Asymptotically Optimal Mobile Ad-Hoc Routing

Fabian Kuhn

Roger Wattenhofer

Aaron Zollinger

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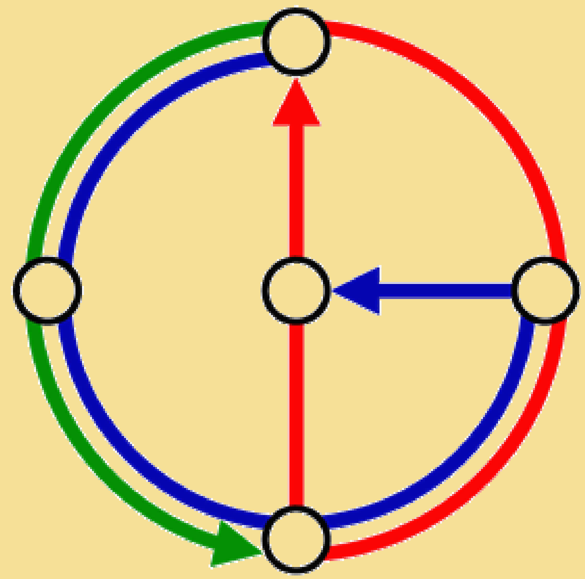
Swiss Federal Institute of Technology Zurich



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Overview

- Introduction
- Model
- Face Routing
- Adaptive Face Routing
- Lower Bound
- Conclusion

Model I (Ad-Hoc Network)

- Nodes are points in $\mathbb{R}^2$
- All nodes have the same transmission range (normalized to 1)
  - $\Rightarrow$ network is a unit disk graph
- Distance between any two nodes is lowerbounded by a constant d_0
  - $\Rightarrow$ $\Omega(1)$-model

Model II (Geometric Routing)

- Nodes know the geometric positions of themselves and of their neighbors
- Source s knows the coordinates of destination t
- Nodes not allowed to store anything
- In the message, only $O(\log n)$ additional bits can be stored

Cost Model I

- Cost of sending a message over a link (edge) e is $c(e)$
- Cost of a path p is the sum over the costs of its edges
- Cost of a routing algorithm $\mathcal{A}$ is the sum over the costs of the traversed edges

Cost Model II

3 different cost metrics:

- Link distance metric ($c_\ell(e) \equiv 1$)
- Euclidean distance ($c_d(e)$)
- Energy metric ($c_E(e) := c_d^2(e)$)

  more general: $c_E(e) := c_d^\alpha(e)$ for an $\alpha \geq 2$

Costs are Equivalent

Lemma:

In the $\Omega(1)$-model the link, Euclidean, and energy metrics of a path or an algorithm are equivalent up to a constant factor.

Face Routing

geometric routing algorithm for planar graphs

Compass Routing on Geometric Networks
[Kranakis, Singh, Urrutia; 1999]

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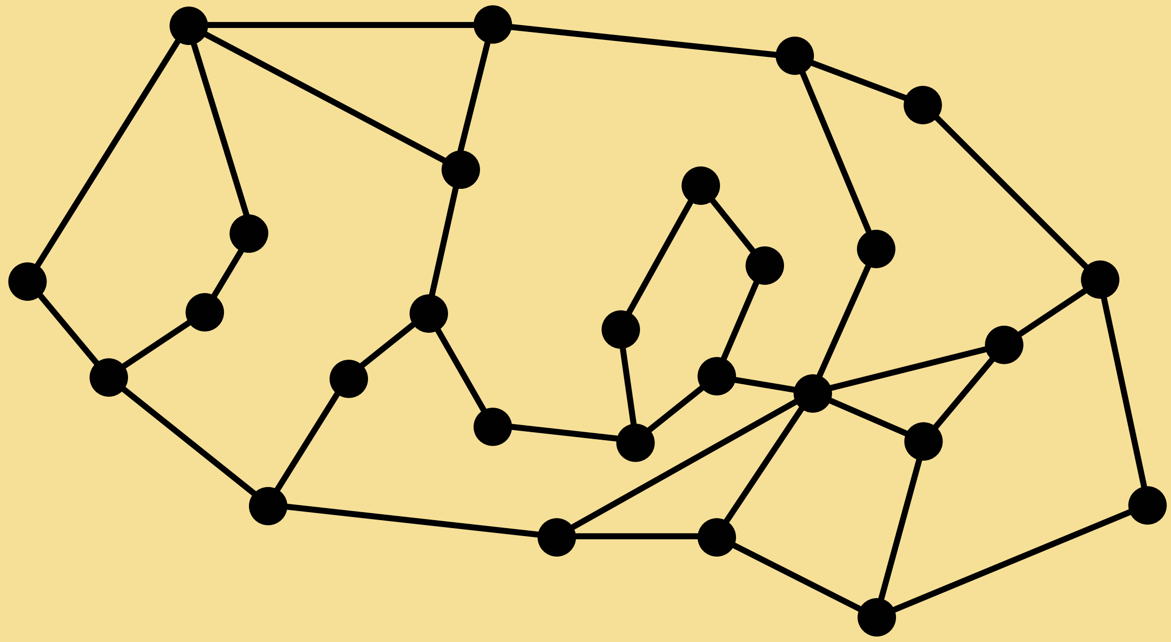

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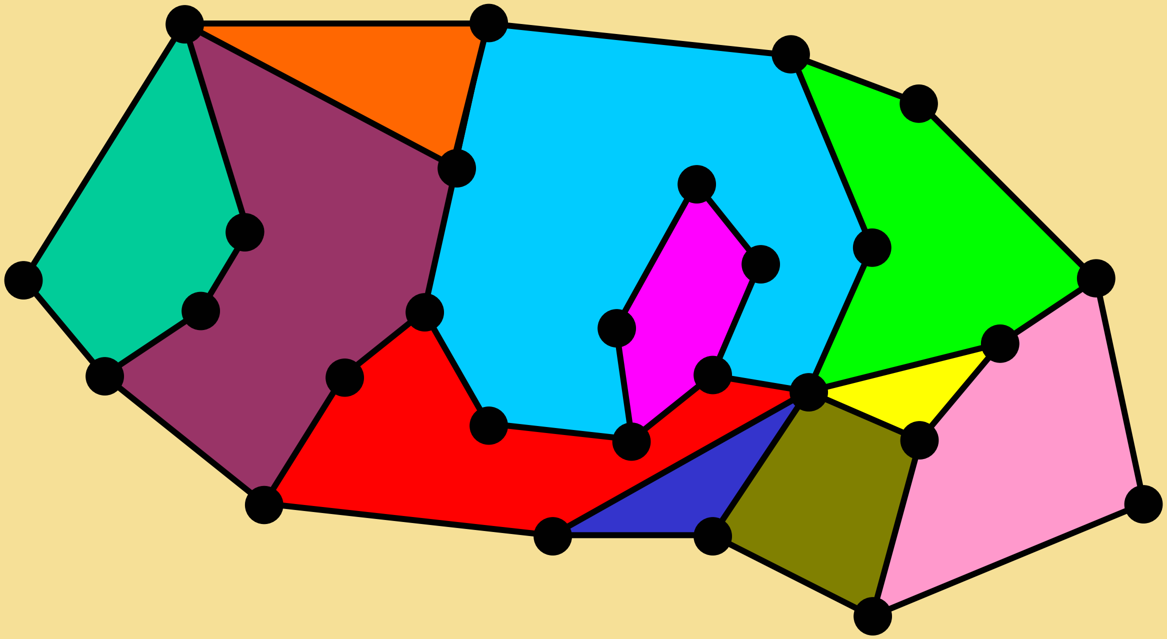
Face Routing (Faces)



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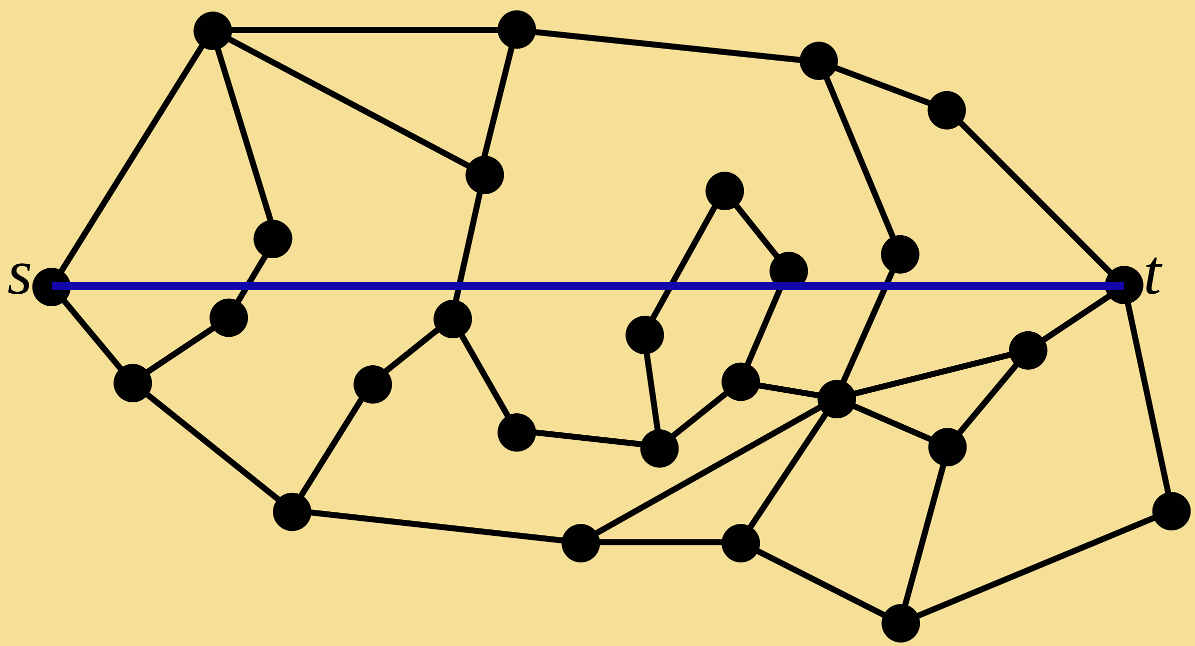

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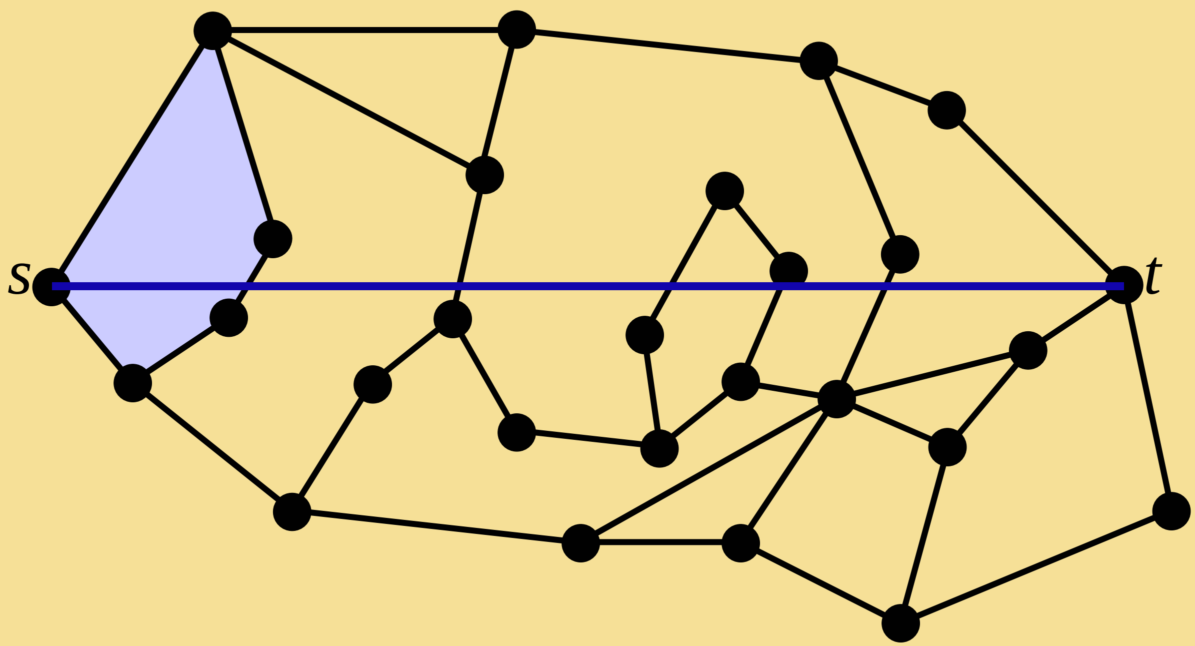

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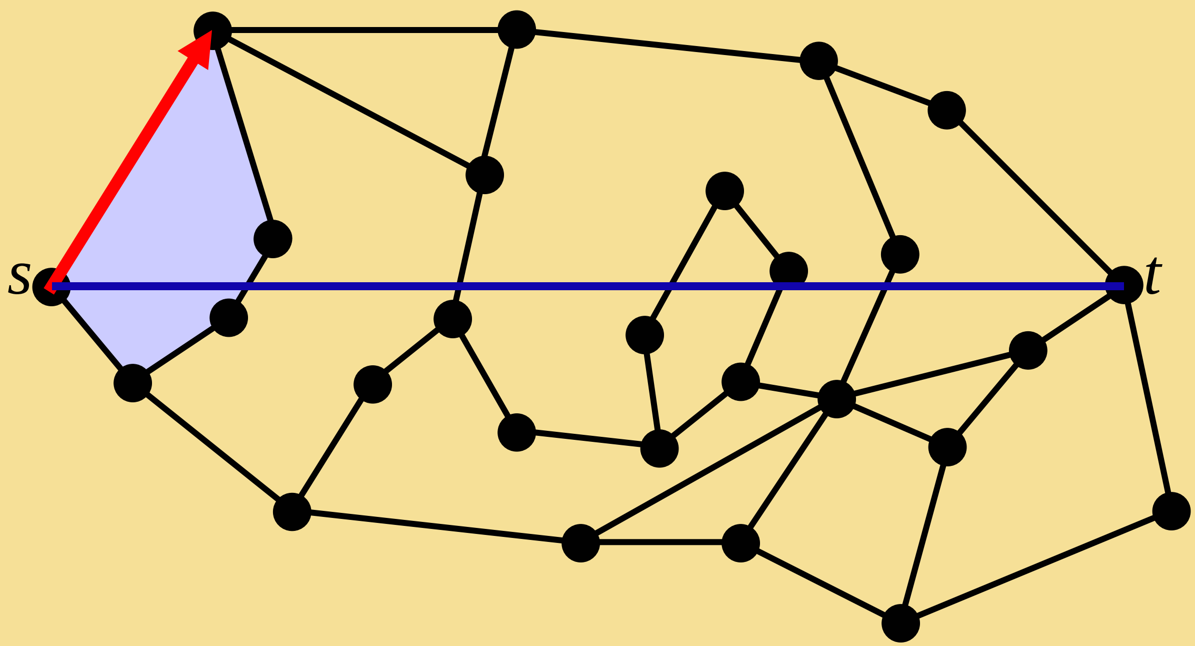

