Multidimensional Approximate Agreement with Asynchronous Fallback

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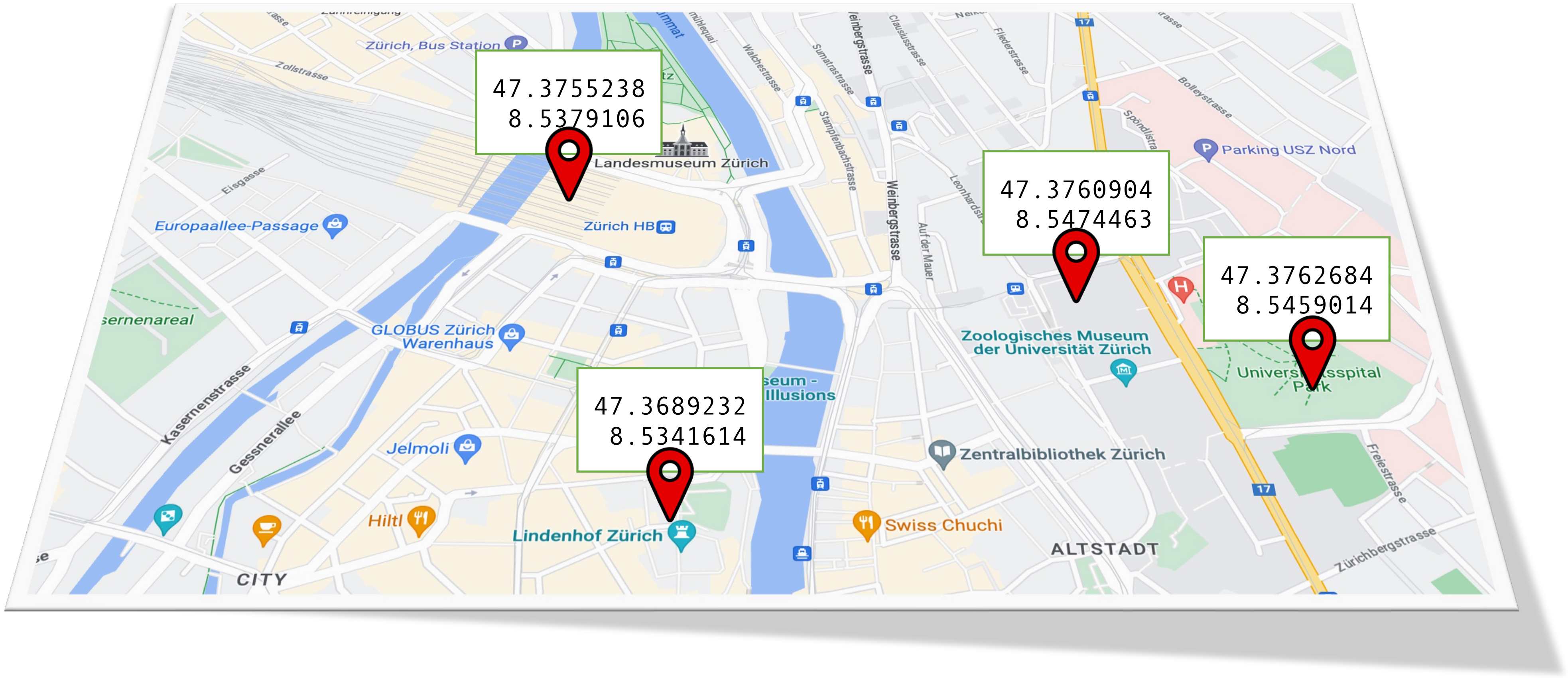

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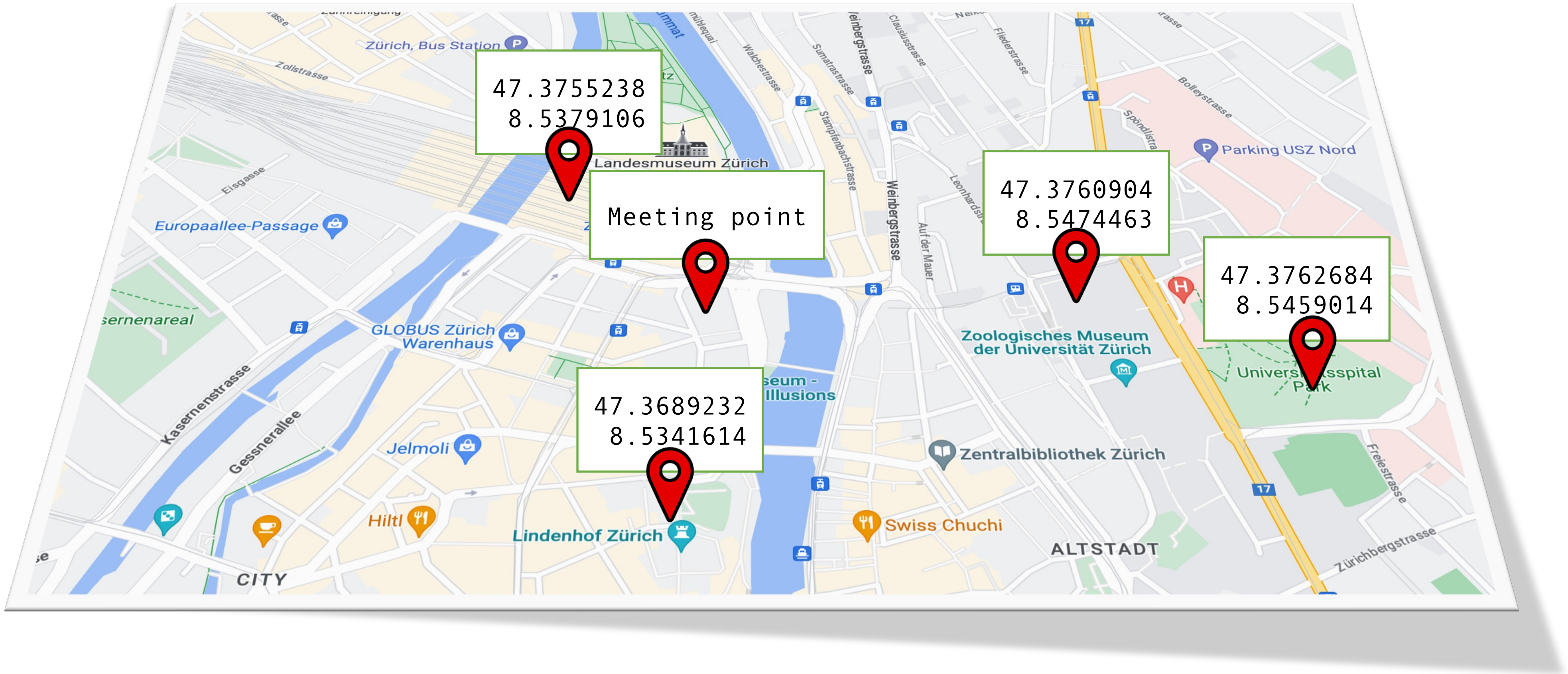
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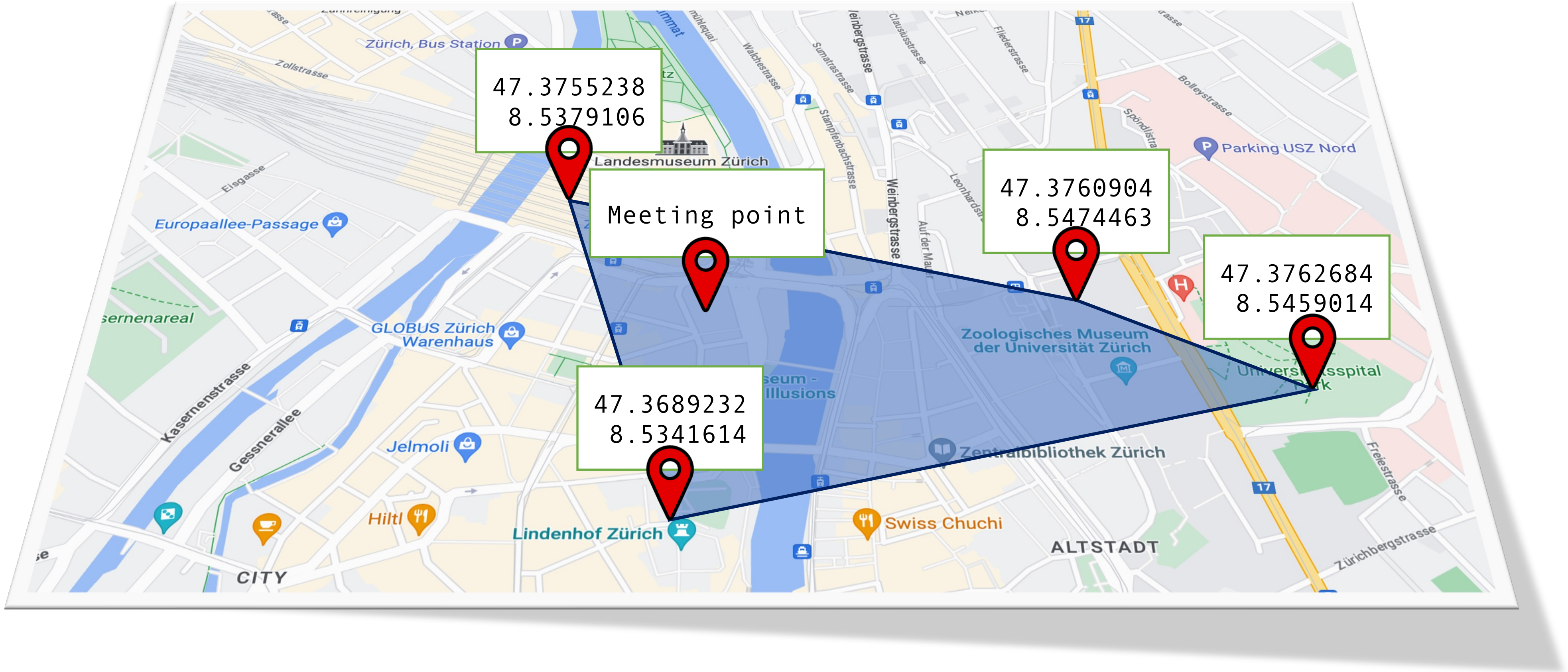

