ size\ at\ most\ k$

Pr[at most half messages collide]

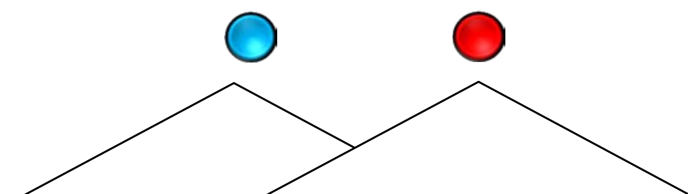
$> 1 - \frac{1}{n^{-c}}$

Unique Subset

Not too big …

Not too small …

Just right!



Send on random channel $\in \left\{1, \ldots, 2^{\frac{\log n}{2k}}\right\}$.

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

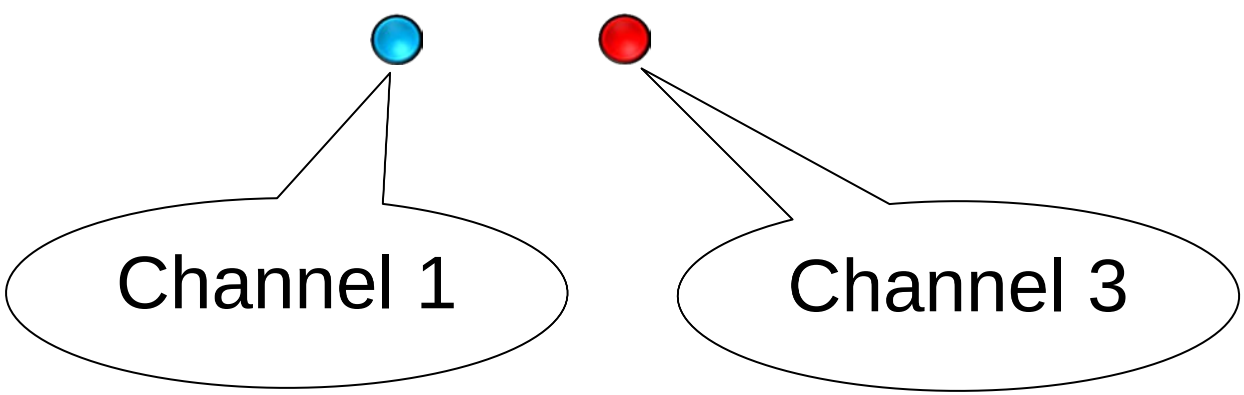
k:= # information

Assume: k known

Example: 3 channels

- {1}
- {2}
- {3}
- {1,2}
- {1,3}
- {2,3}

Unique Subset



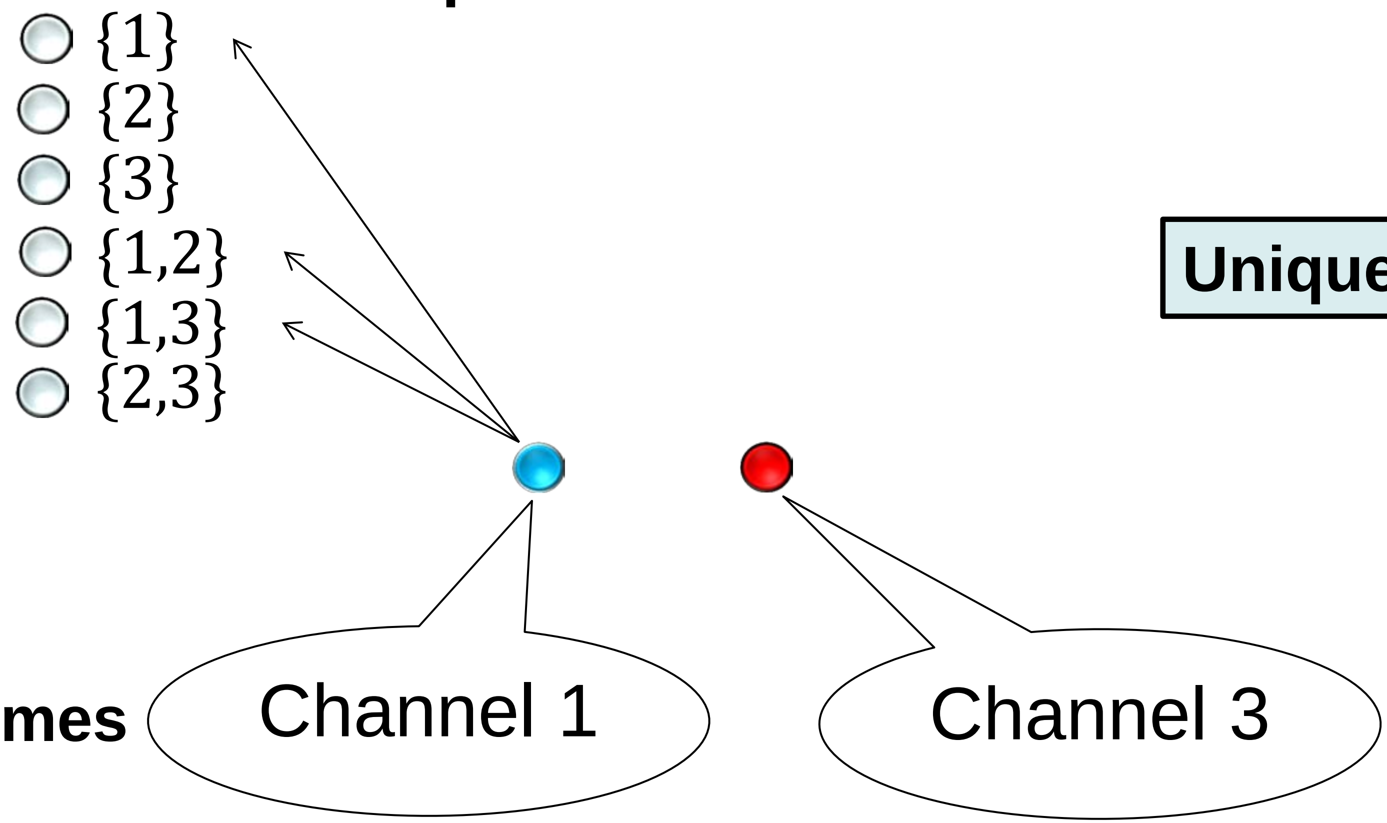
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