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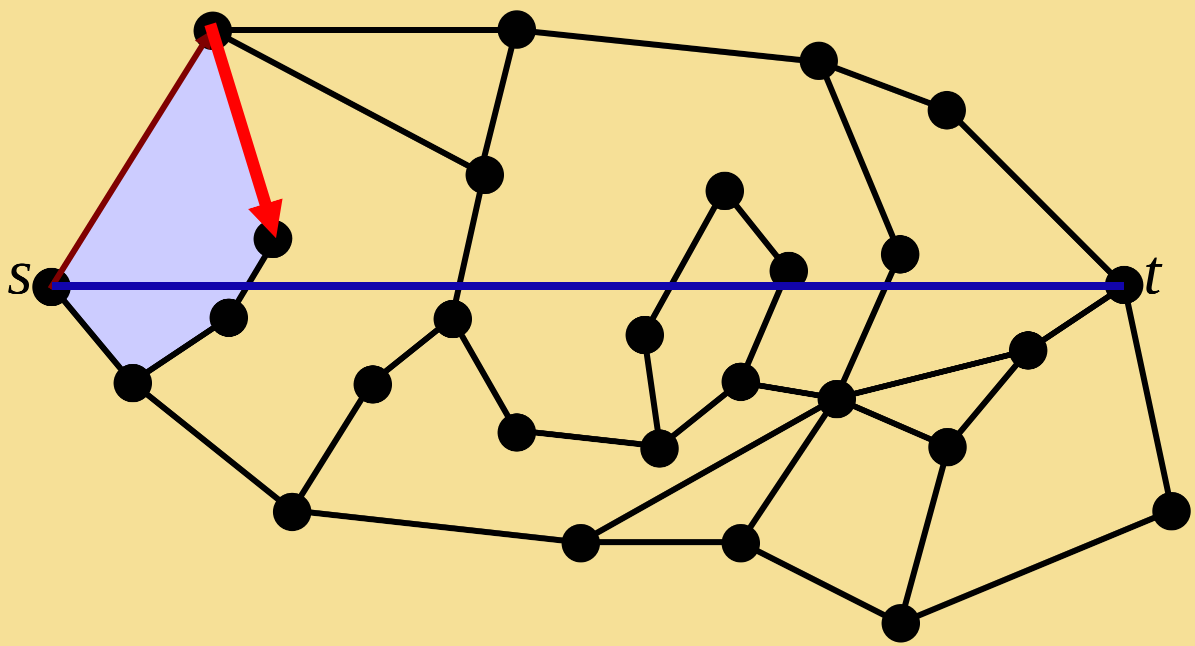

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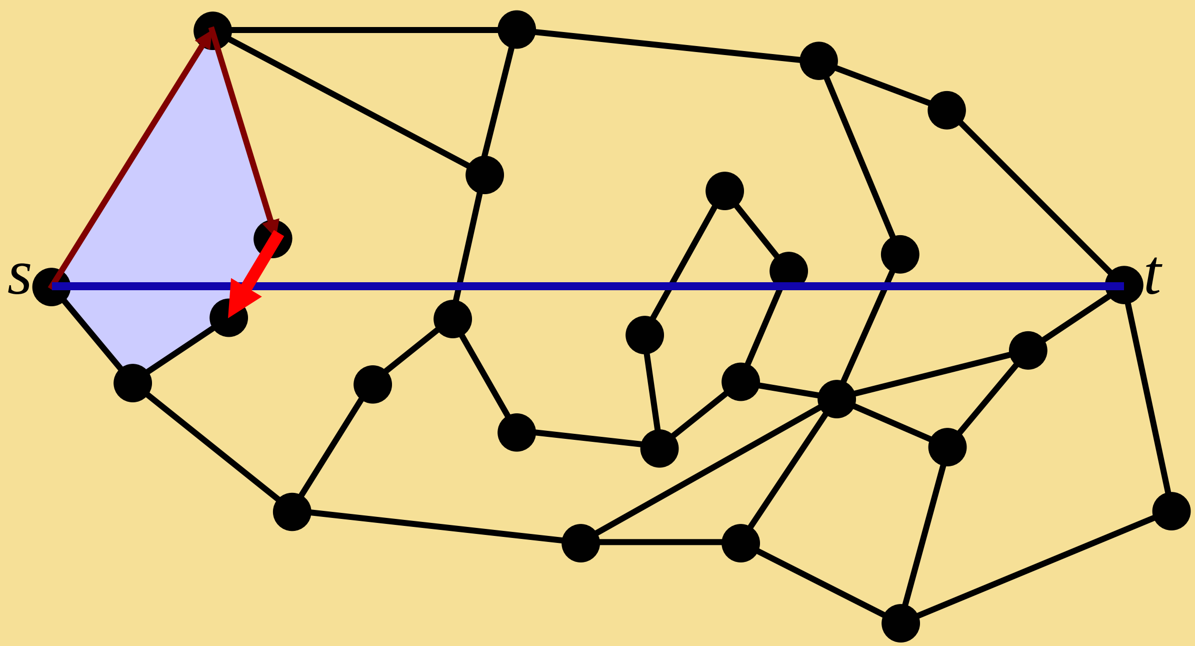

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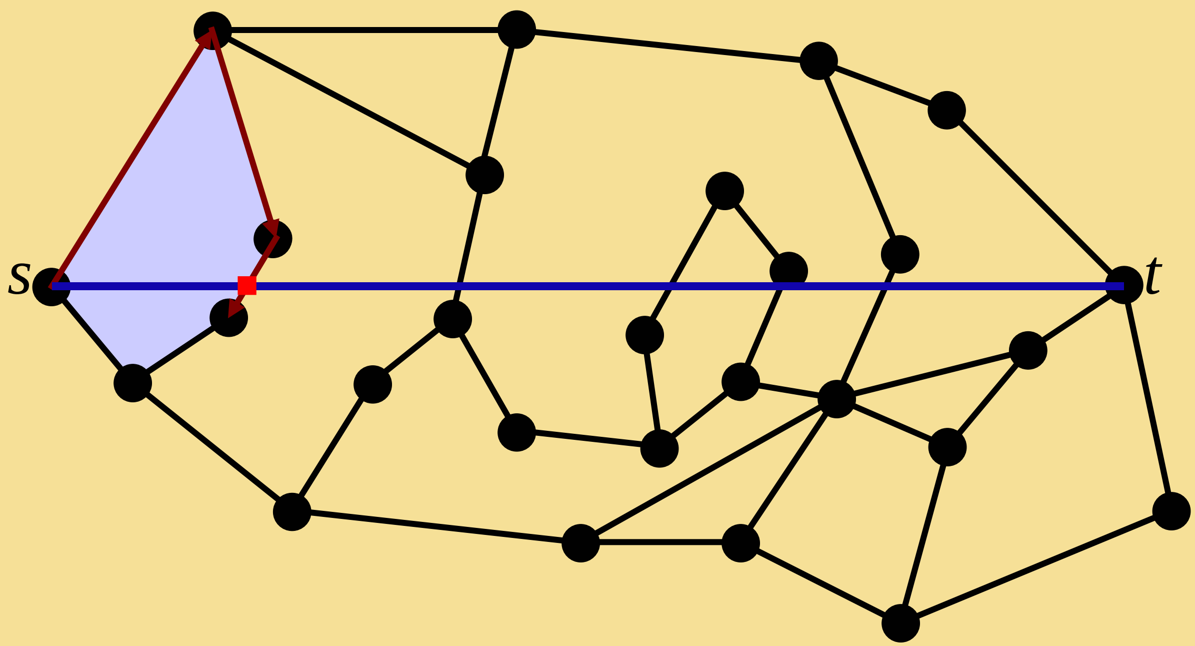

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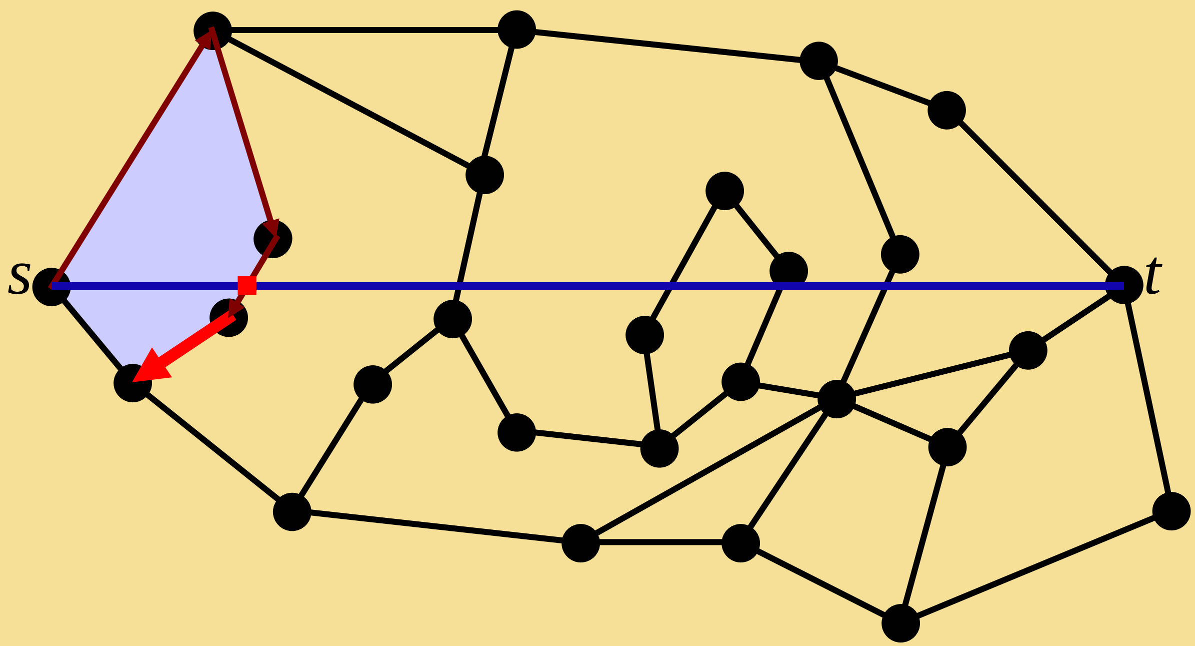

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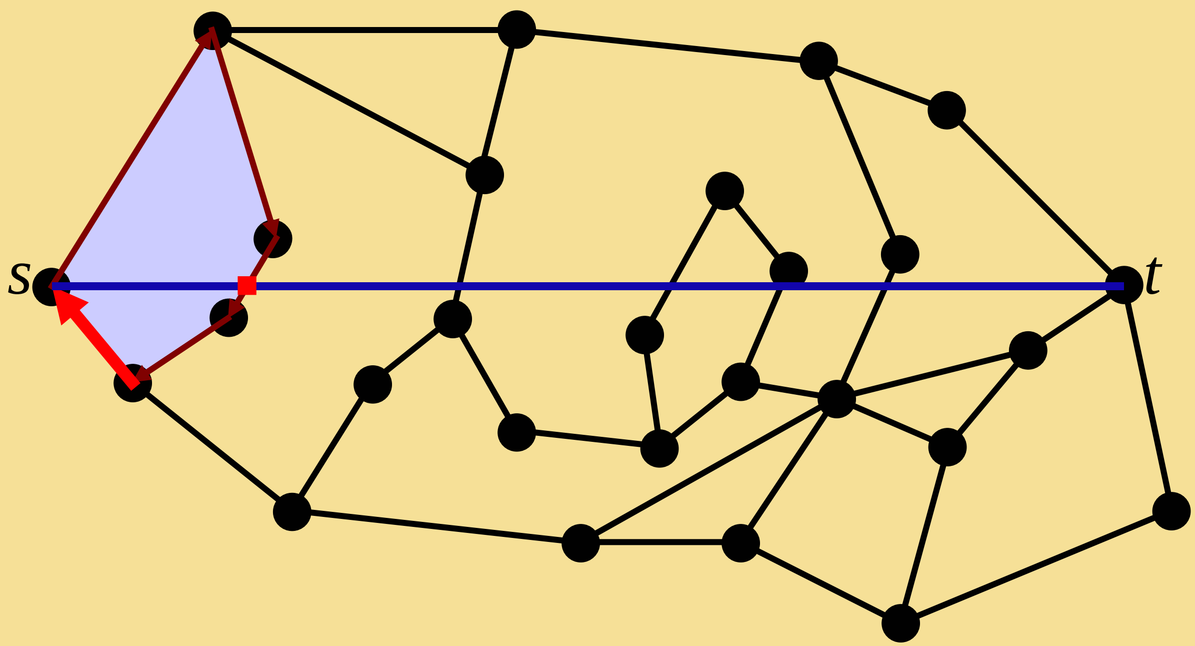

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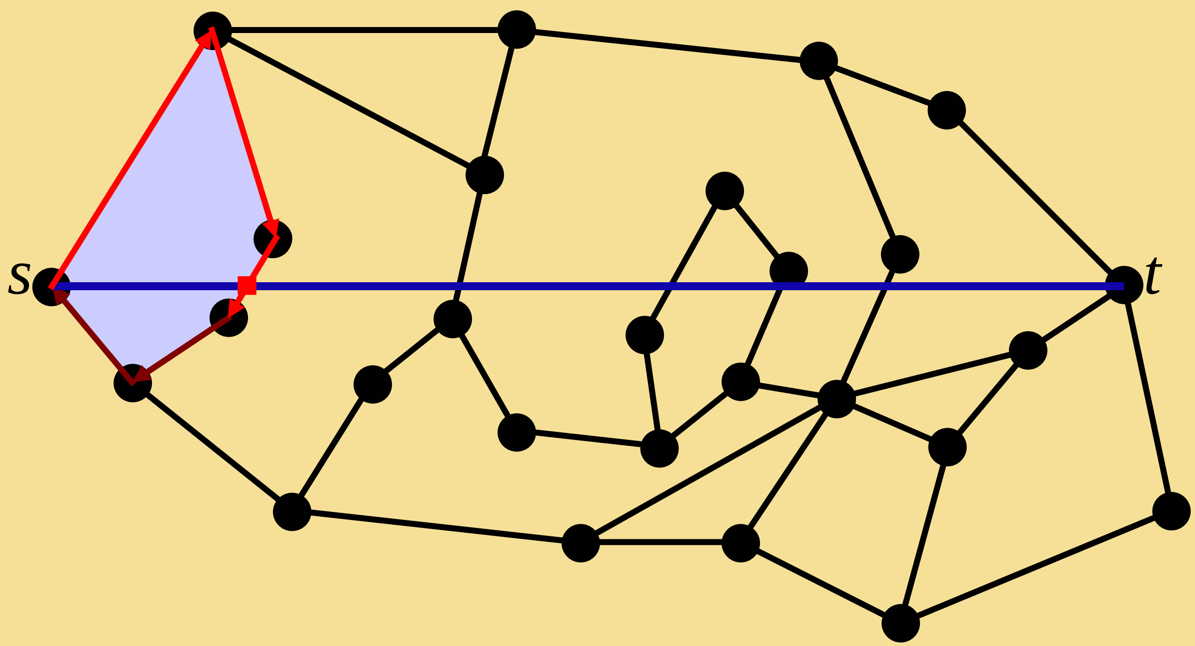

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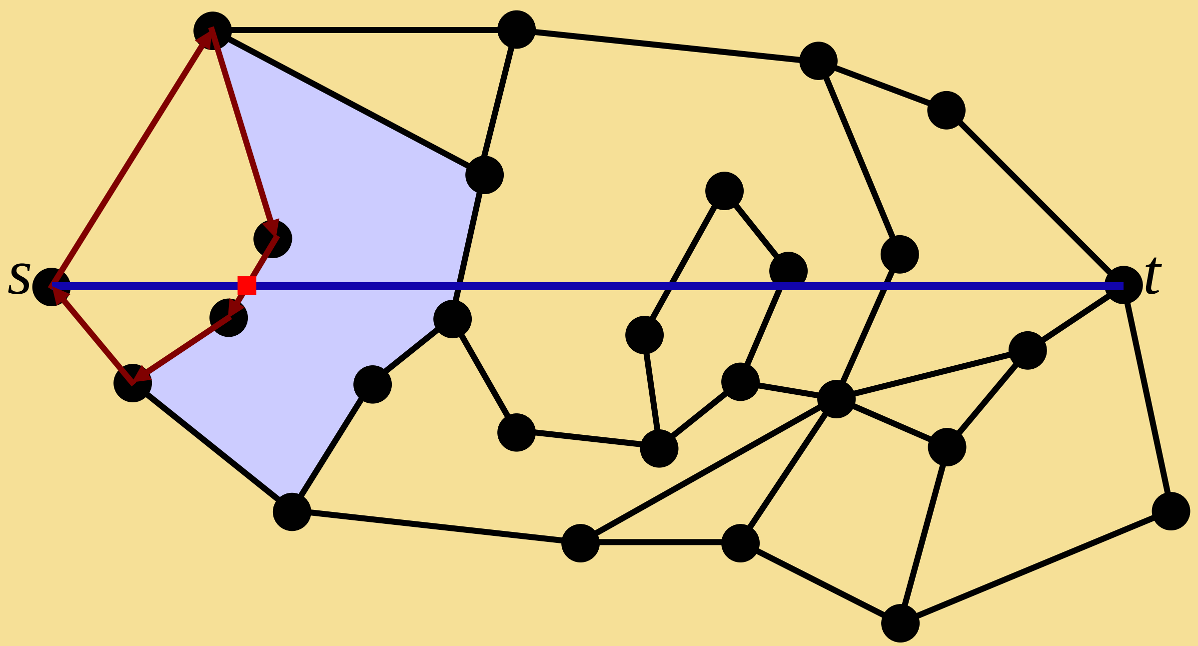