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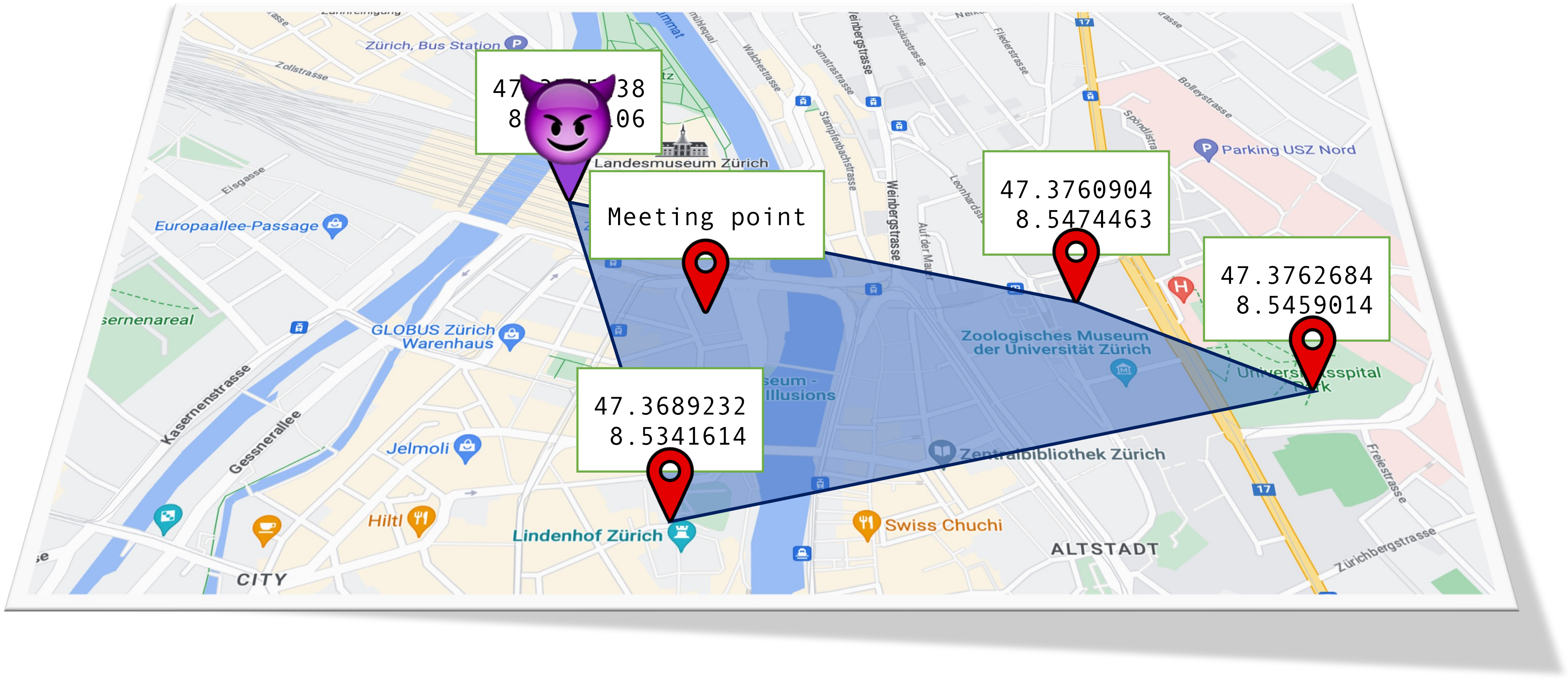