Example: 3 channels



Unique Subset

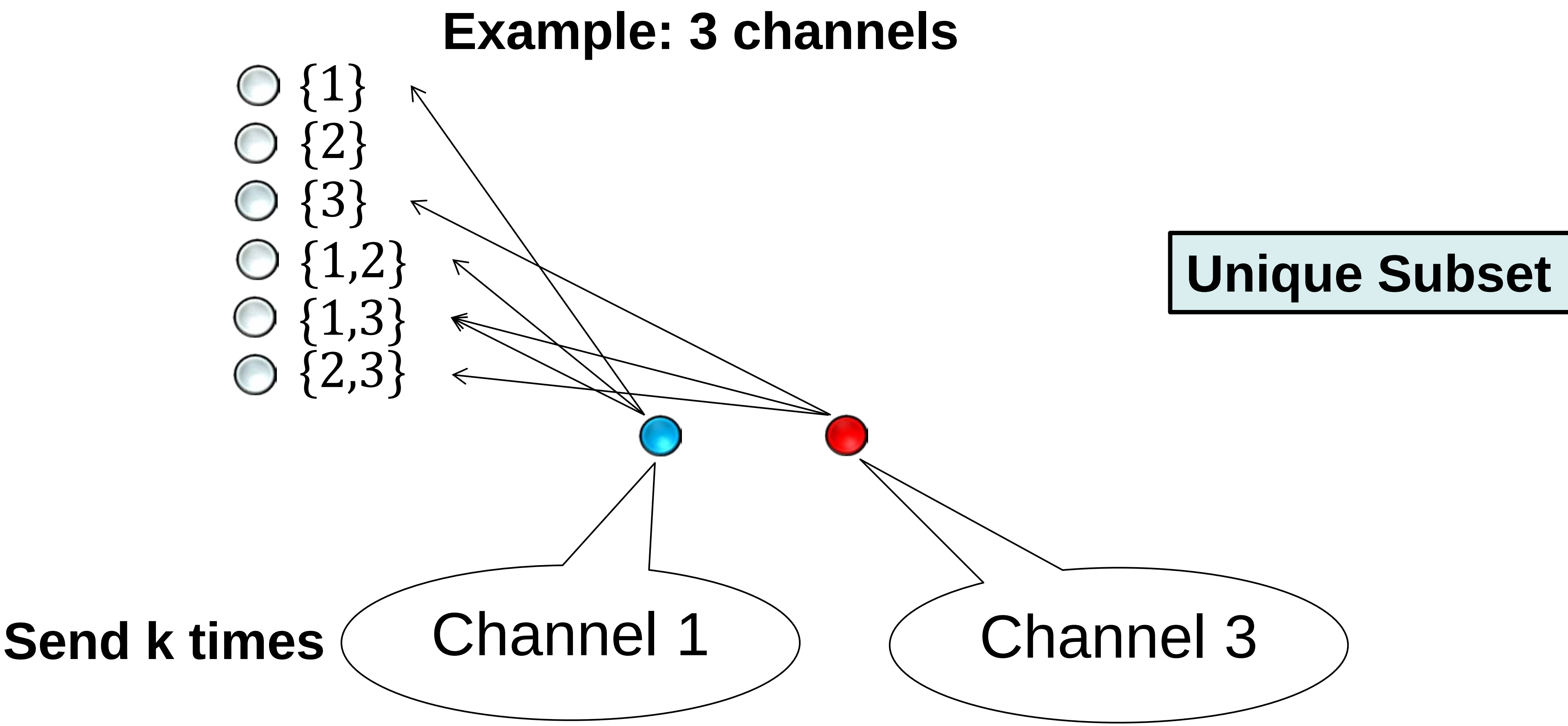
Send k times

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