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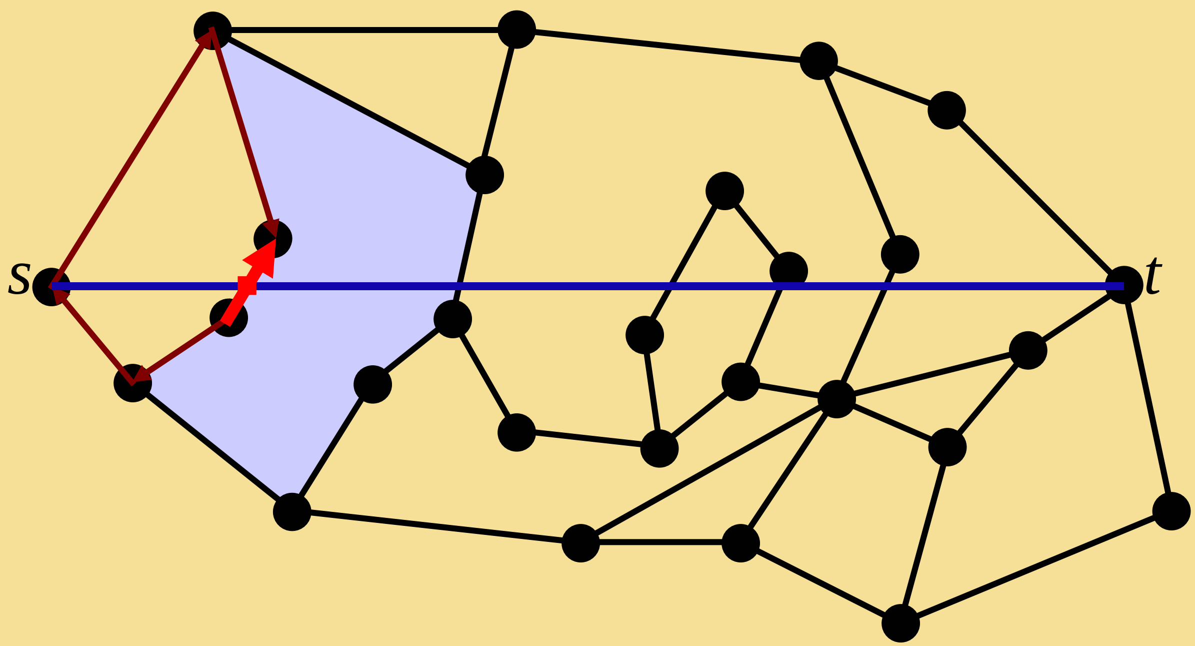

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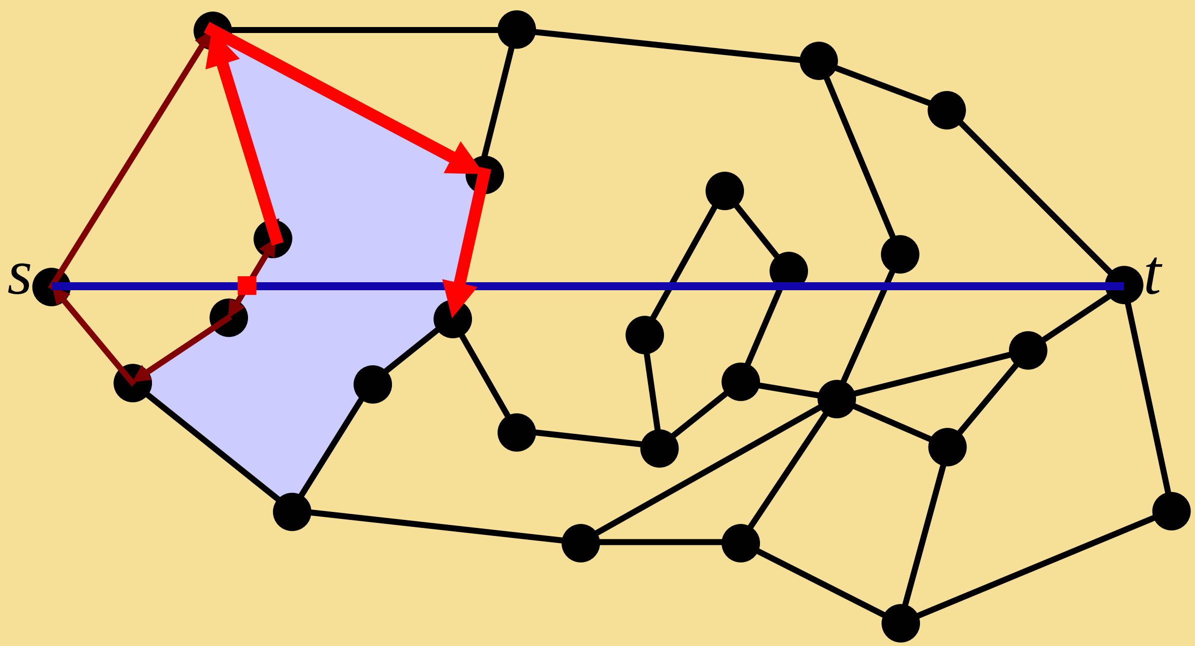

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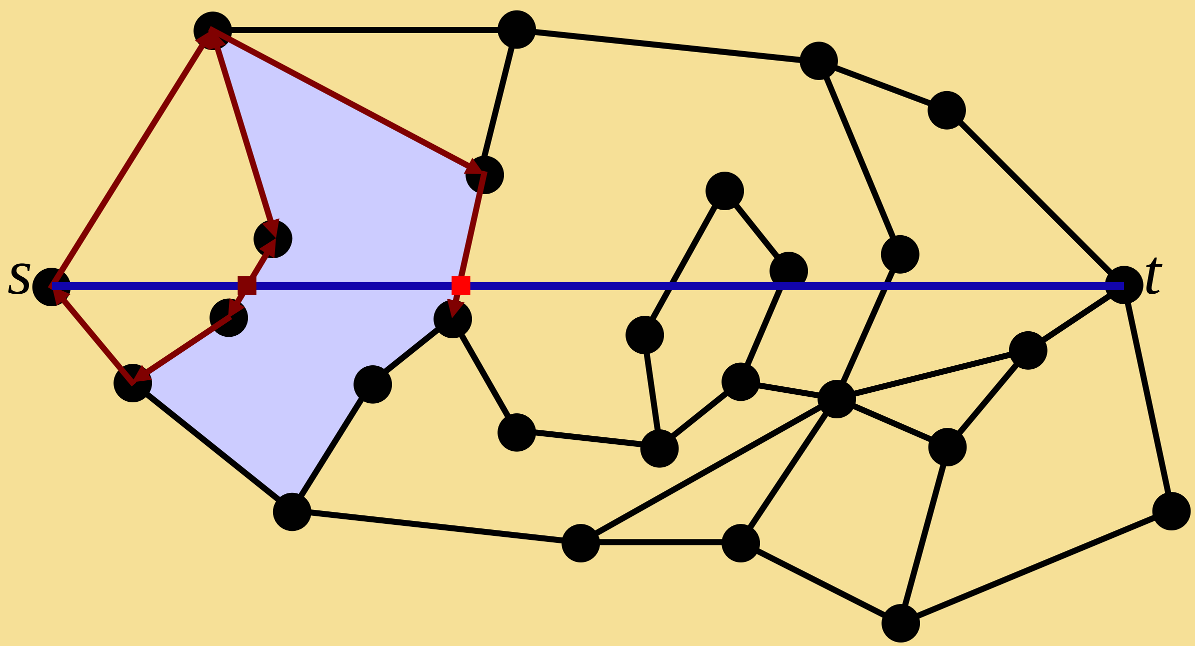

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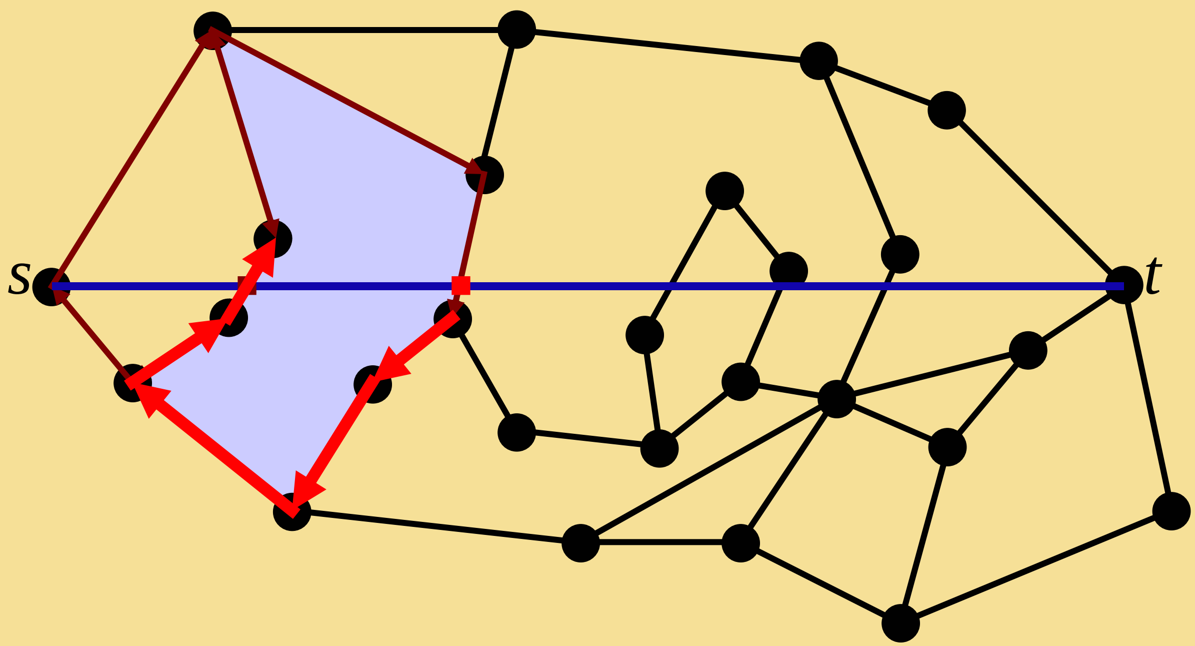

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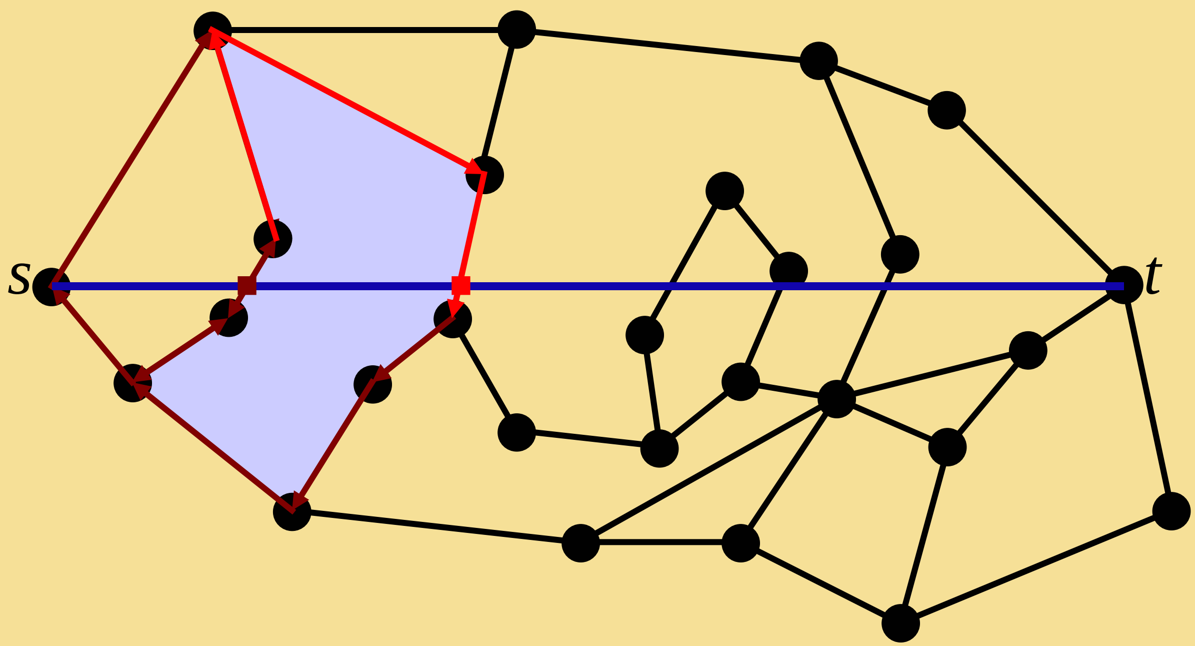

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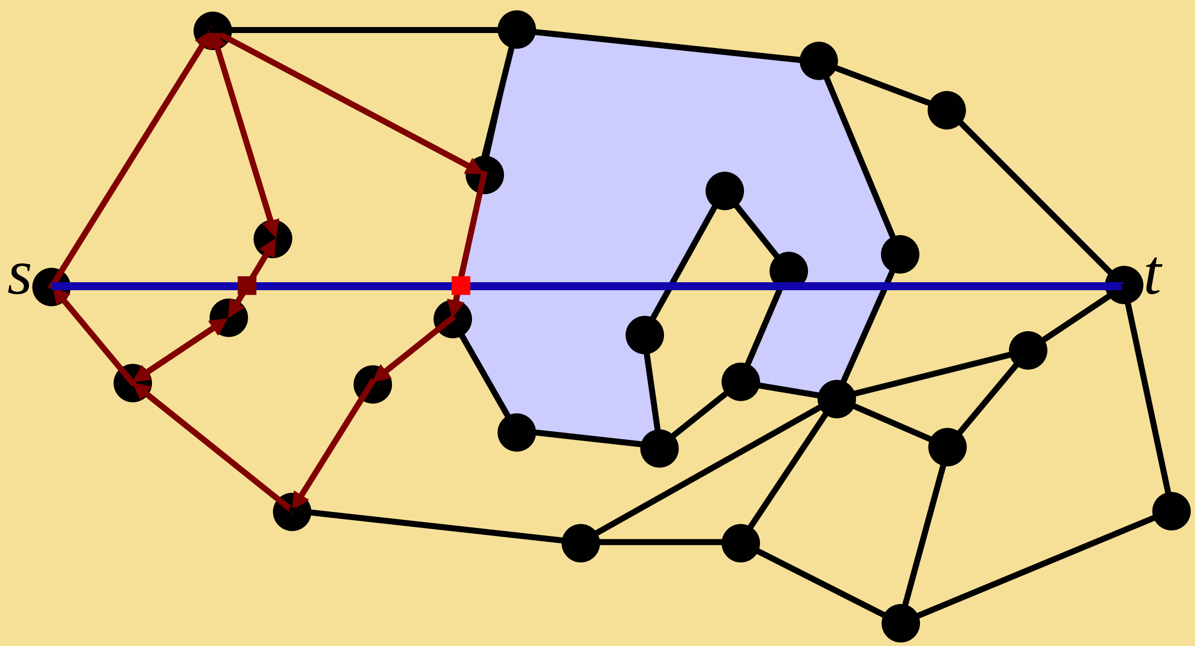