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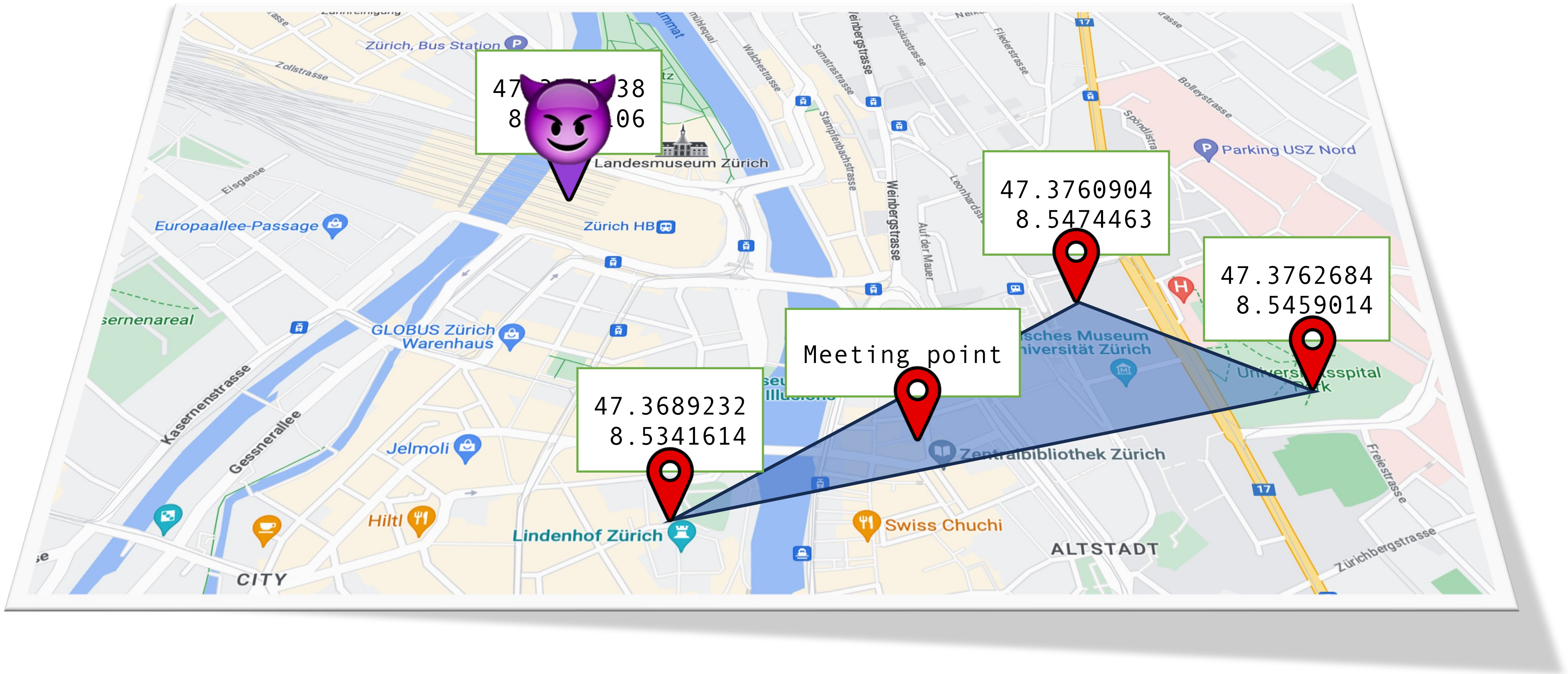

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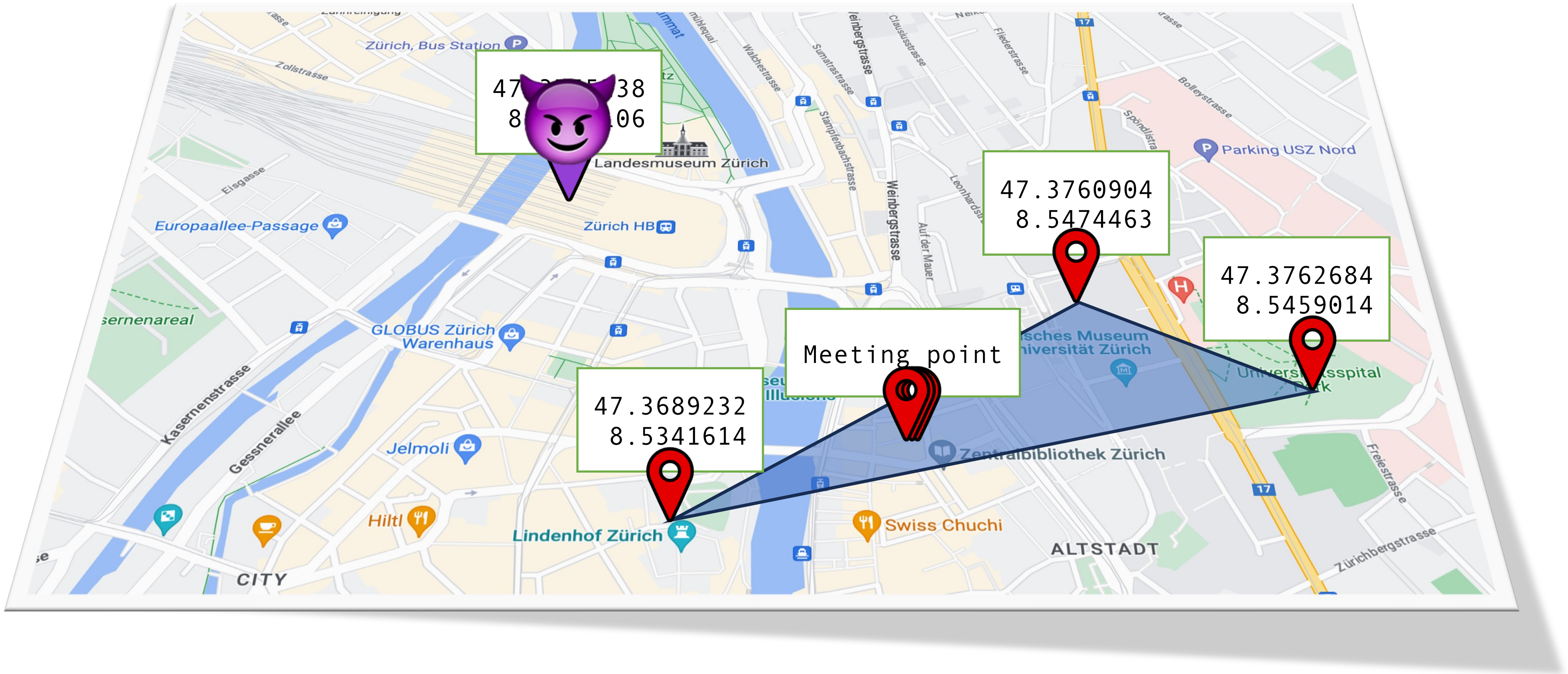