known



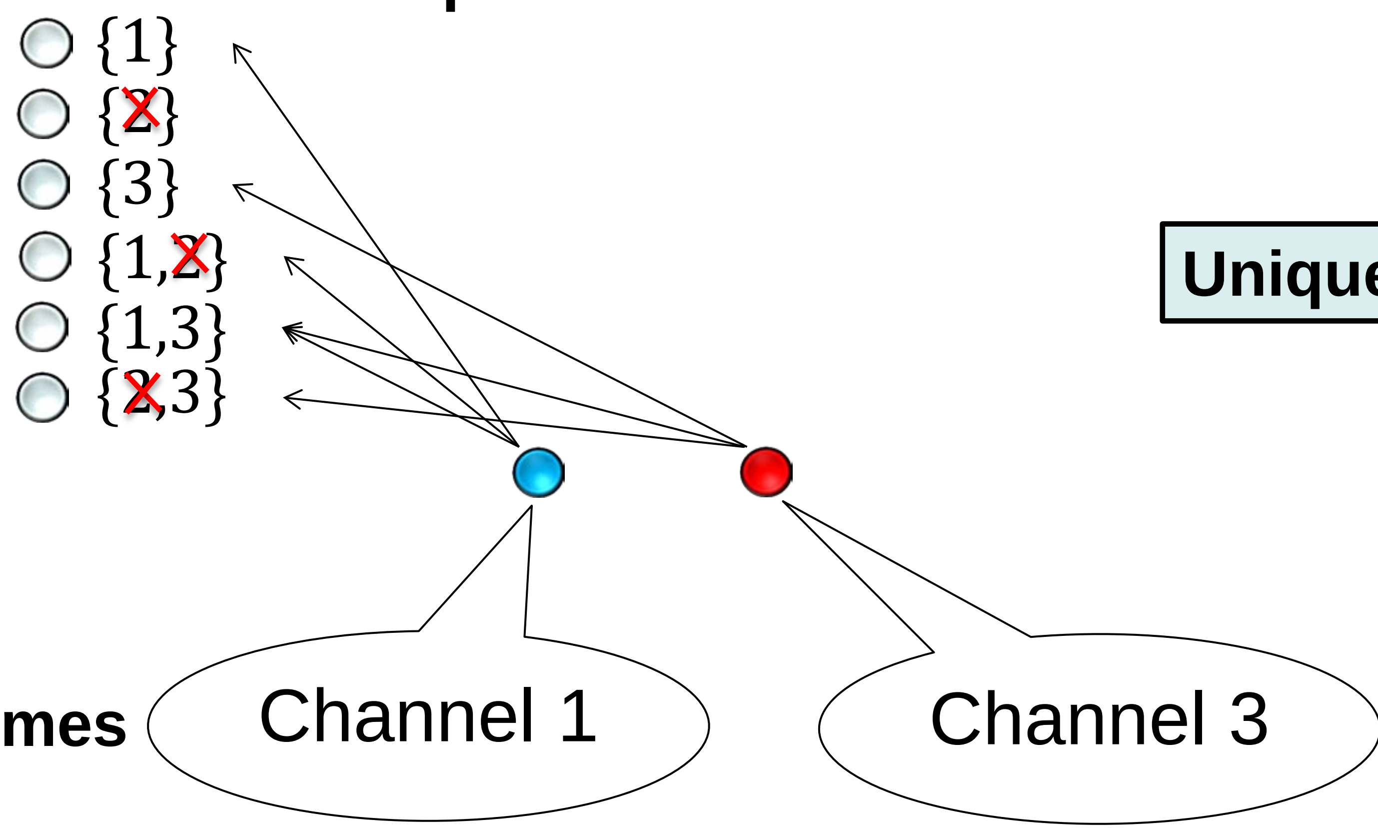
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