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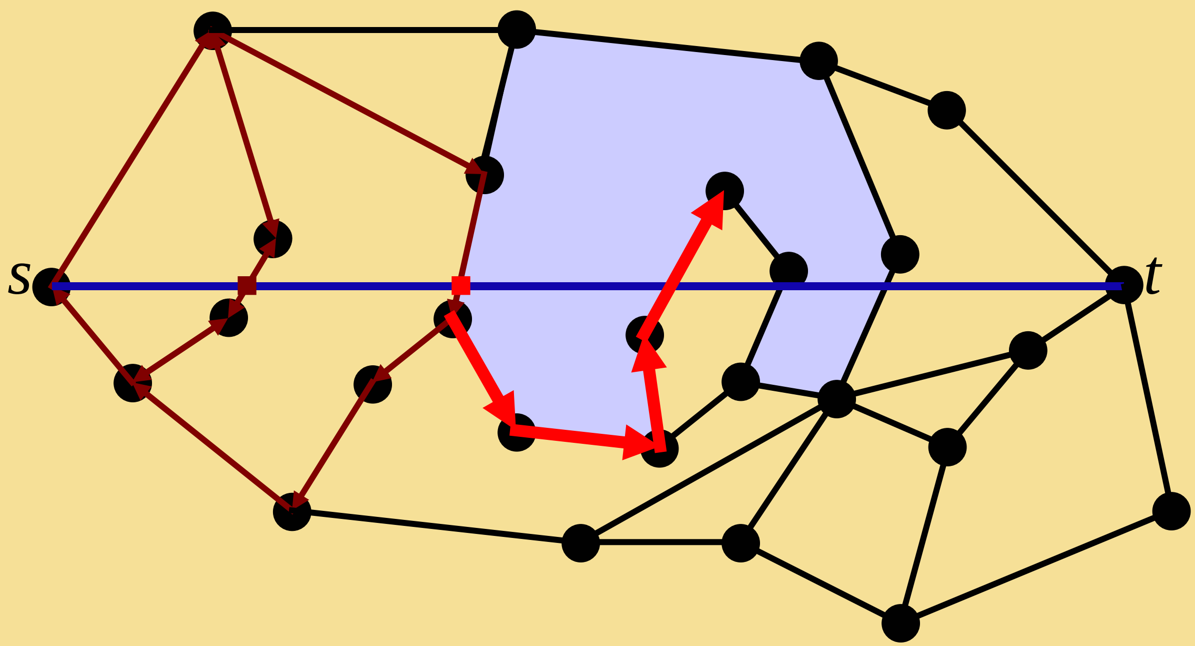

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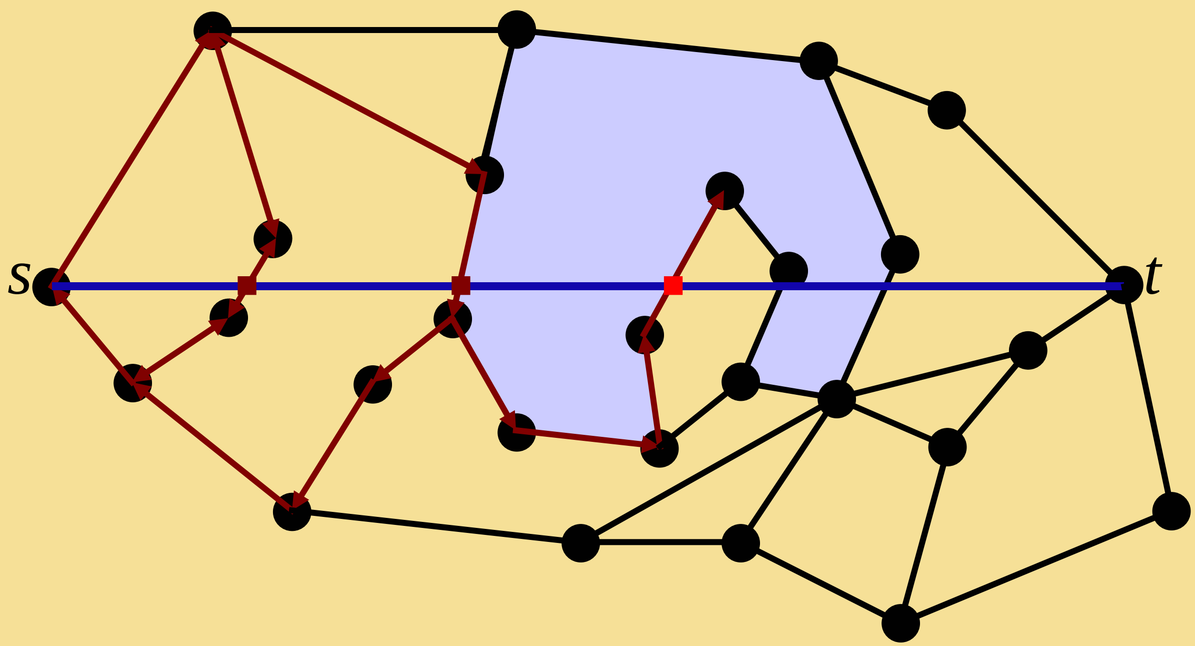

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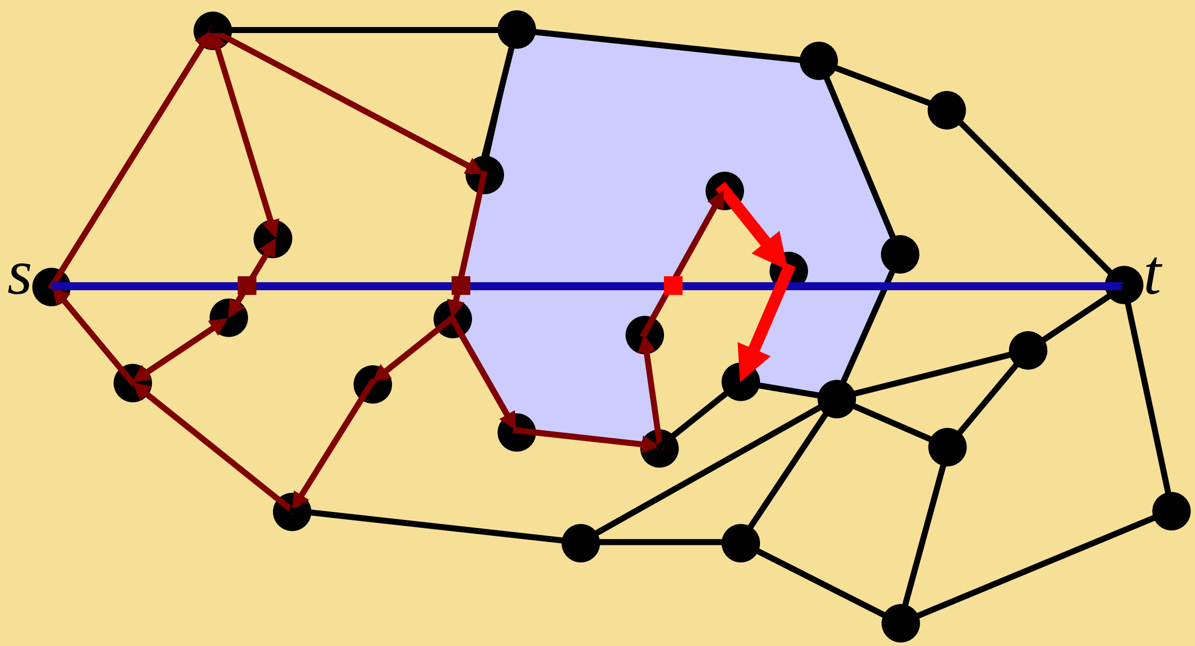

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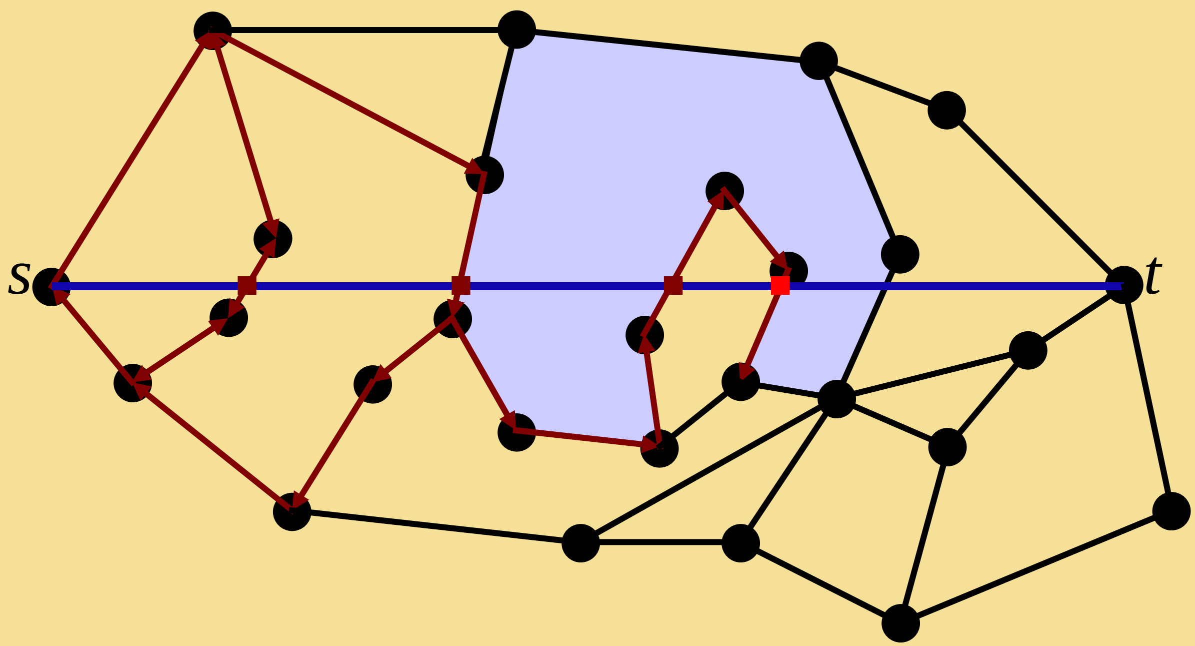

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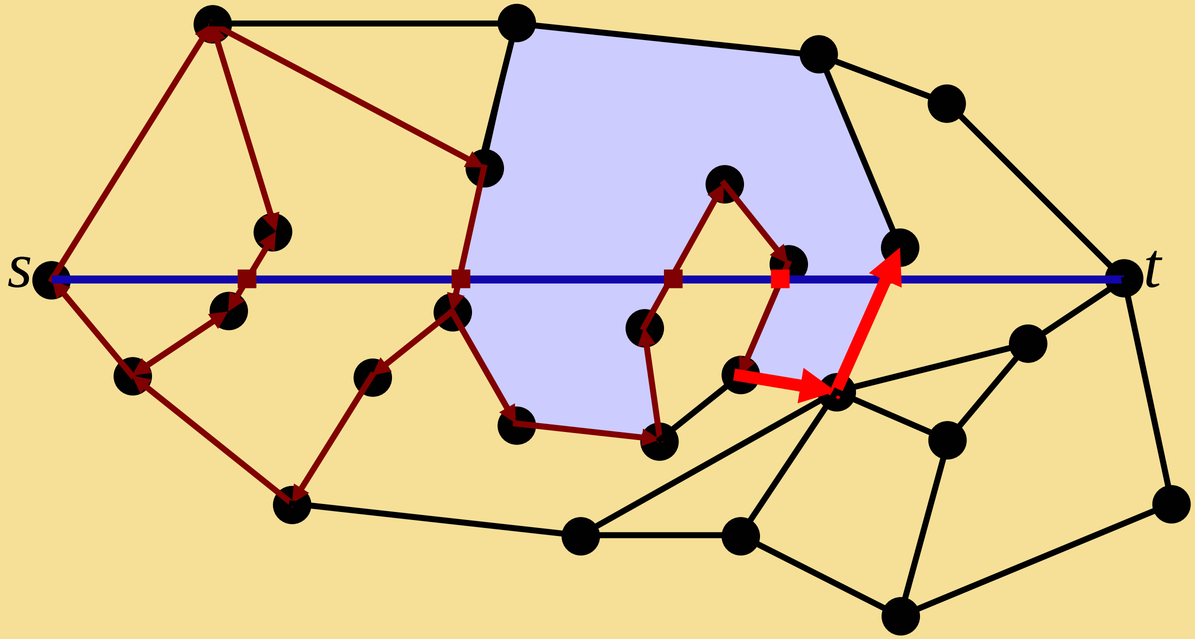

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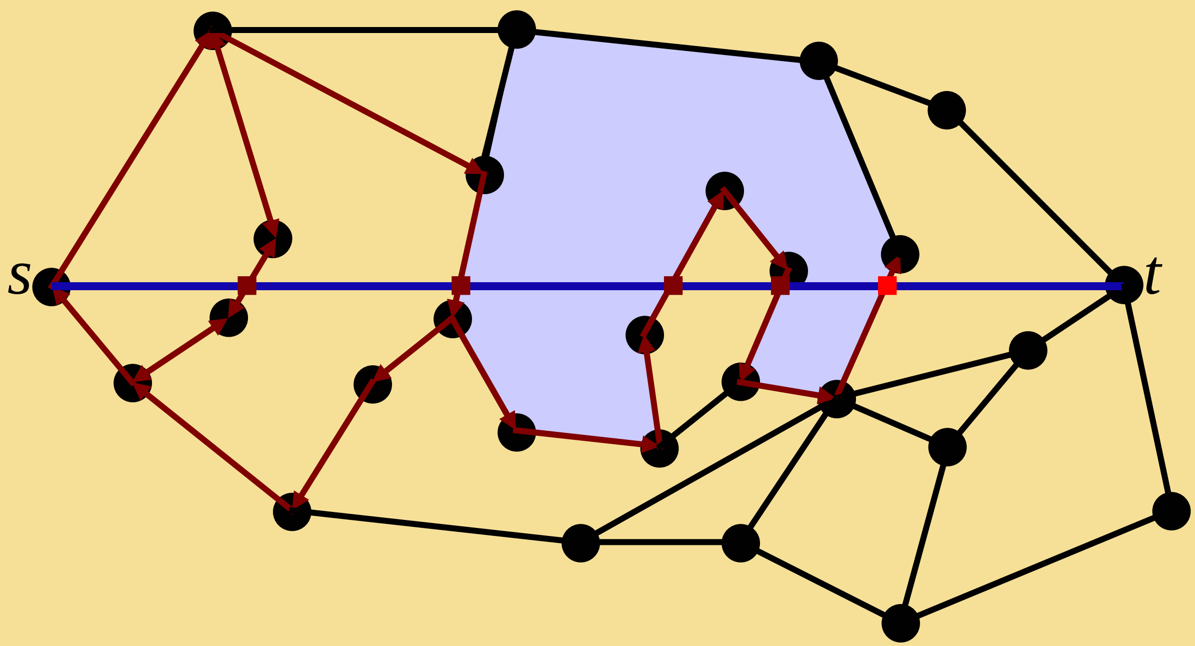