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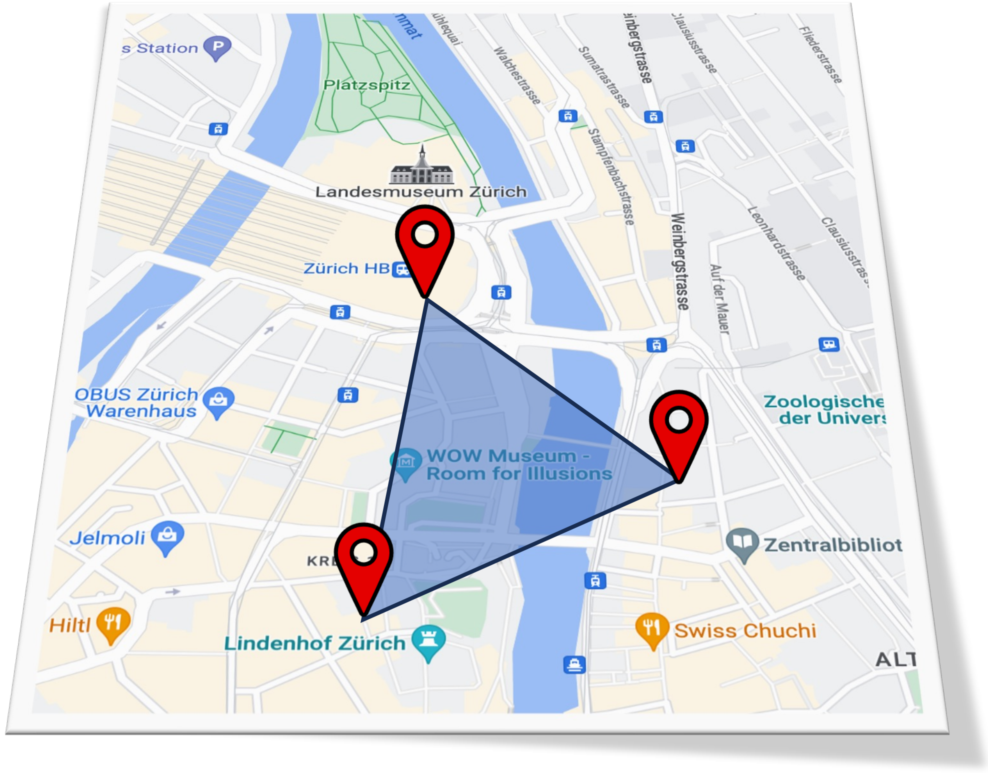
Multidimensional Approximate Agreement (***D-AA***)

Consider some $\mathcal{E} > 0$, and a setting of n parties holding inputs in $\mathbb{R}^D$.

Even when t of the n parties are Byzantine, honest parties obtain:

- $\mathcal{E}$-close outputs in $\mathbb{R}^D$ ($\mathcal{E}$-*Agreement*).
- That are in the convex hull of their inputs (*Validity*).

How many corruptions can be tolerated?



In $\mathbb{R}^2$, t $\geq n/3$ is too much.

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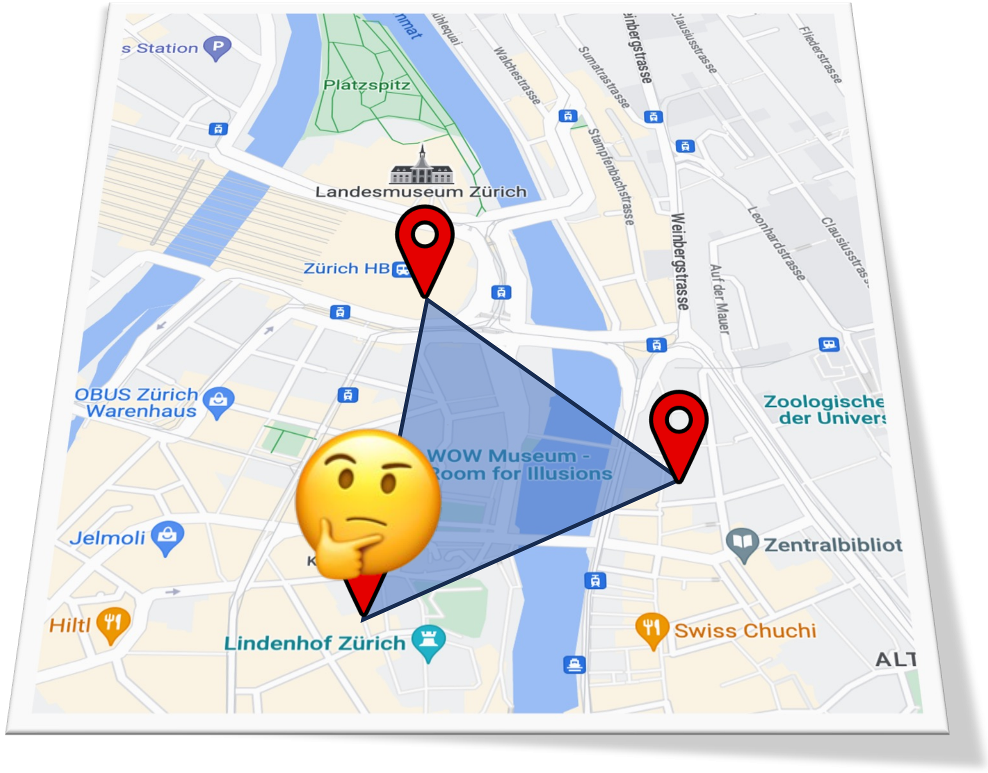

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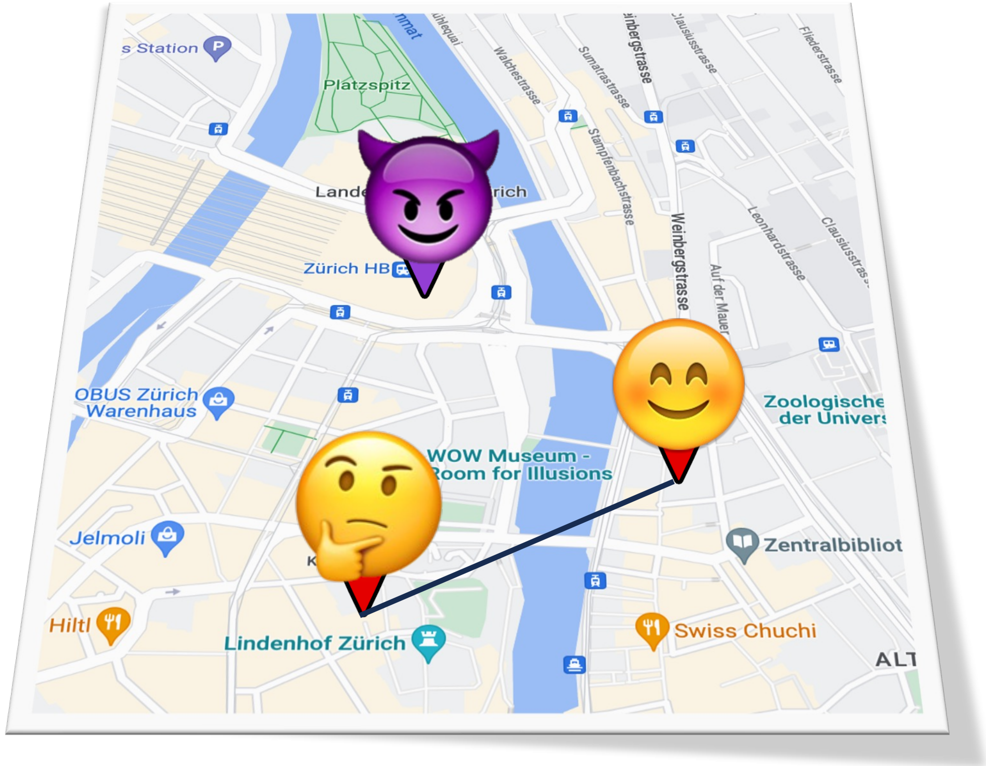