Example: 3 channels



Unique Subset

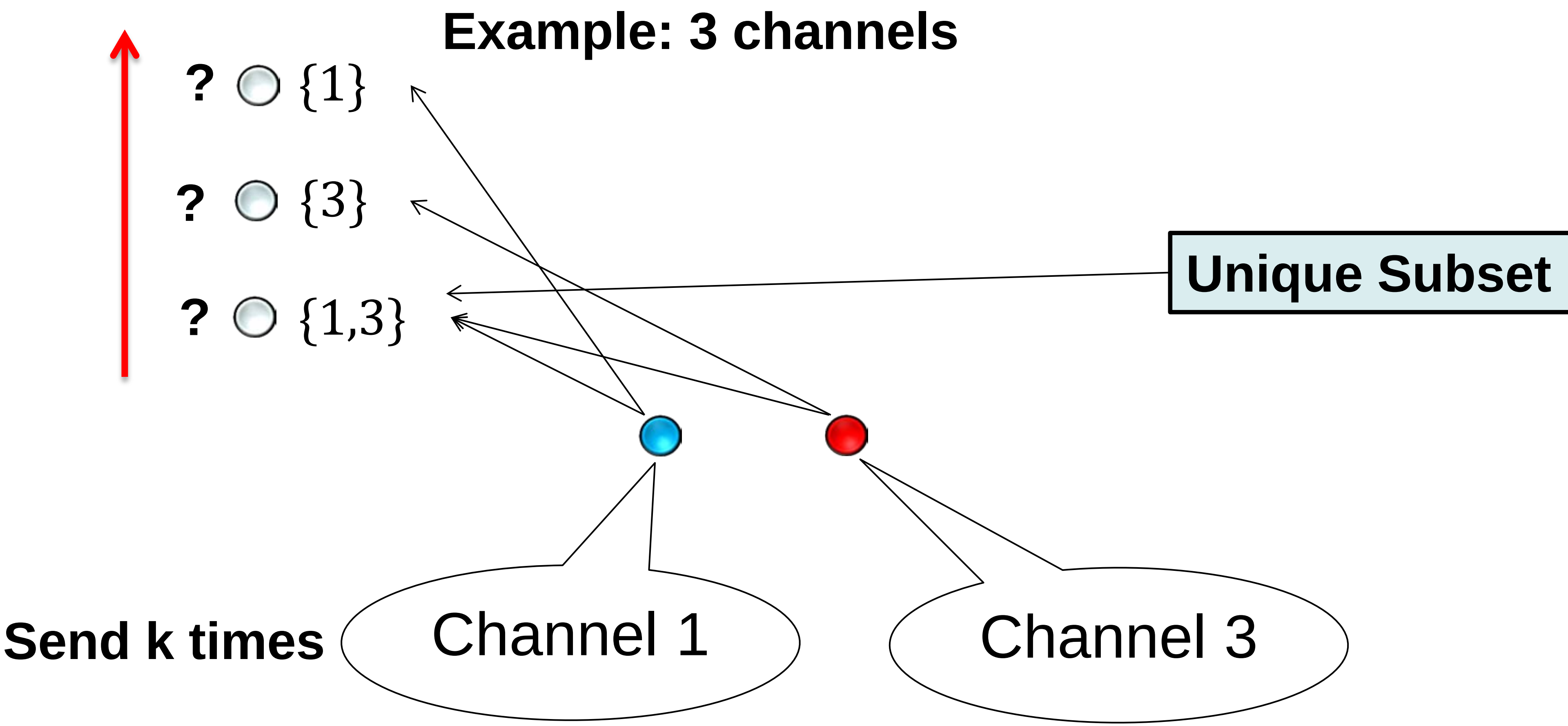
Send k times

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