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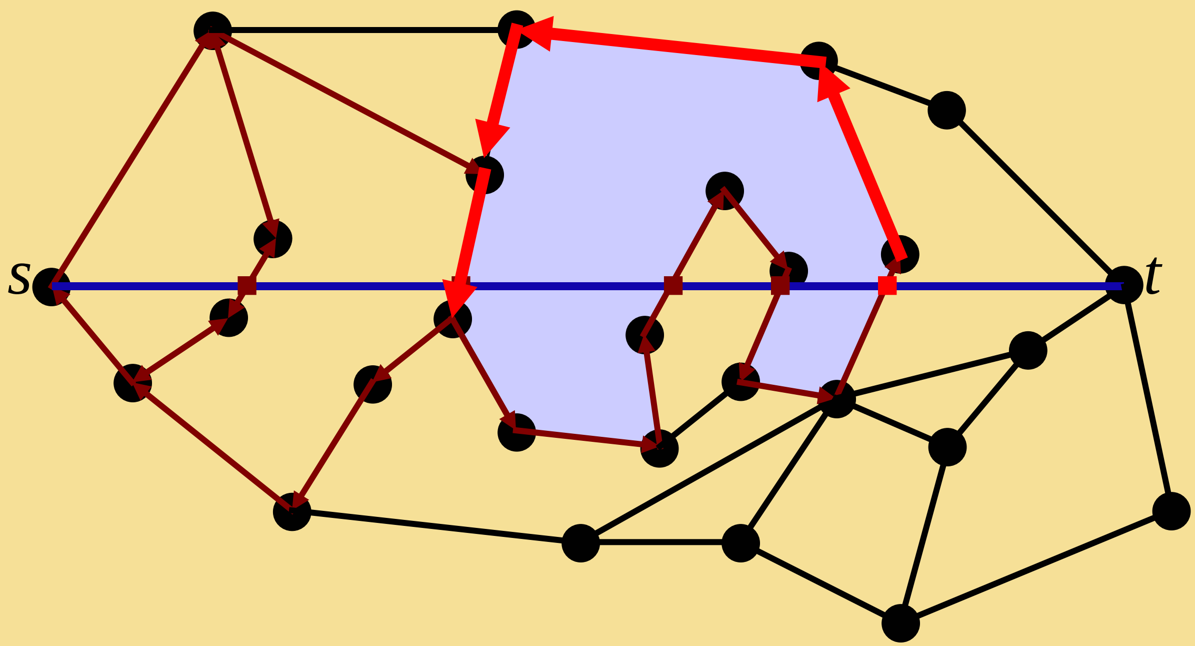

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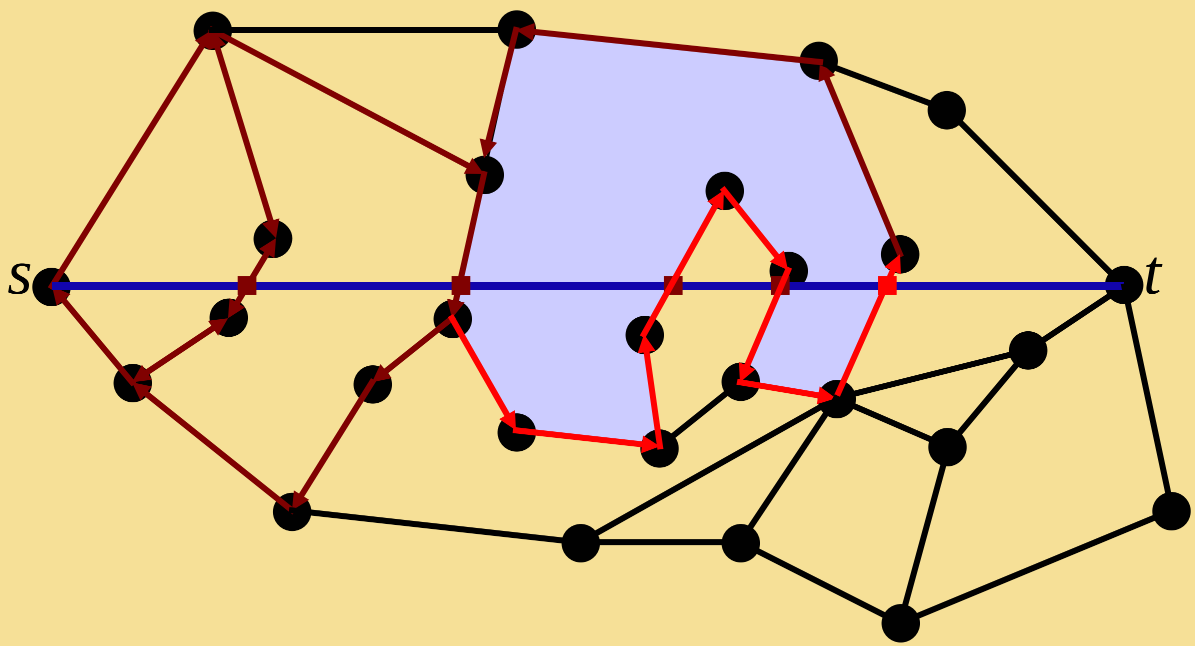

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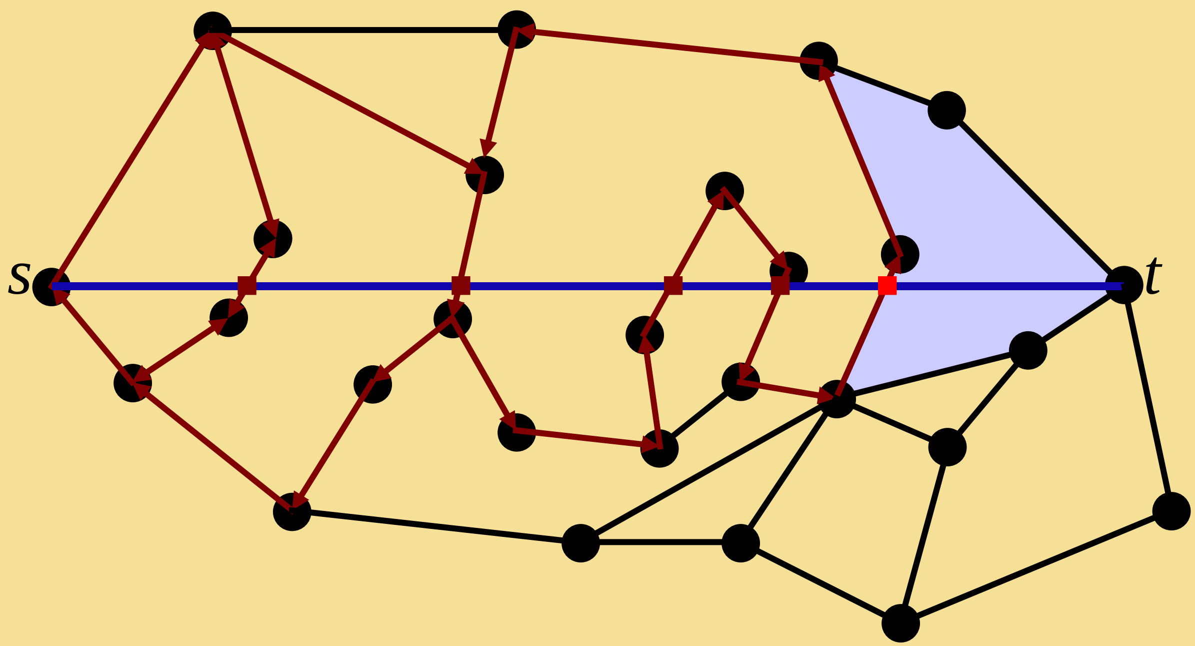

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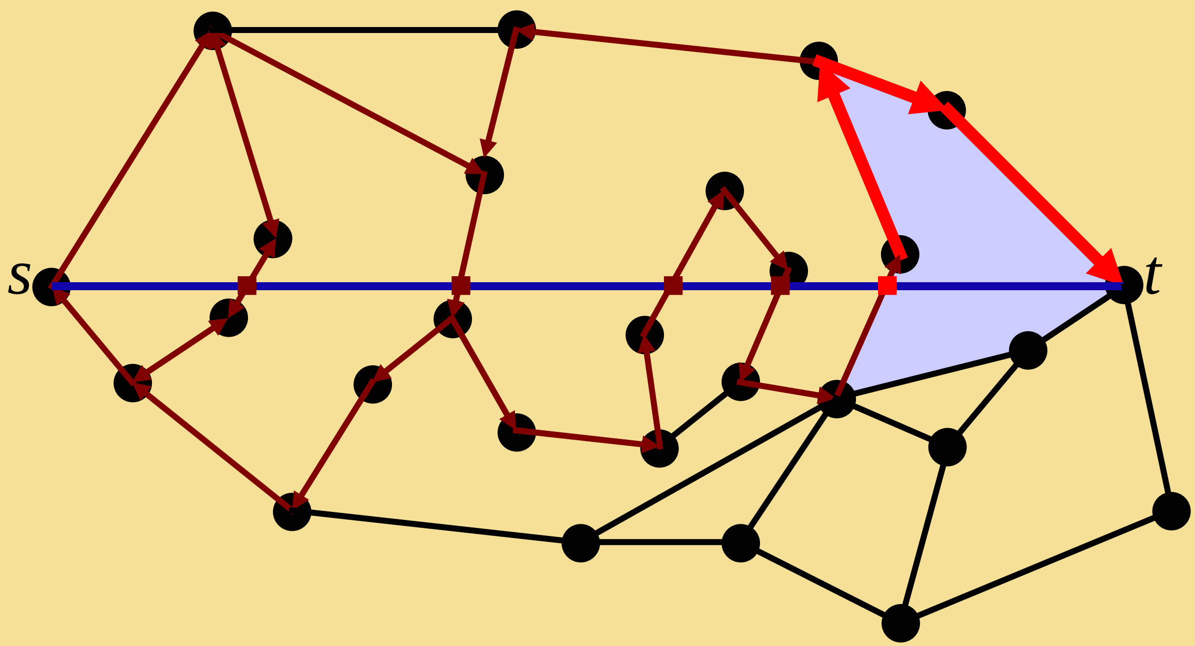


Face Routing (Analysis)

Lemma:

Face Routing always finds a path to the destination. The total cost of Face Routing is $O(n)$.

Face Routing (Analysis)

Proof Sketch:

- each face is explored at most once

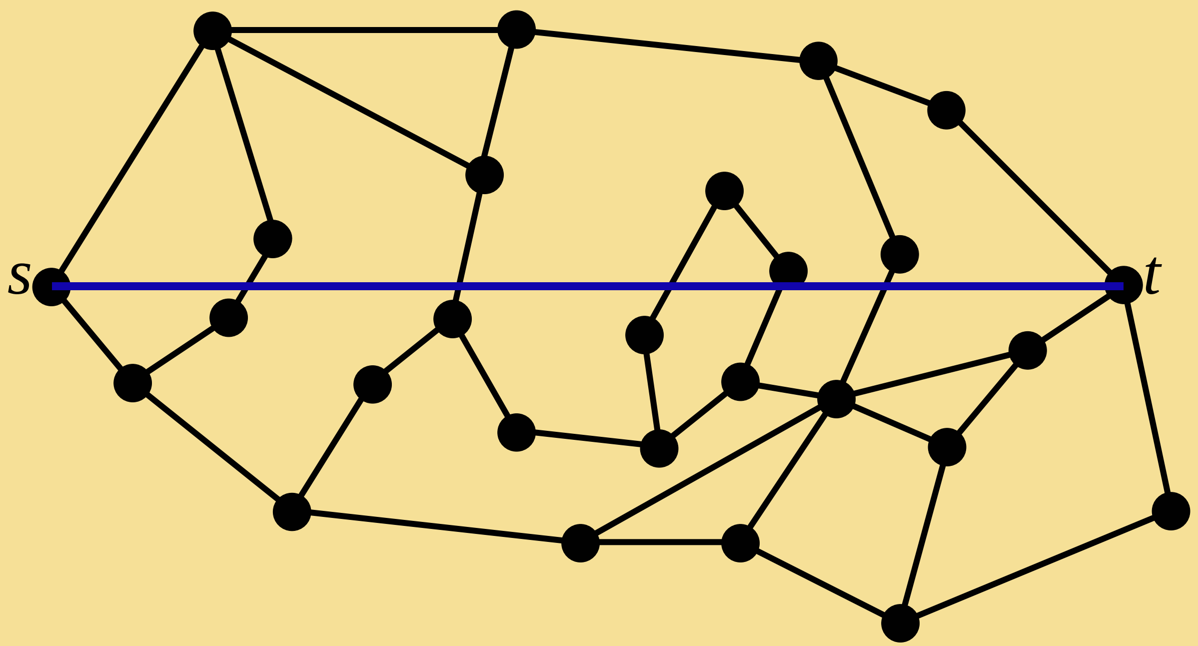
  $\Rightarrow$ each edge is traversed at most four times

- there are at most $3n$-6 edges (Euler formula)

Problem of Face Routing

- Face Routing always reaches destination
- However, even if source and destination are close to each other, Face Routing can take $O(n)$ steps.
- We would like to have an algorithm, whose cost is a function of the cost of an optimal path.

Adaptive Face Routing



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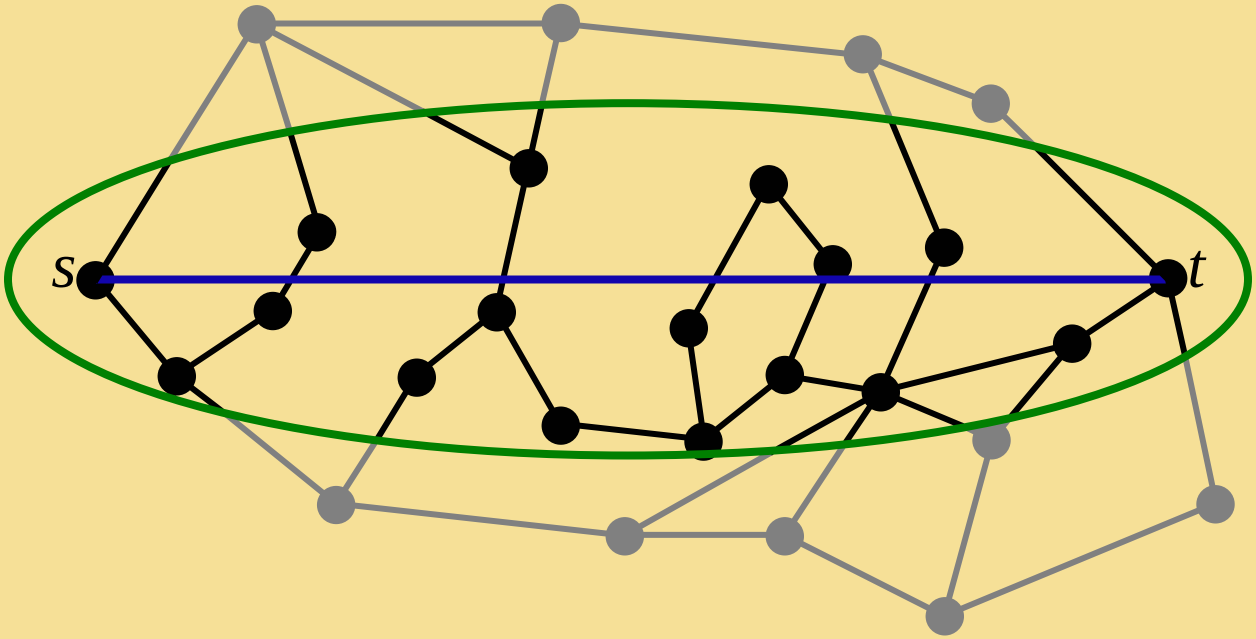