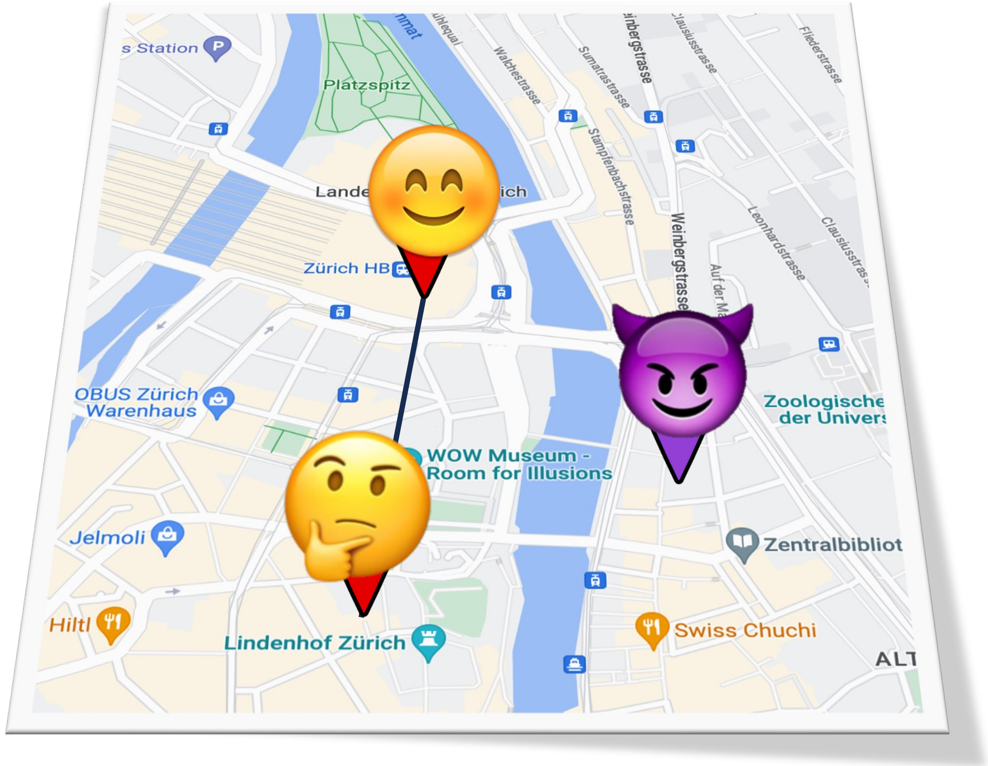

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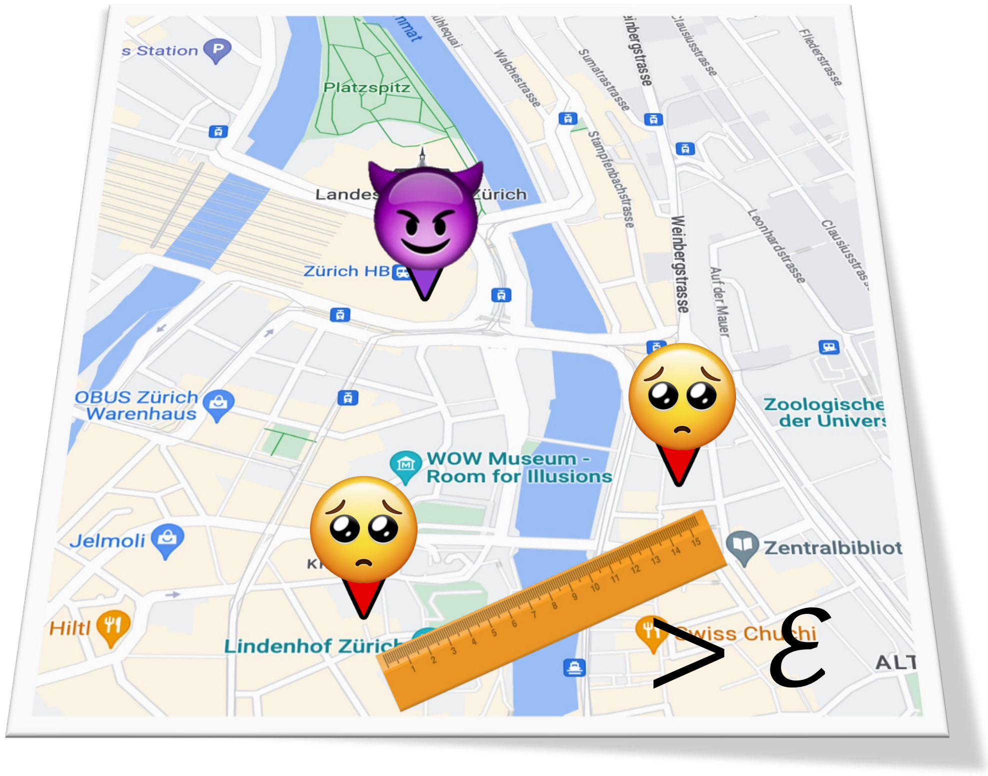

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How many corruptions can be tolerated?

- $t < n/(D+1)$: necessary.

[PODC:VaiGar13]

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- $t < n/(D+2)$: necessary and sufficient **in the asynchronous model.**
  [STOC:MenHer13, PODC:VaiGar13]

Main question

The parties do not know whether the network is synchronous or not:

- synchronous $\Rightarrow t_s < n/(D+1)$ corruptions.
- asynchronous $\Rightarrow t_a < n/(D+2)$ corruptions ($t_a \leq t_s$).

Can we achieve D-AA in this model?

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Can we achieve D-AA in this model?