known



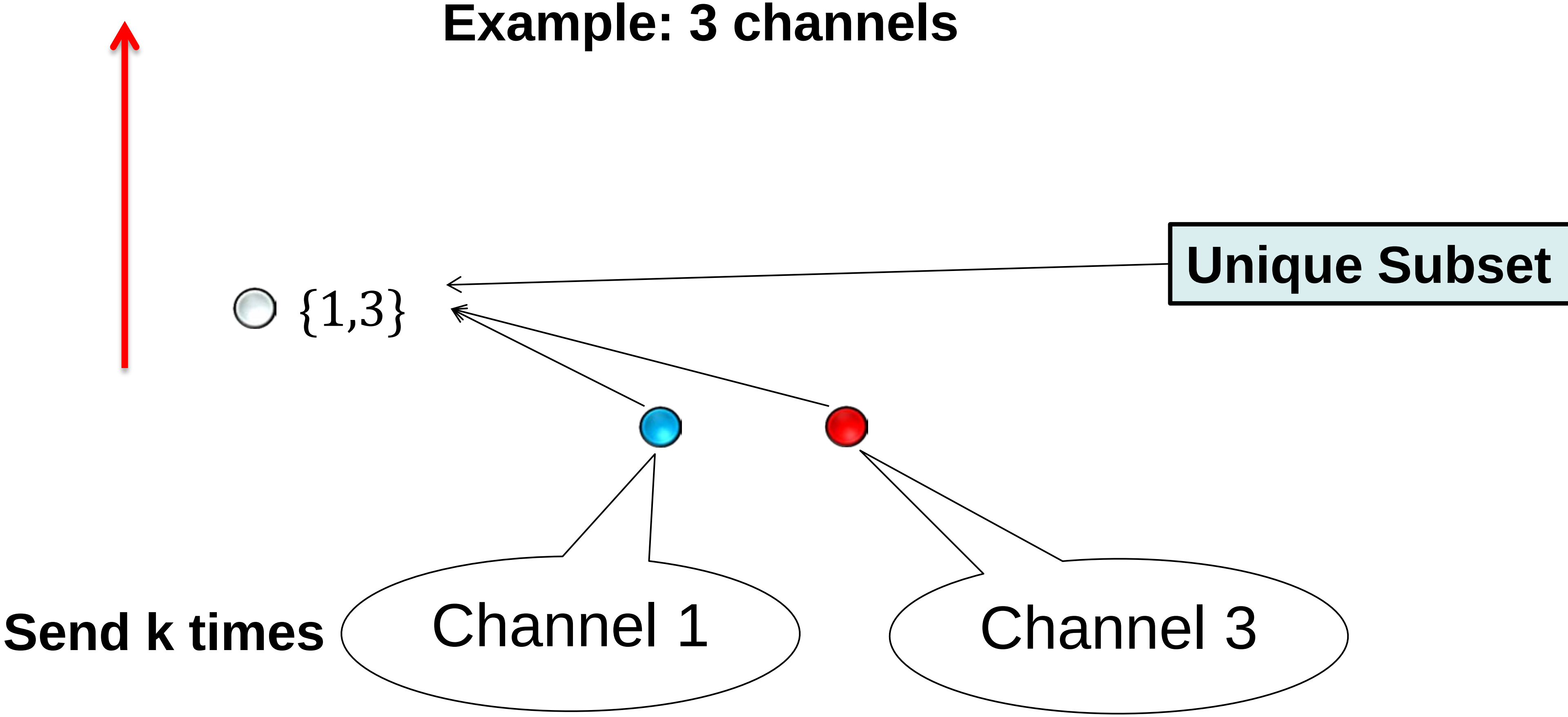
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