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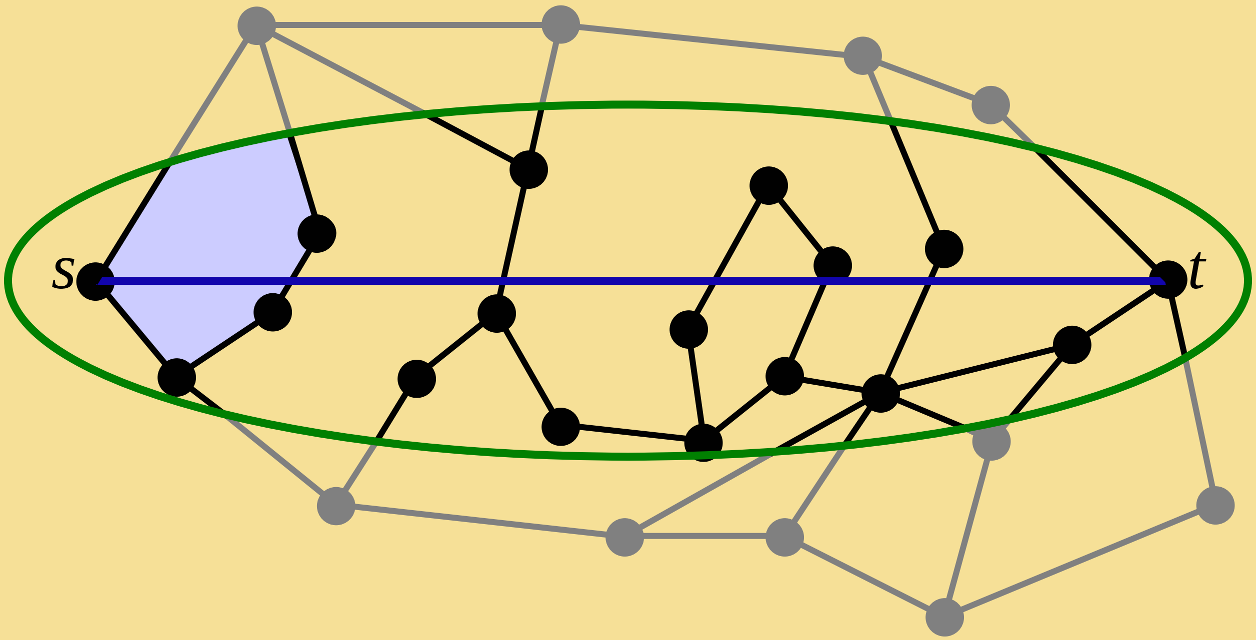

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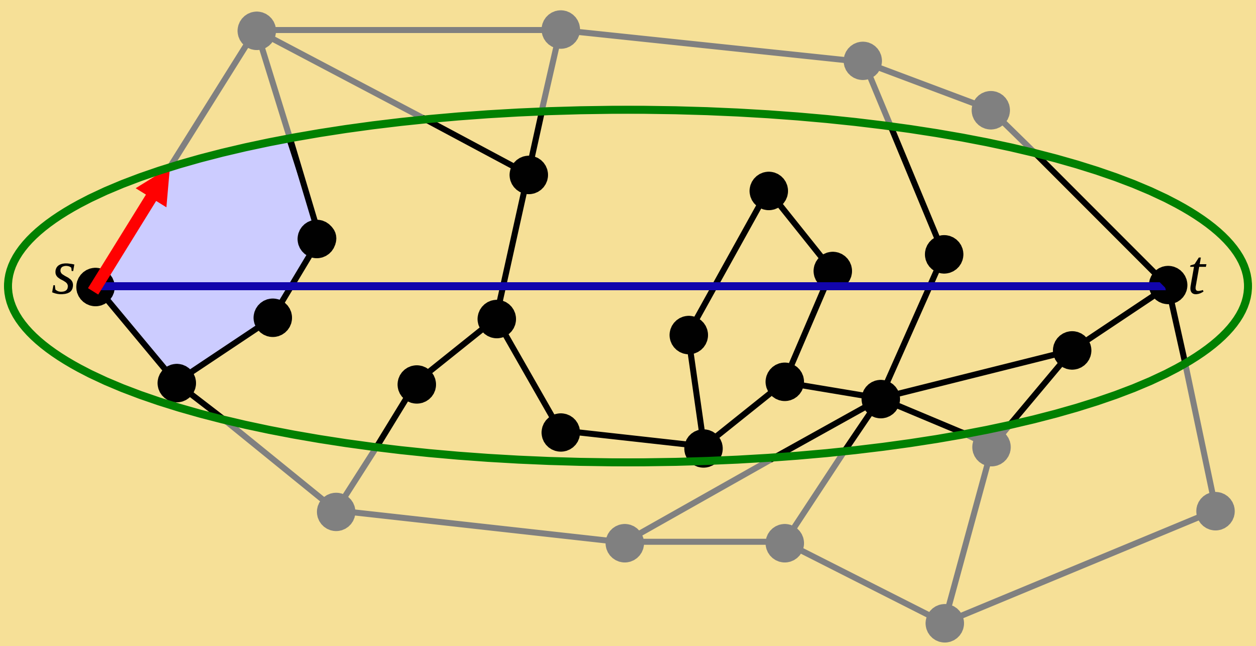

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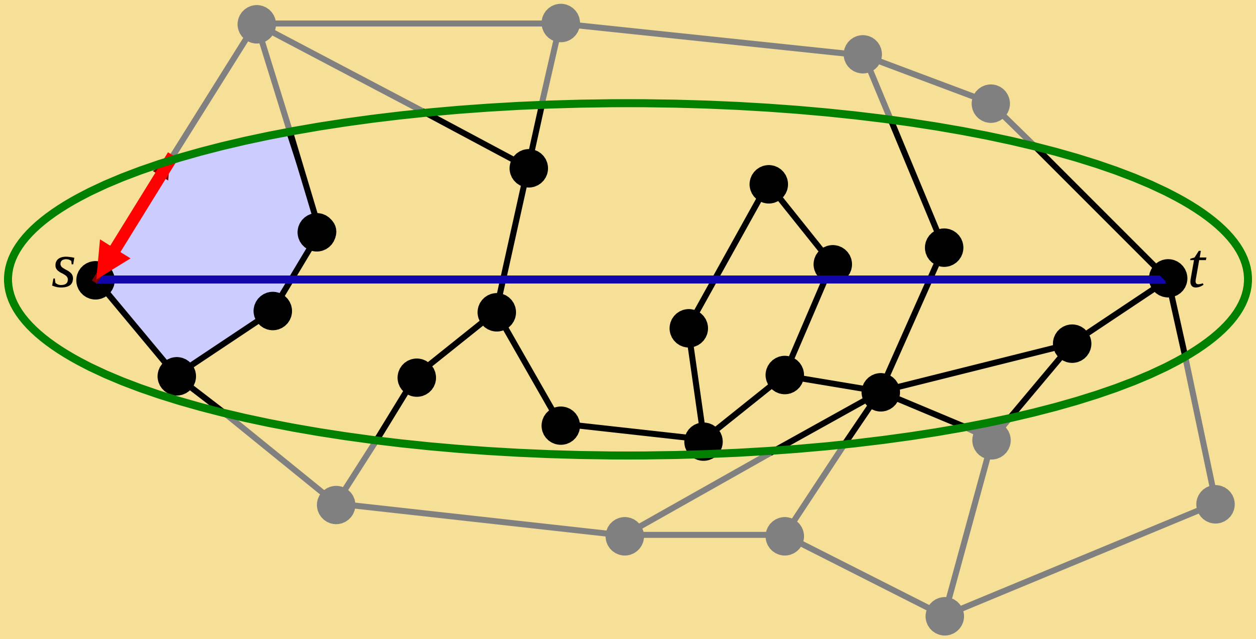

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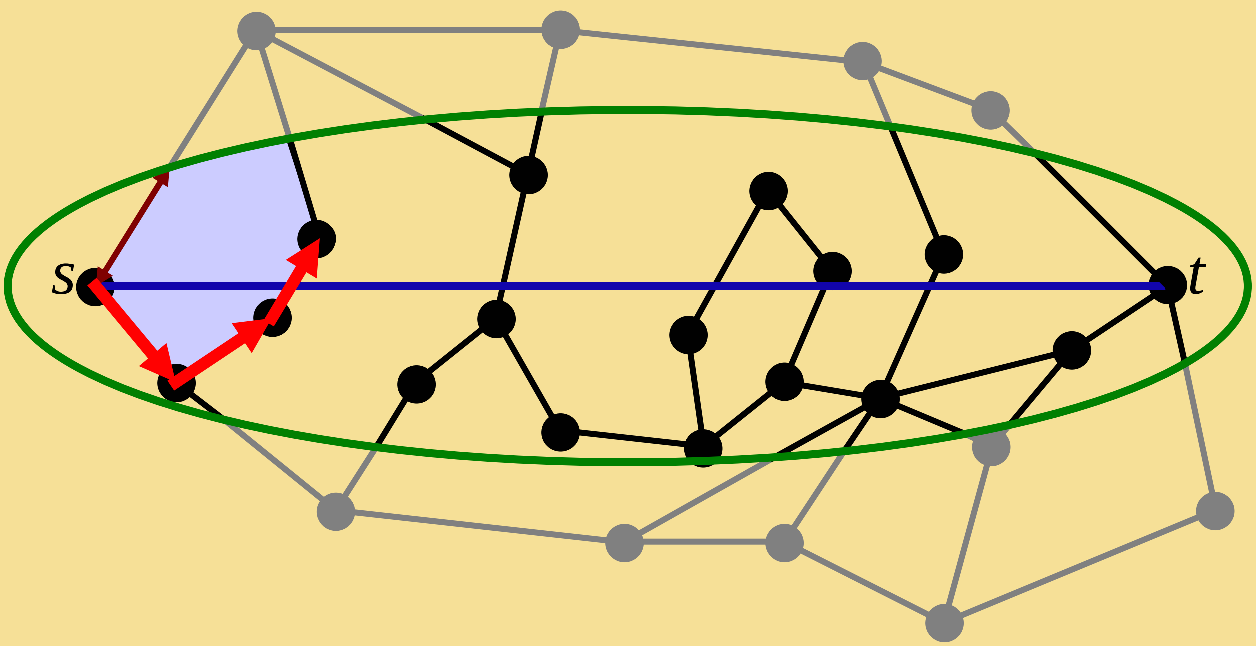

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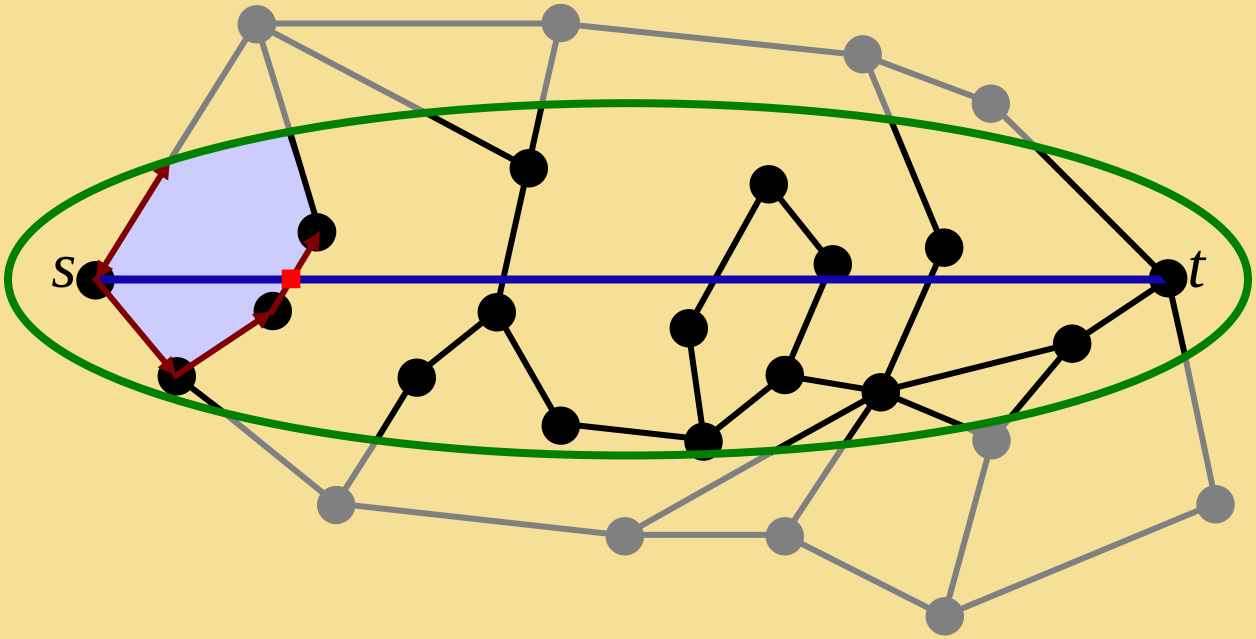

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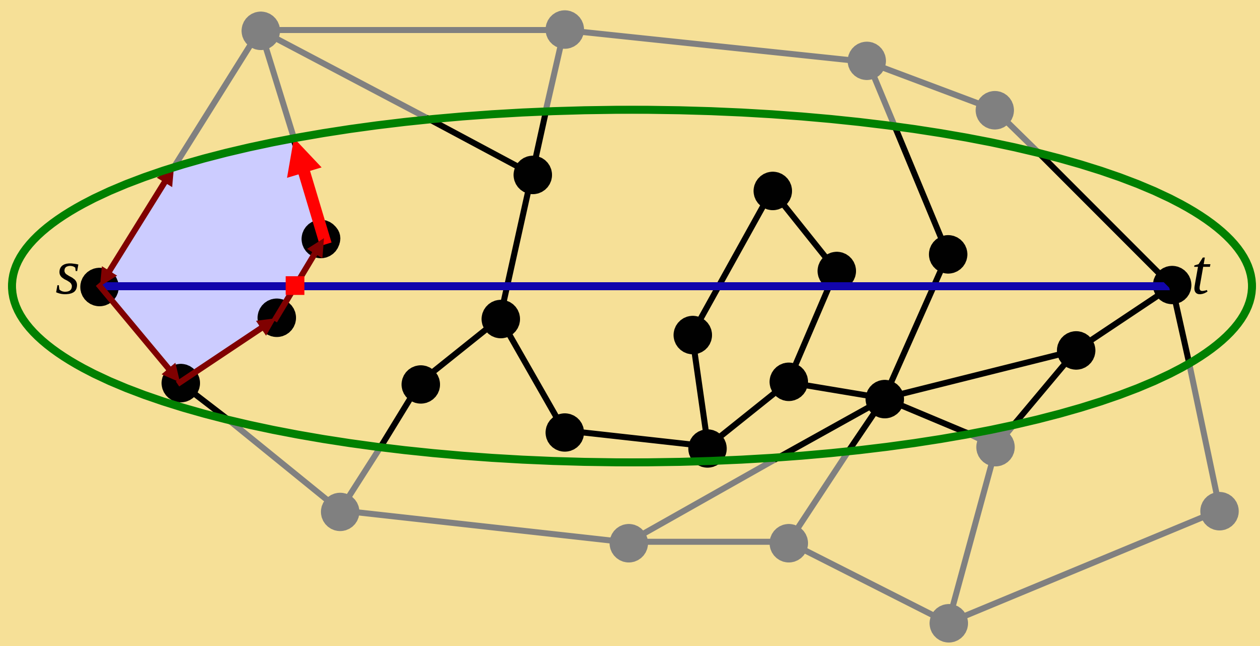