- $D = 1$: yes, iff $2 \cdot t_s + t_a < n$ (with PKI). [PODC:GhLiWa22]
- $D > 1$: yes, if $(D+1) \cdot t_s + t_a < n$ (without setup). [this work]

Algorithm outline

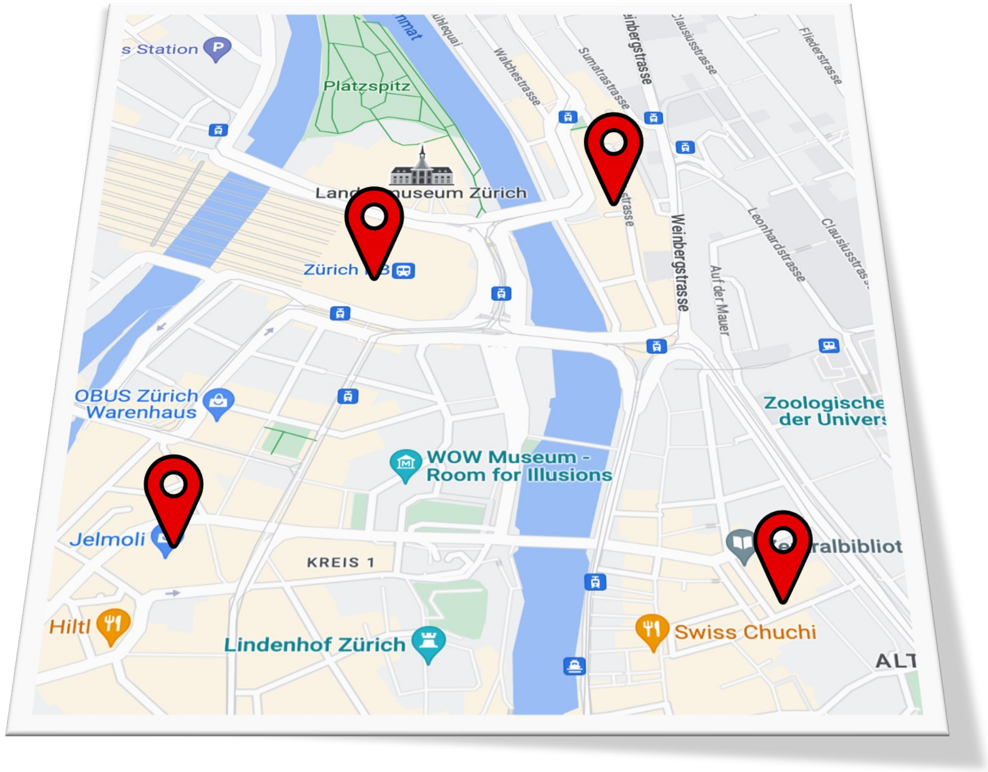
- Iterations:

    1. Parties distribute their current values via Overlap All-to-All Broadcast (OBC).

    2. Based on the values received, compute a *safe area.*

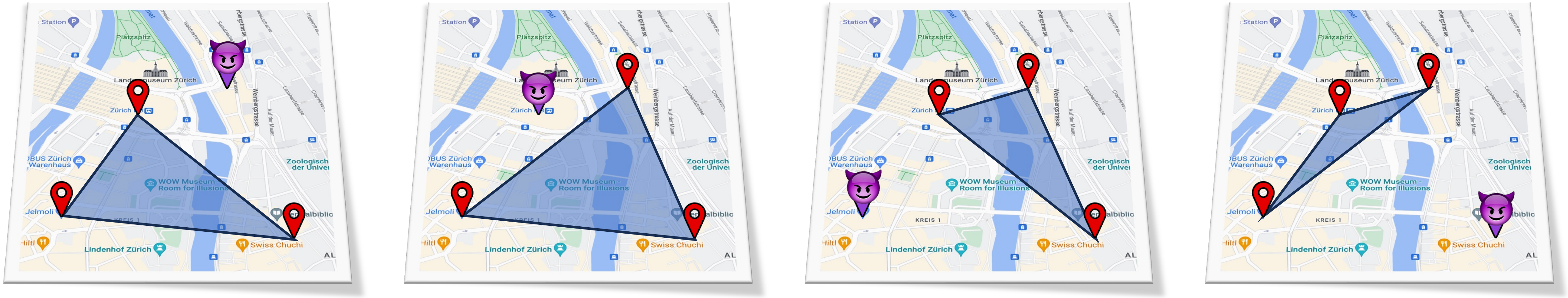
    3. Choose a new value from the safe area for the next iteration.

Safe areas



[STOC:MenHer13, PODC:VaiGar13]

Safe areas



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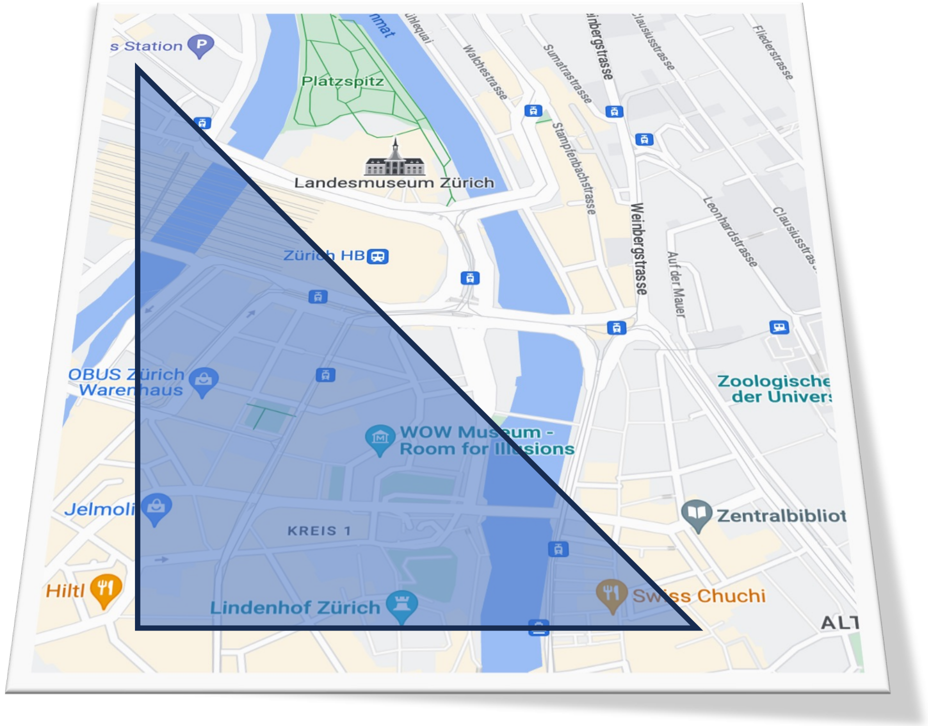
- Safe areas are:
  - non-empty.
  - included in the honest values' convex hulls.

New values from the safe areas

- New value: the midpoint of a segment realizing the safe area's diameter.