Example: 3 channels



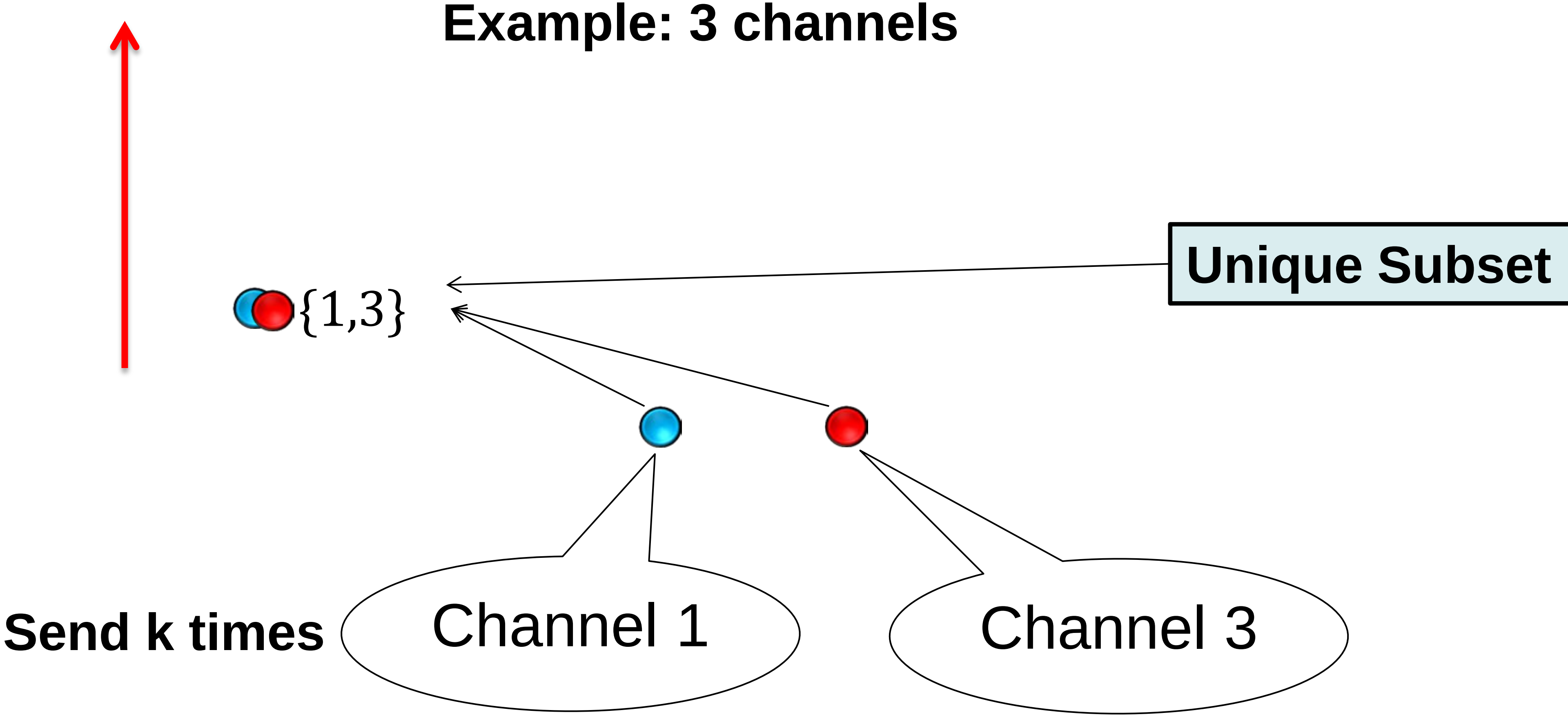
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