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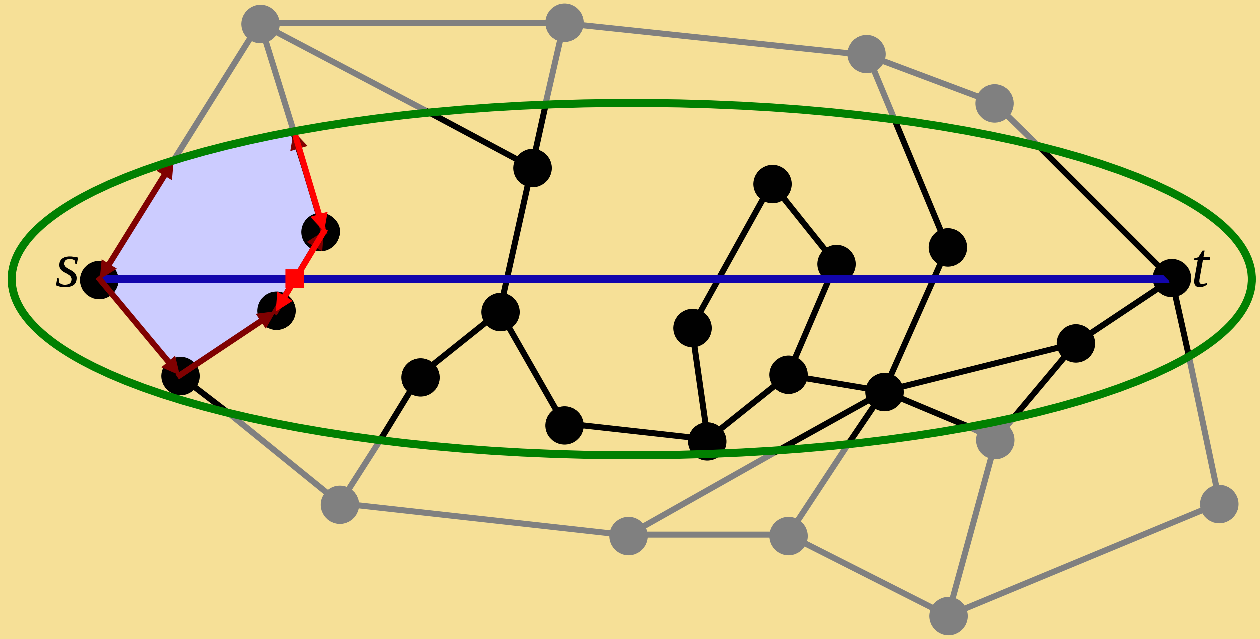

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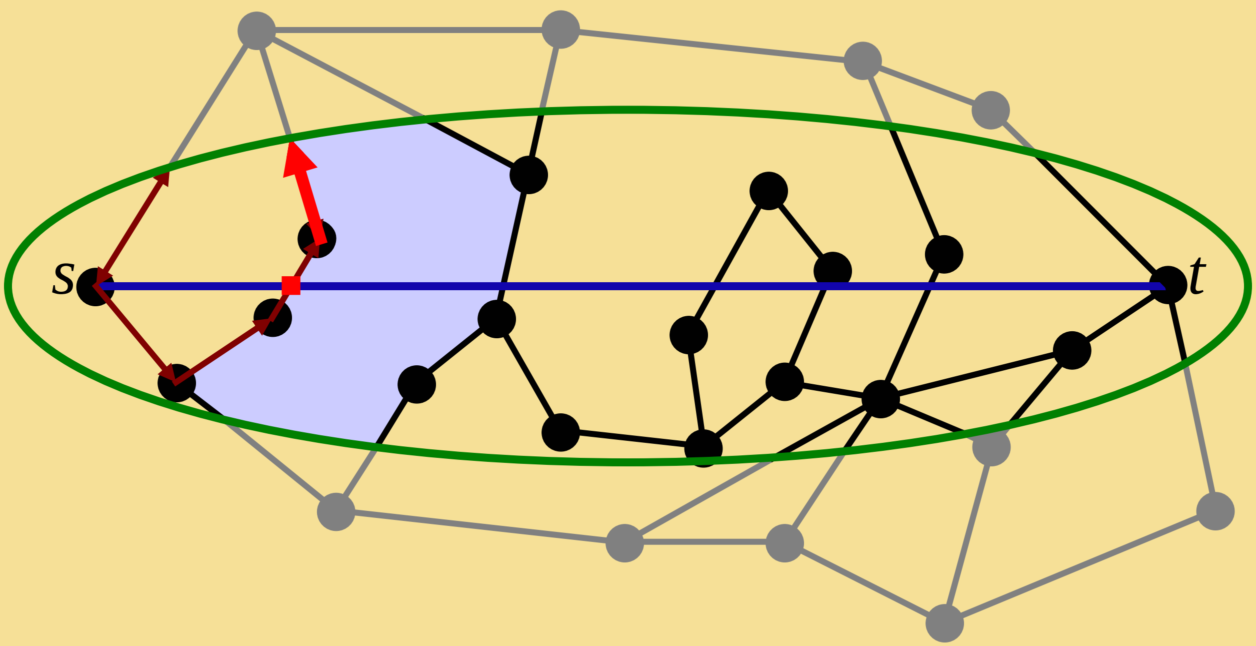

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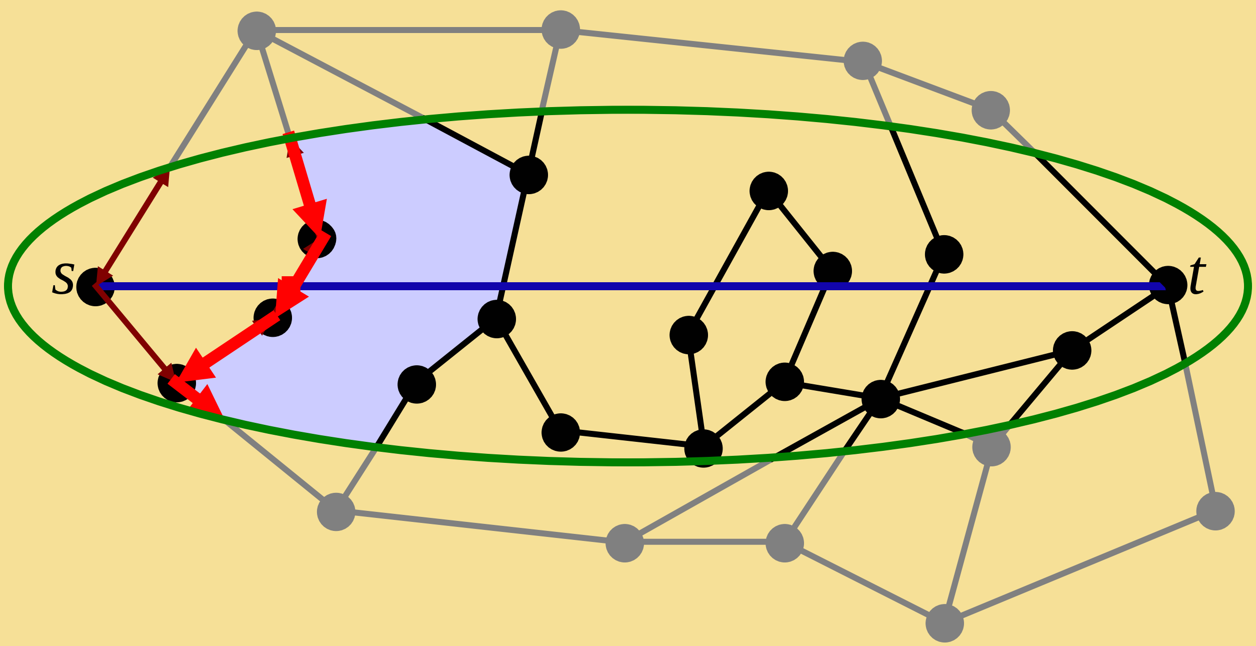
Ellipse is too small, go back!



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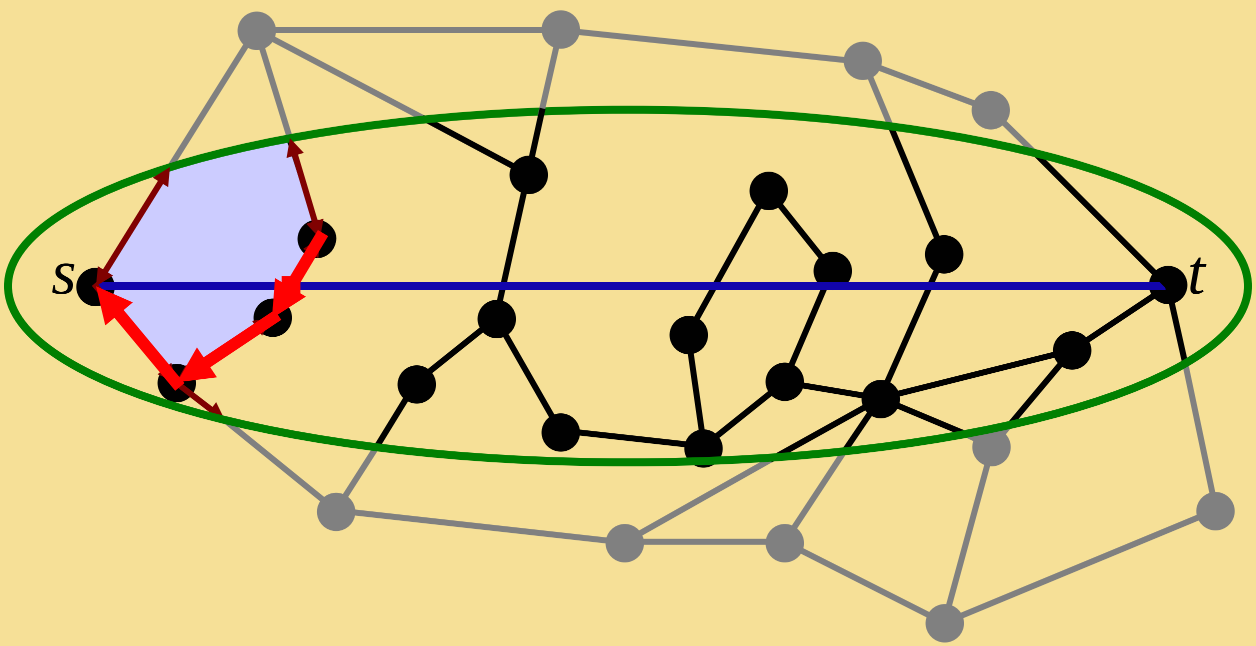

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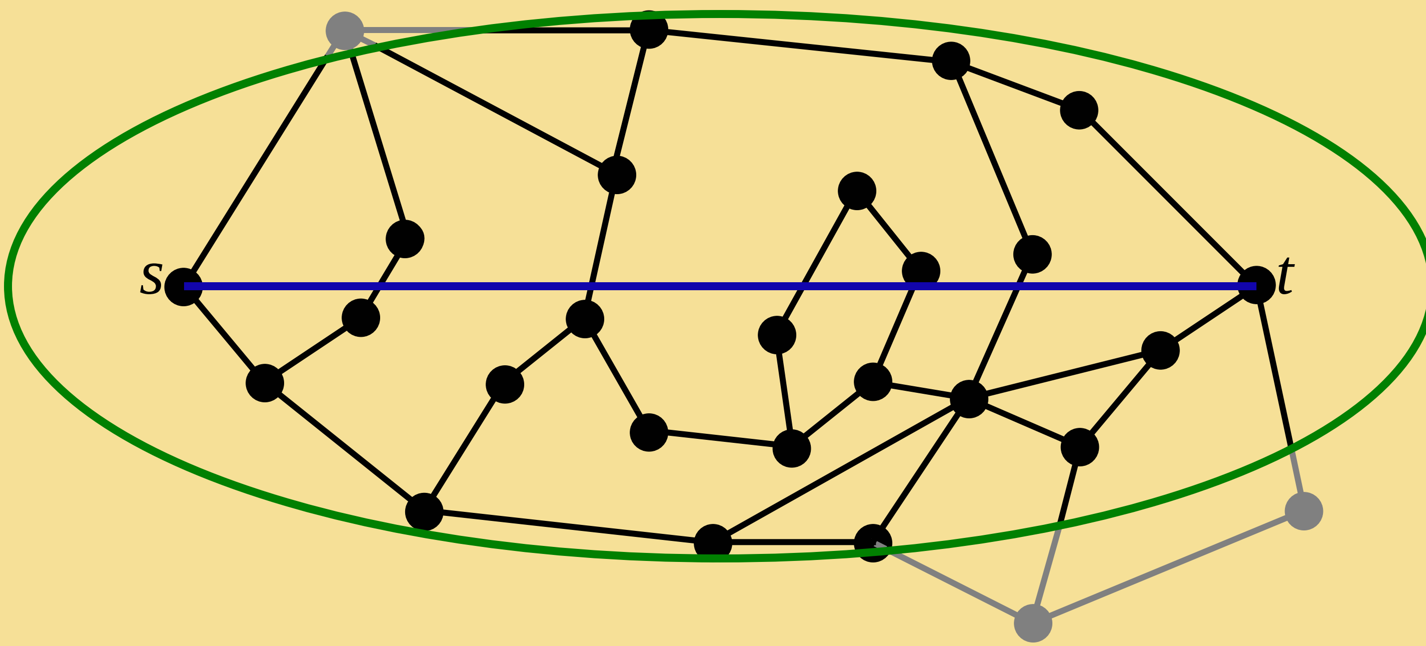

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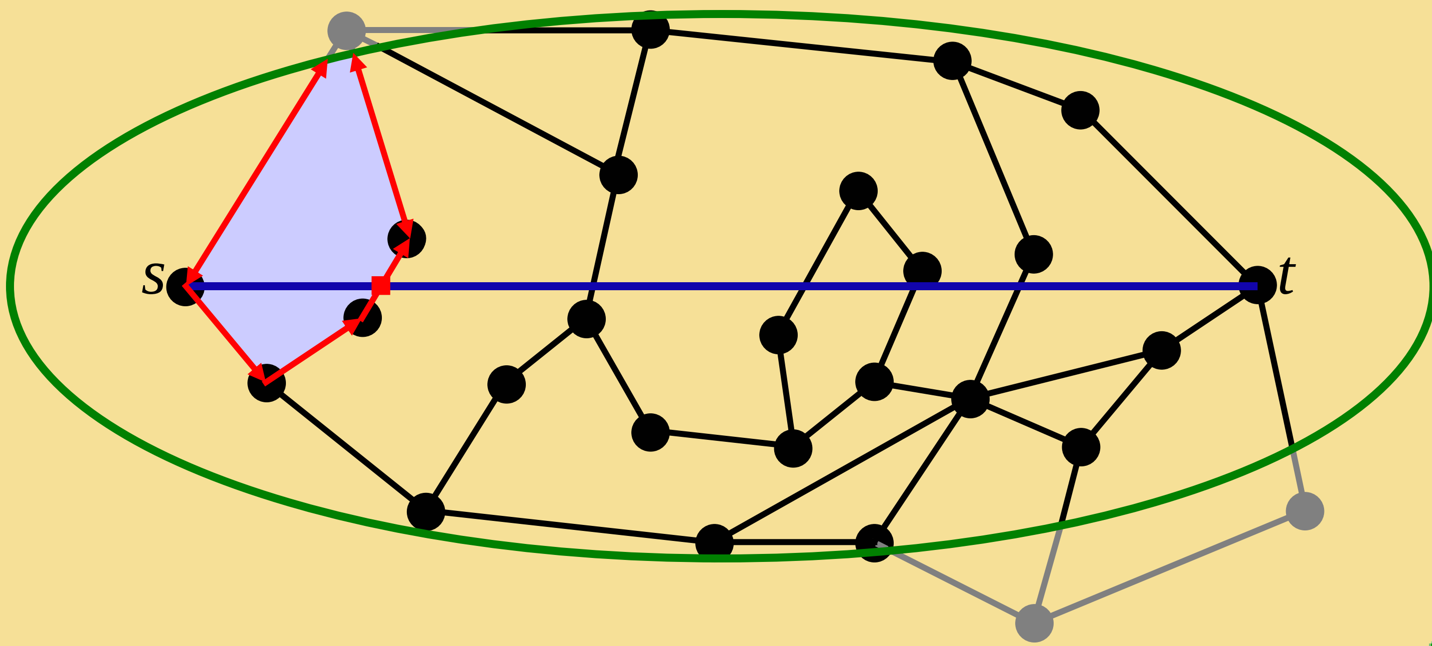