[DISC:FügNow18]

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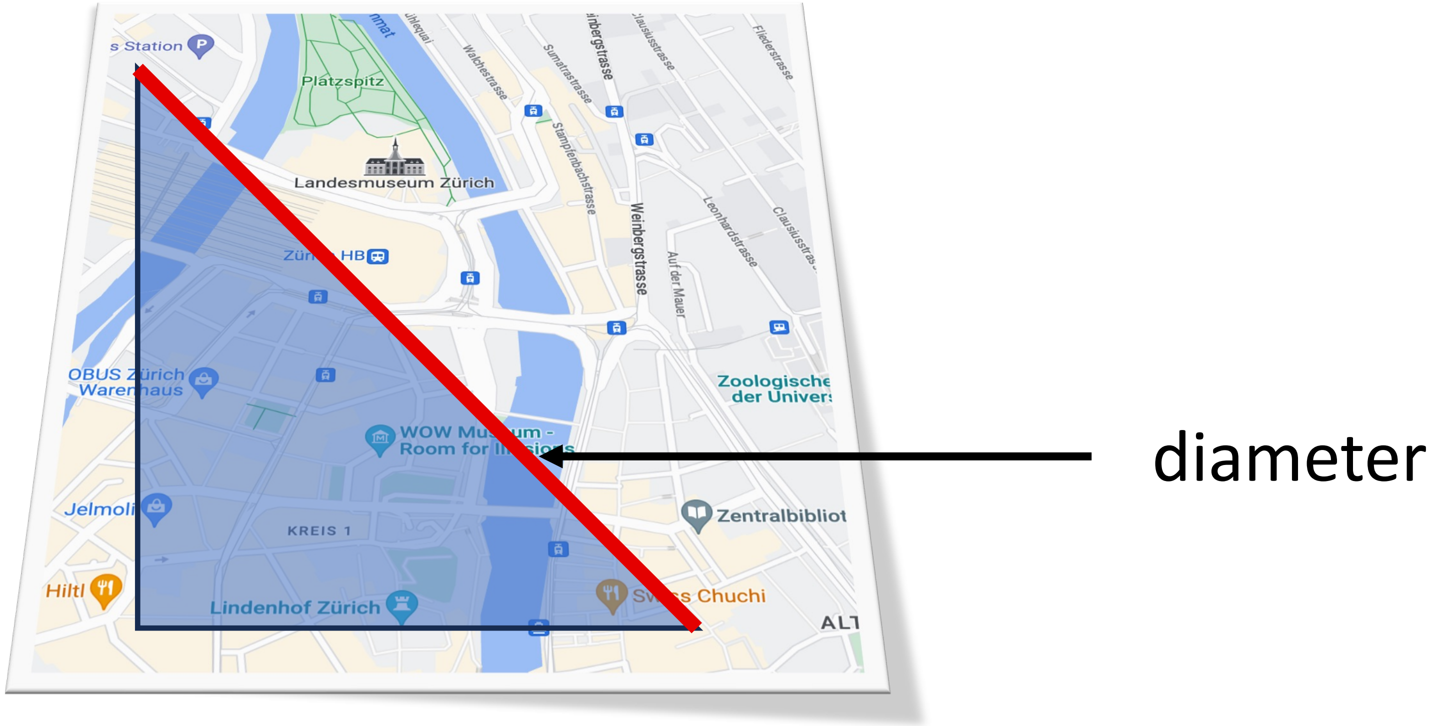

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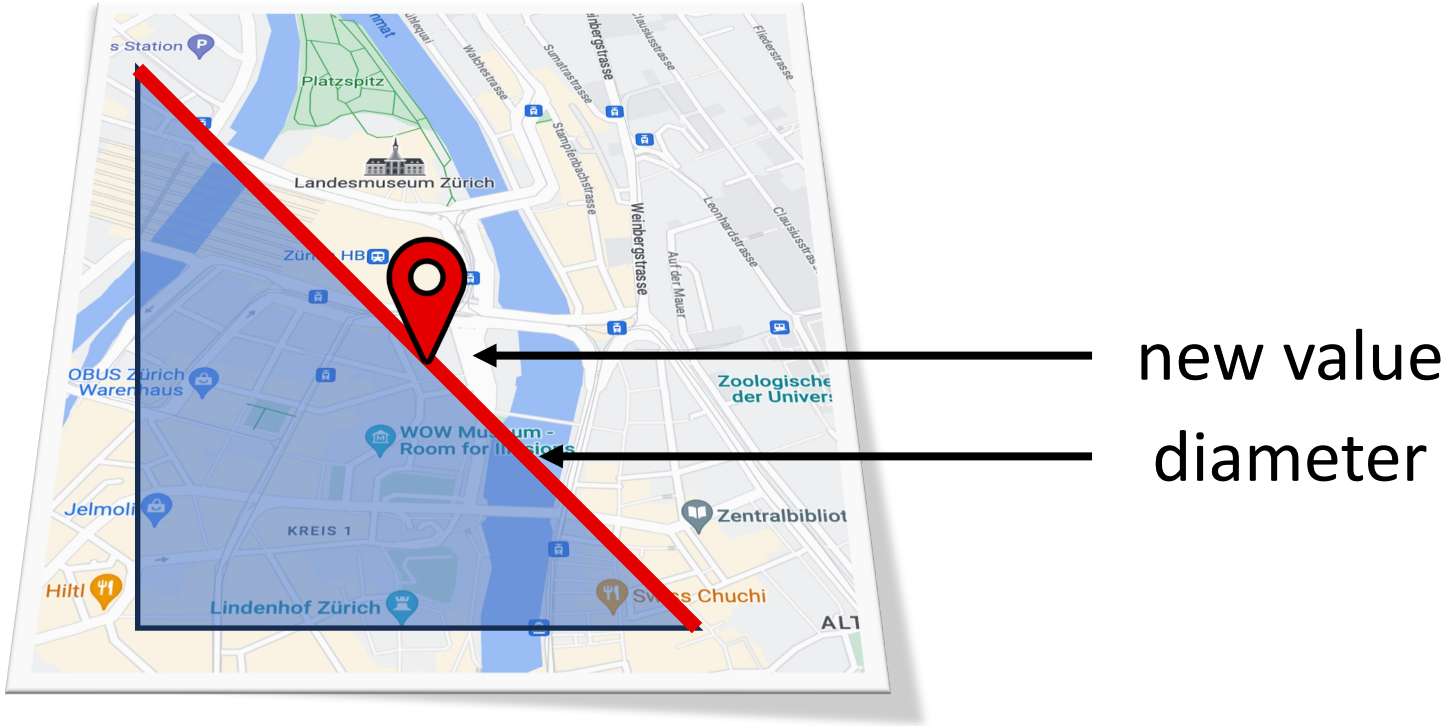