Example: 3 channels



Time-Optimal Information Exchange on Multiple Channels

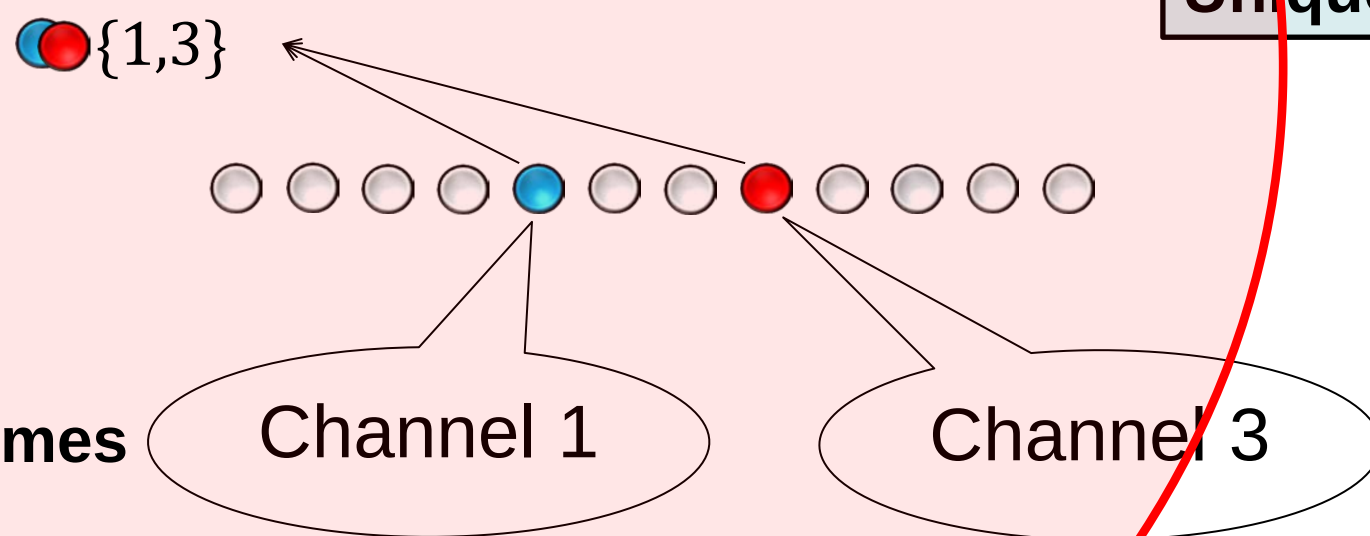
n:= # nodes

k:= # information

Assume: k known

Example: 3 channels

Unique Subset



Send k times

Time-Optimal Information Exchange on Multiple Channels

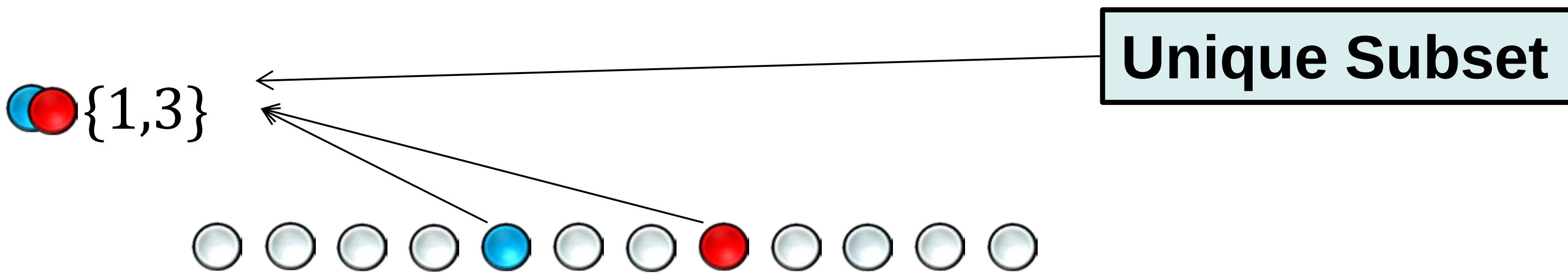
n:= # nodes

k:= # information

Assume: k known

Example: 3 channels

Pr[at most half messages collide]> 1- $\frac{1}{n^{-c}}$



O(k)

Time-Optimal Information Exchange on Multiple Channels

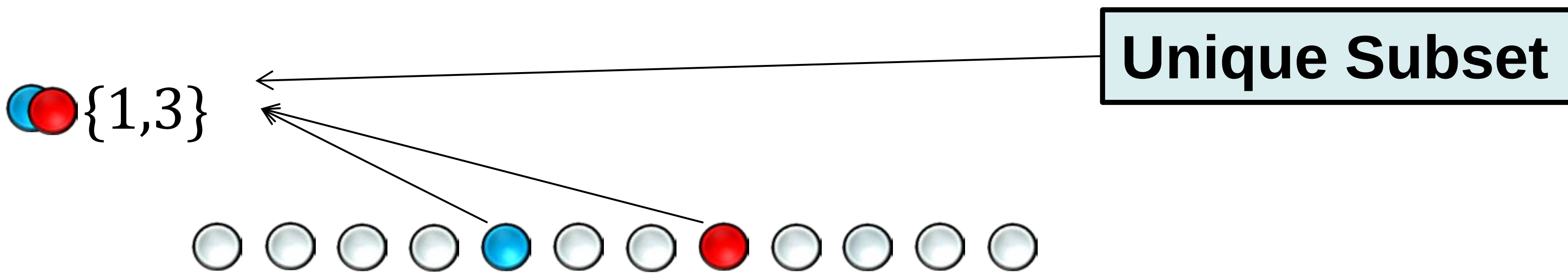
n:= # nodes

k:= # information

Assume: k known

Example: 3 channels

Pr[at most half messages collide]> 1- $\frac{1}{n^{-c}}$



O(k + k/2 + k/4 …

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

Example: 3 channels

Pr[at most half messages collide]> 1- $\frac{1}{n^{-c}}$

$2^{\frac{\log n}{2k}} = \sqrt{\boldsymbol{n}}$ **channels**

Unique Subset

O(k)