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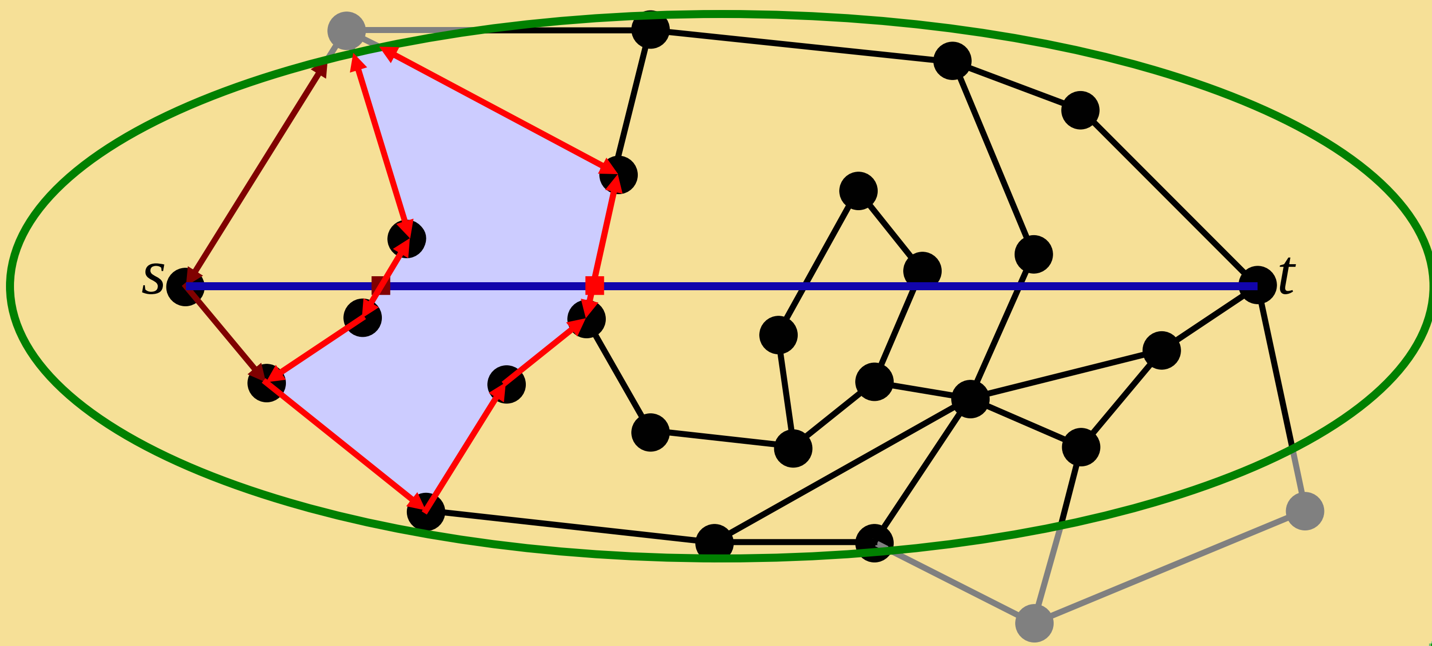

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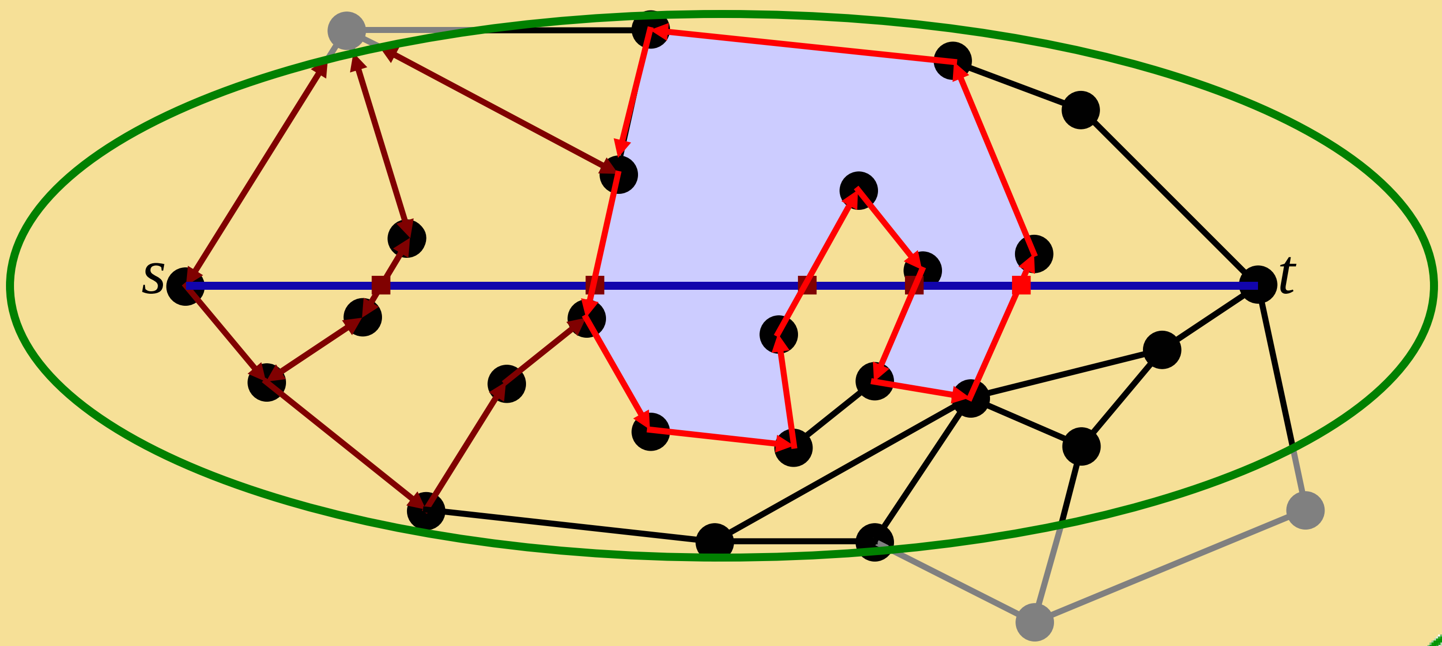

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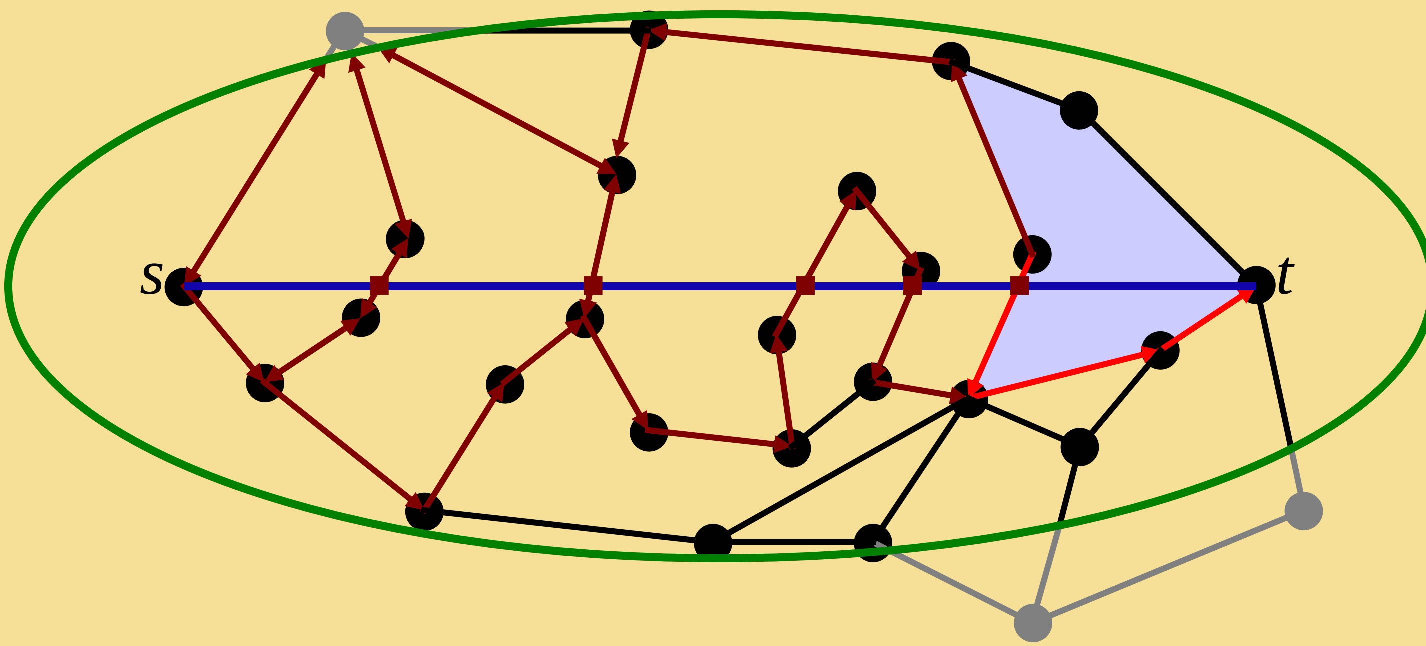


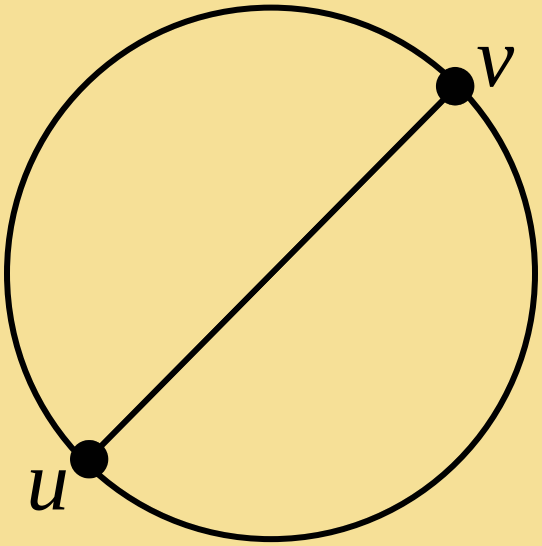
AFR on the Unit Disk Graph

- AFR needs a planar graph, UDG is not planar
- need a planar subgraph of UDG:
  - simple distributed construction
  - spanner for link distance, Euclidean, and energy metric

Gabriel Graph

Definition:

Two nodes u and v are connected by an edge iff the circle with $\overline{uv}$ as diameter contains no other node.



Properties of GG ∩ UDG

- For each pair of nodes, the GG (∩ UDG) contains an energy optimal path (on UDG).

  ⇒ spanner for link, Eucl. dist., energy ($\Omega(1)$-model)
- planar
- no additional communication

⇒ meets all our requirements

AFR Complexity

Theorem 1:

Let c_* be the cost (link, Euclidean, or energy) of an optimal path between two nodes on the UDG. Applying AFR on GG∩UDG then terminates with cost $O(c_*^2)$.

AFR Complexity, Proof I

Lemma:

For each used ellipse $\mathcal{E}$, the cost is linear in the number of nodes in $\mathcal{E}$.

Collorary:

In the $\Omega(1)$-model, for each used ellipse $\mathcal{E}$, the cost is linear in area covered by $\mathcal{E}$.

AFR Complexity, Proof II

Ellipses grow exponentially

$$\Downarrow$$

Lemma:

The cost, AFR needs to route a packet, is linear in the area covered by the last used ellipse.

AFR Complexity, Proof III

Lemma:

Using an ellipse $\mathcal{E}$, AFR finds a path from s to t iff there is such a path inside $\mathcal{E}$.

Lemma:

All paths of (Euclidean) length smaller or equal to c are inside an ellipse whose area is in $O(c^2)$.

AFR Complexity, Proof IV

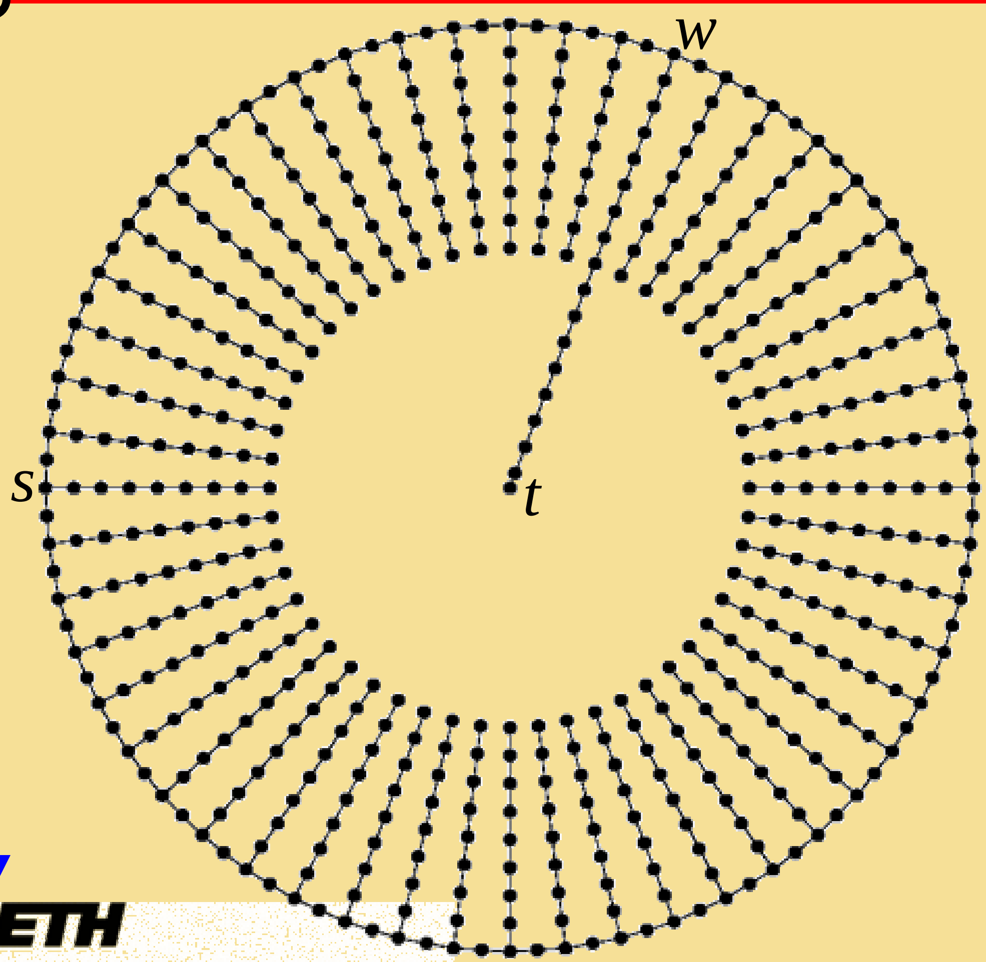
All the lemmas together now prove Theorem 1.

Lower Bound

Theorem 2:

Let c_* be the cost (link, Euclidean, or energy) of an optimal path between two nodes on a UDG $\mathcal{G}$. For each geometric routing algorithm, there is a graph for which the cost is $\Omega(c_*^2)$.

Lower Bound, Proof



Cost of optimal path:

$$c_* \leq (\pi + 1)R$$

Exp. cost for an algorithm $\mathcal{A}$:

$$\mathrm{E}[c(\mathcal{A})] \geq \frac{\pi R}{2} \cdot \Theta(R)$$
$$= \Theta(R^2)$$

Main Theorem

Theorem 3:

On the Unit Disk Graph in the $\Omega(1)$-model, AFR is asymptotically optimal.

(follows directly from Theorems 1 and 2)

Conclusion I

$\Omega(1)$ restriction can be dropped by clustering:

- works fine for link and Euclidean distance (still $\Theta(c_*{}^2)$)
- For energy, it can be shown that the cost of a geometric routing alg. cannot be bounded by a function of c_* alone.

Conclusion II

- The lower bound holds for all routing algorithms which have only local knowledge at the beginning.
- if coordinates of dest. are not known, but if each node can store some bits: Then there is a simple flooding variant which achieves $O(c_*^2)$.