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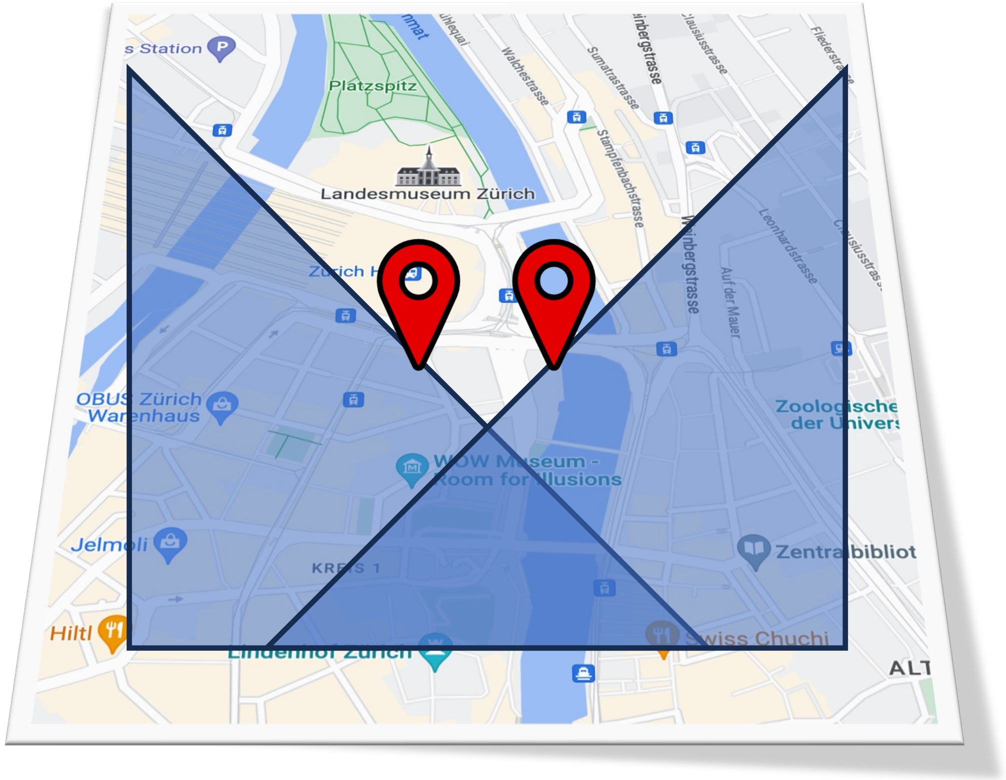
Why do honest parties’ values get closer?

The safe areas obtained by every two honest parties intersect.



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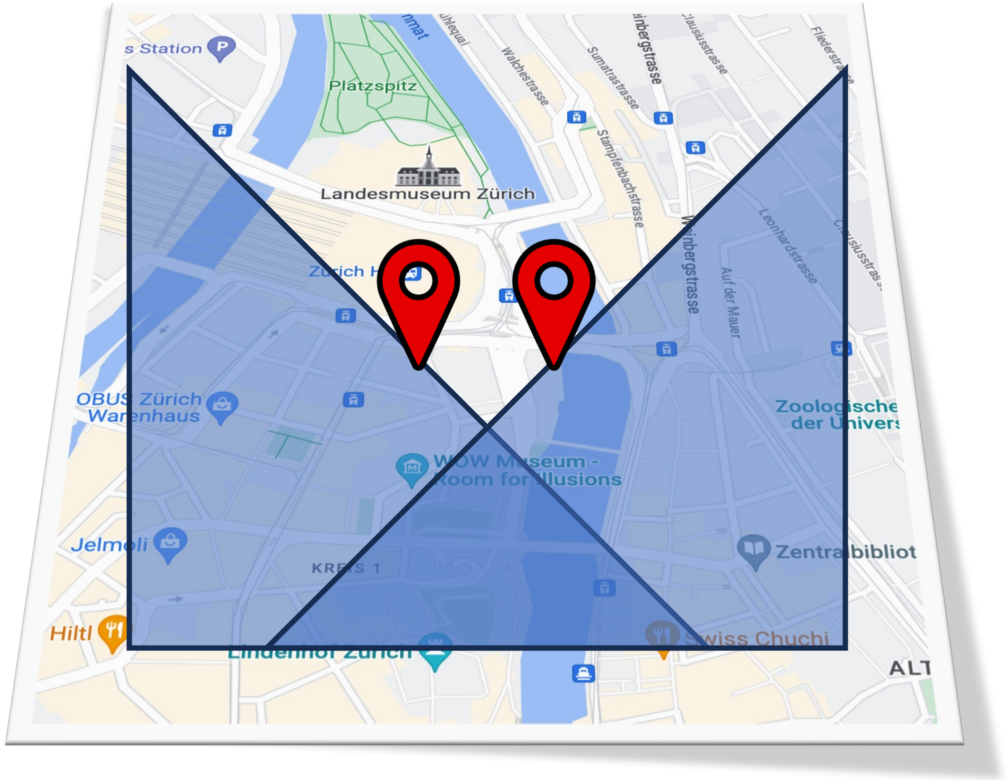
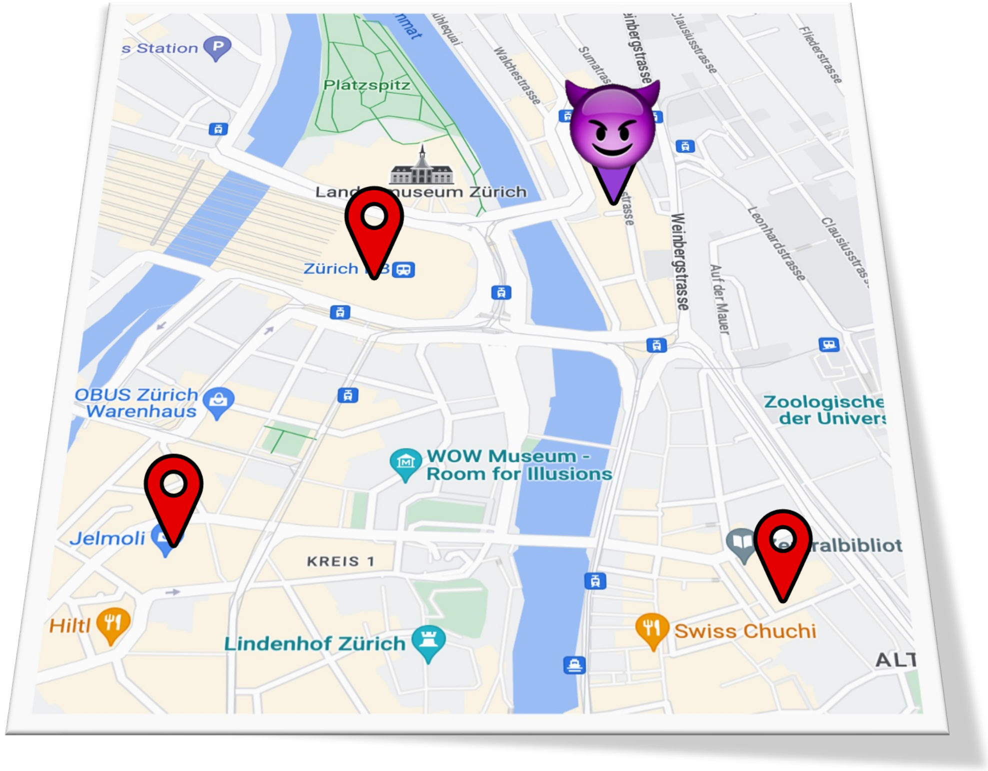
The safe areas obtained by every two honest parties intersect.

=> the diameter of the honest values’ convex hull decreases by a factor of $\sqrt{7/8}$ in each iteration.



How many iterations are needed?

- We estimate the honest inputs’ convex hull.



Summary

We are working in a model where the parties do not know whether the network is synchronous or asynchronous.

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