Pr[this works] > 1- $\frac{1}{n^{-c}}$

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k known

Unique Subset	$k < \sqrt{\log n}$
Balls into Bins	$\sqrt{\log n} \leq k < \log n$
TREE	$\log n \leq k$

Time-Optimal Information Exchange on Multiple Channels

n:= # nodes

k:= # information

Assume: k ~~known~~ unknown

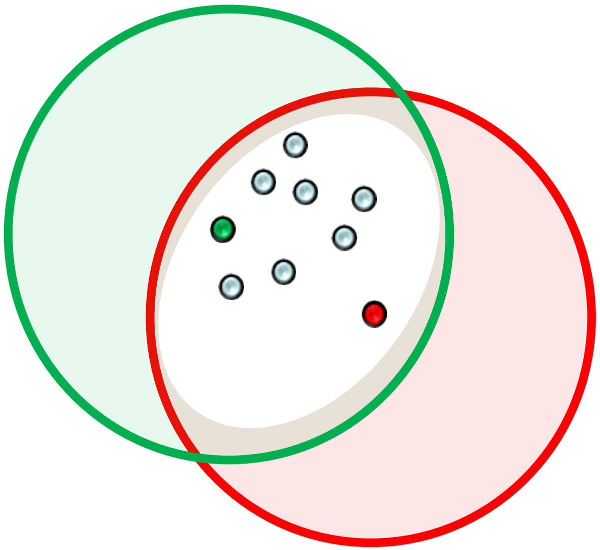
$\sqrt{n}$ channels	Unique Subset	$k < \sqrt{\log n}$
2^k channels	Balls into Bins	$\sqrt{\log n} \leq k < \log n$
n channels	TREE	$\log n \leq k$
n channels		

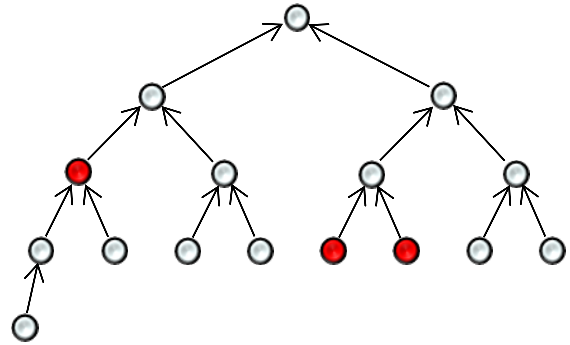
Future directions: deterministic
less channels
lower bounds

Time-Optimal Information Exchange on Multiple Channels

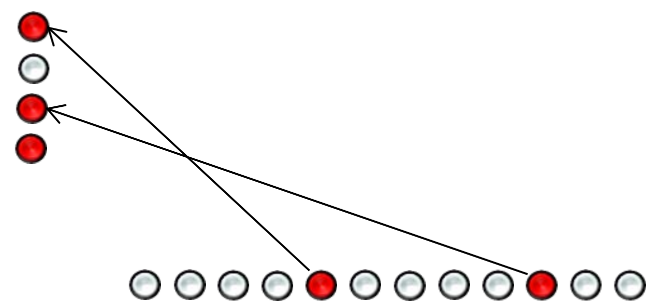
in Summary …

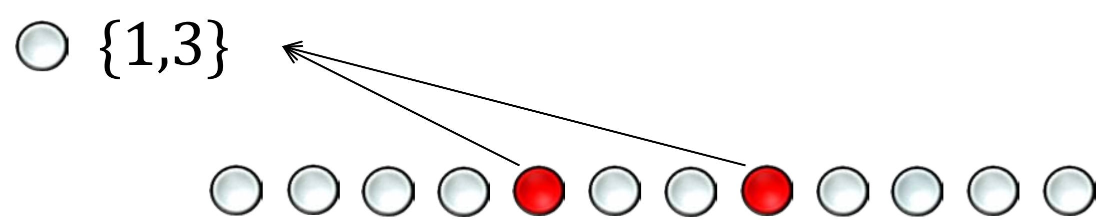
Detect / Disseminate Information!





101 Mhz
117 Mhz
132 Mhz
…





$$\Theta(k)$$

Thank You!

Questions & Comments?

Stephan Holzer
Yvonne Anne Pignolet
Jasmin Smula
Roger Wattenhofer